



# Black Holes' Dark Dress

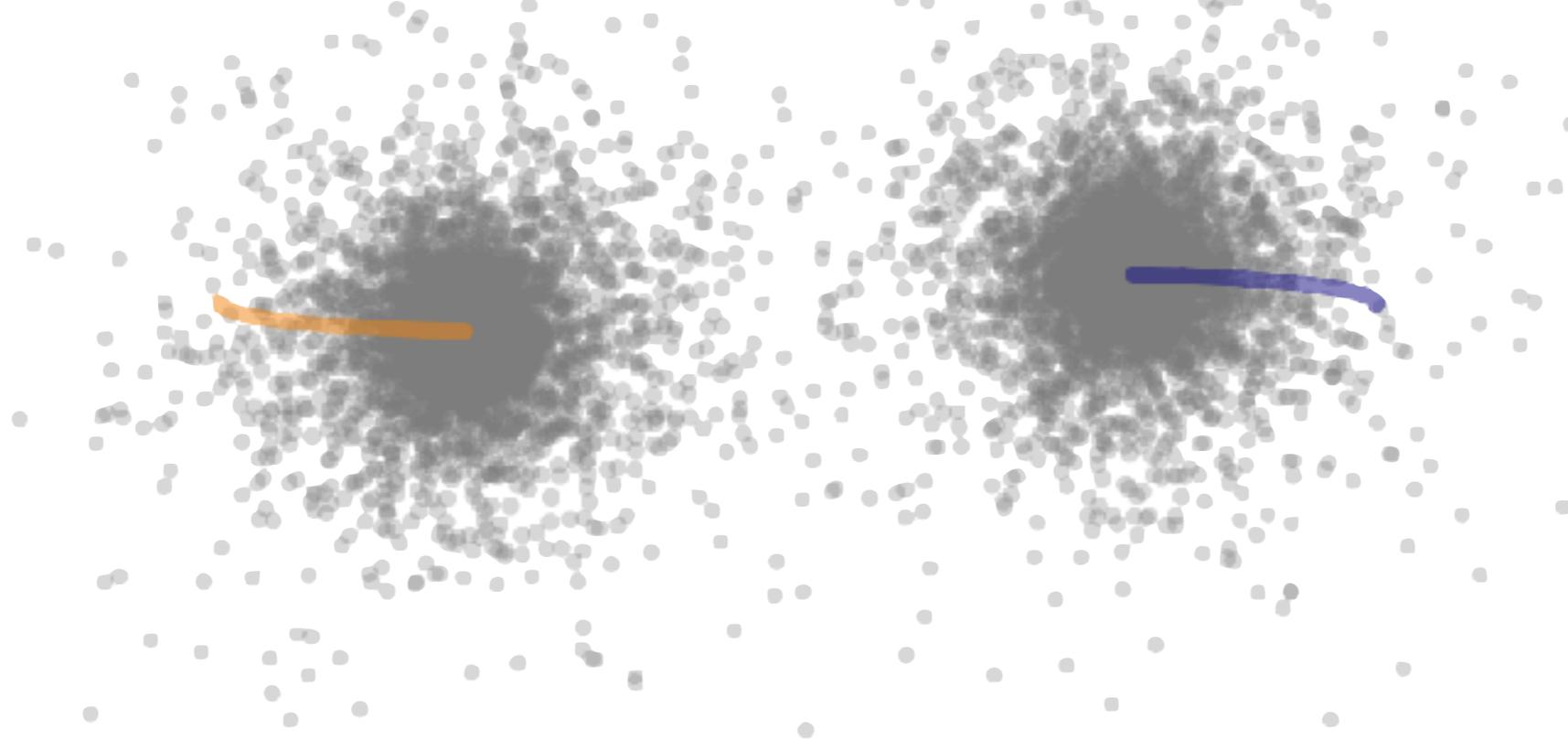
Mergers of primordial black holes and the impact of Dark Matter halos

Bradley J Kavanagh  
GRAPPA, University of Amsterdam

16th November 2018

“Like I say, I very much hope you choose to come here.  
We're a young and vibrant group that is making a big impression.”

- Tony Padilla, 9 March 2011



arXiv:1805.09034, PRD 98, 023536 (2018)

**BJK**, Daniele Gaggero & Gianfranco Bertone

**Movies at [tinyurl.com/BlackHolesDarkDress](https://tinyurl.com/BlackHolesDarkDress)**

Could the observed LIGO events be due to merging  
Primordial Black Holes (PBHs)?

How do local Dark Matter (DM) halos affect the  
merger rate of PBHs?

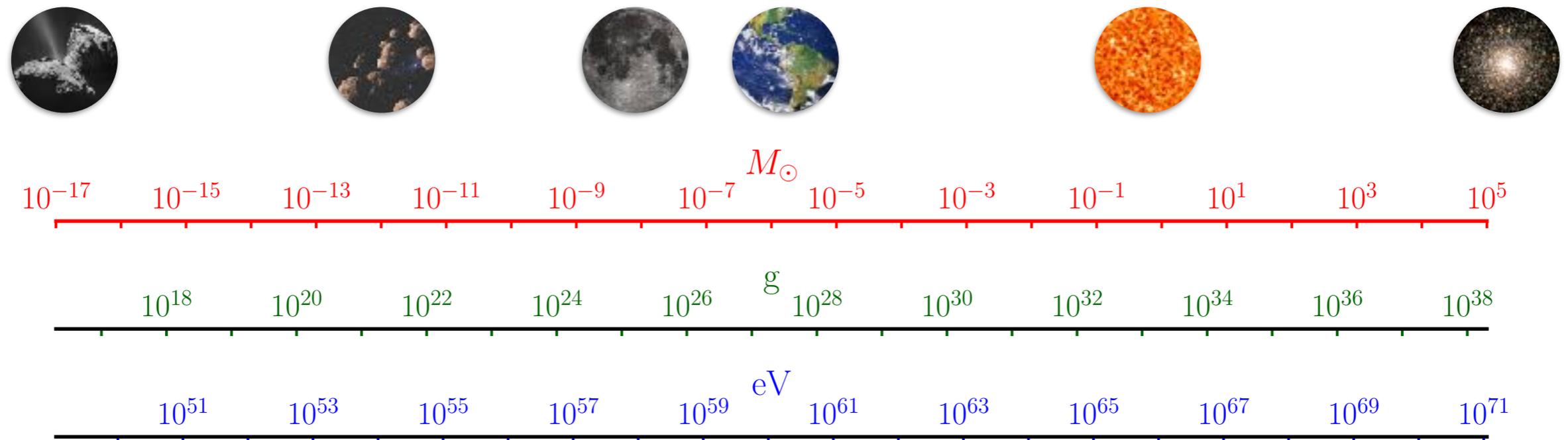
How does DM affect other  
Gravitational Wave (GW) signals?

# What are Primordial Black Holes?

Primordial Black Holes (PBHs) form in the early Universe ( $z \gg 10^8$ ) from large over-densities

Mass roughly given by mass inside horizon at time of formation:

[Green & Liddle, astro-ph/9901268]



[Y. B. Zel'dovich and I. D. Novikov, Soviet Astronomy 10, 602 (1967)]

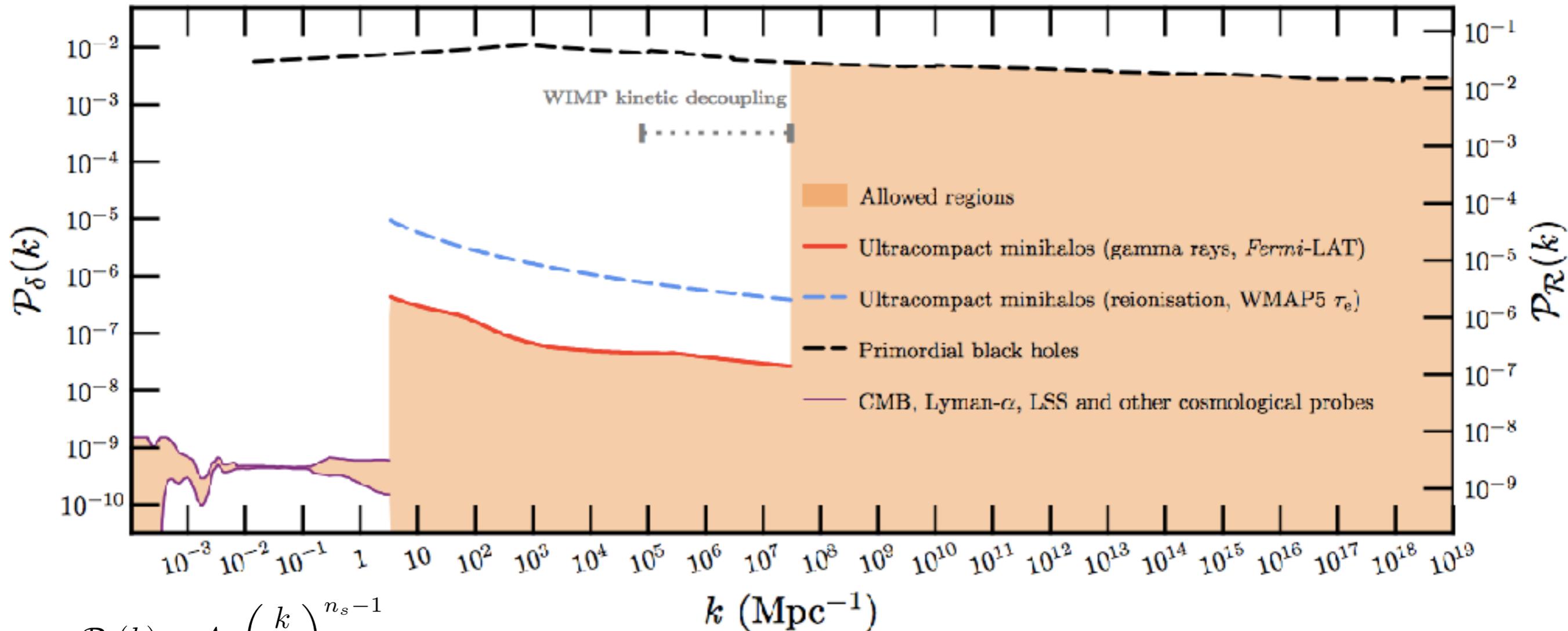
[S. Hawking, Mon. Not. R. Astron. Soc. 152, 75 (1971)]

[Carr and Hawking, MNRAS 168 (1974); Carr, Astrophys. J. 201, 1 (1975)]

# PBH formation

Extrapolating the primordial power spectrum from Planck, fluctuations big enough to produce PBHs should be negligible...

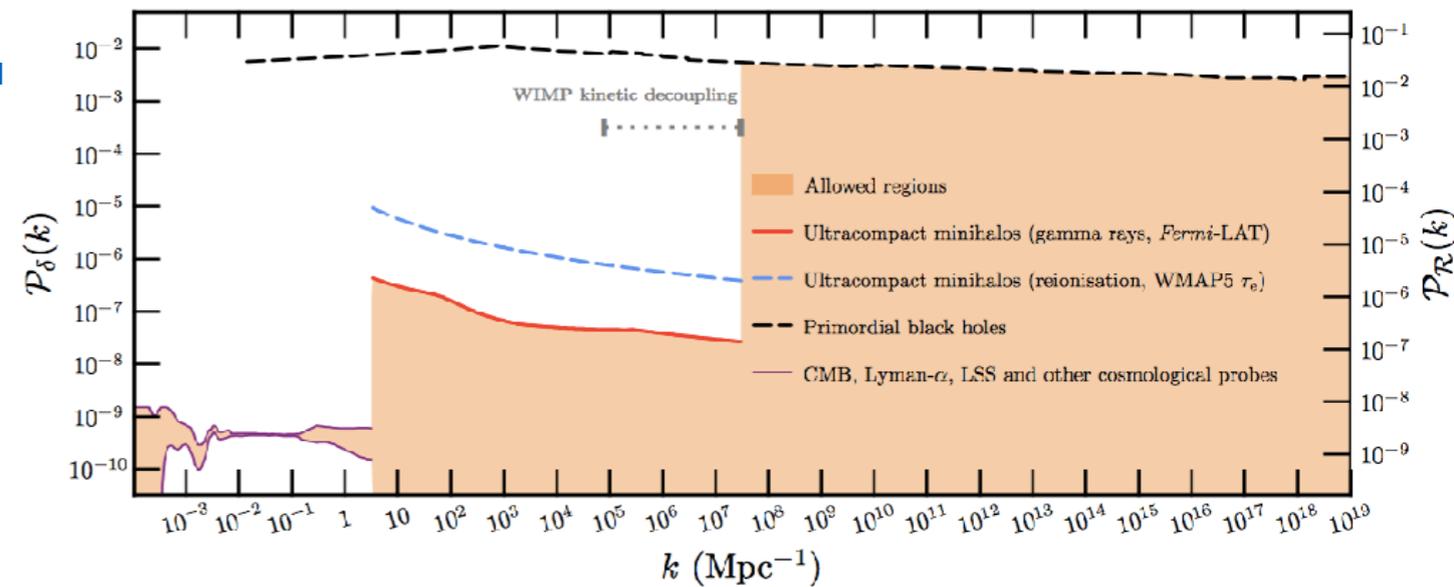
[1110.2484]



...but small scale power spectrum is largely unconstrained...

# PBH formation

How then could we make PBHs?



- Enhancement/feature in power spectrum

[[astro-ph/9509027](#), [astro-ph/9605094](#), [hep-ph/9710259](#), [1206.4188](#), [1709.05565](#)]

- Cosmic String Loops

[[Hawking \(1987\)](#), [Polnarev & Zembowicz \(1991\)](#), [Caldwell & Caspar \(1996\)](#)]

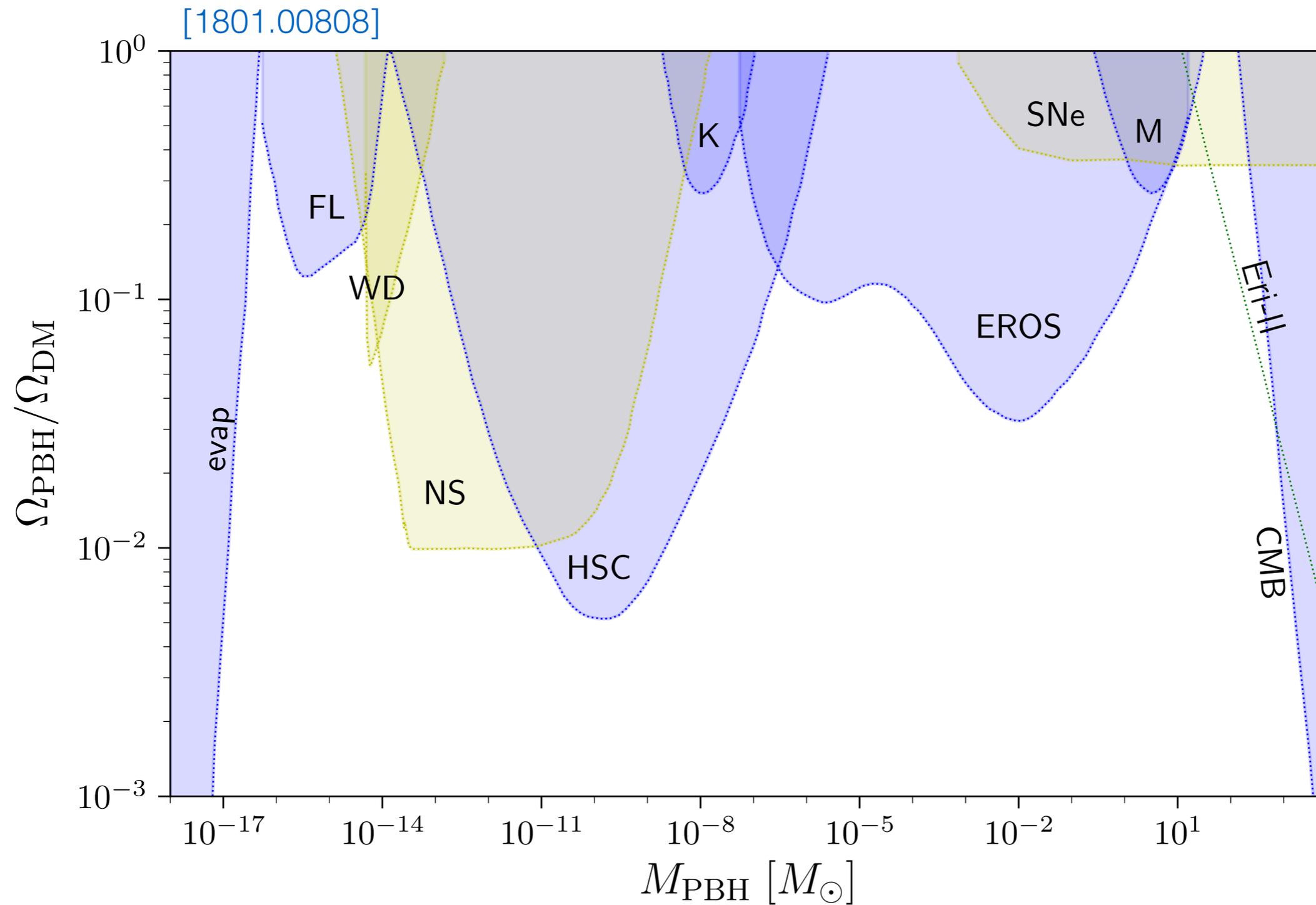
- Bubble collisions

[[Hawking, Moss & Stewart \(1982\)](#); [La & Steinhardt \(1989\)](#)]

PBHs would be a sign of New Physics and a probe of the early universe.

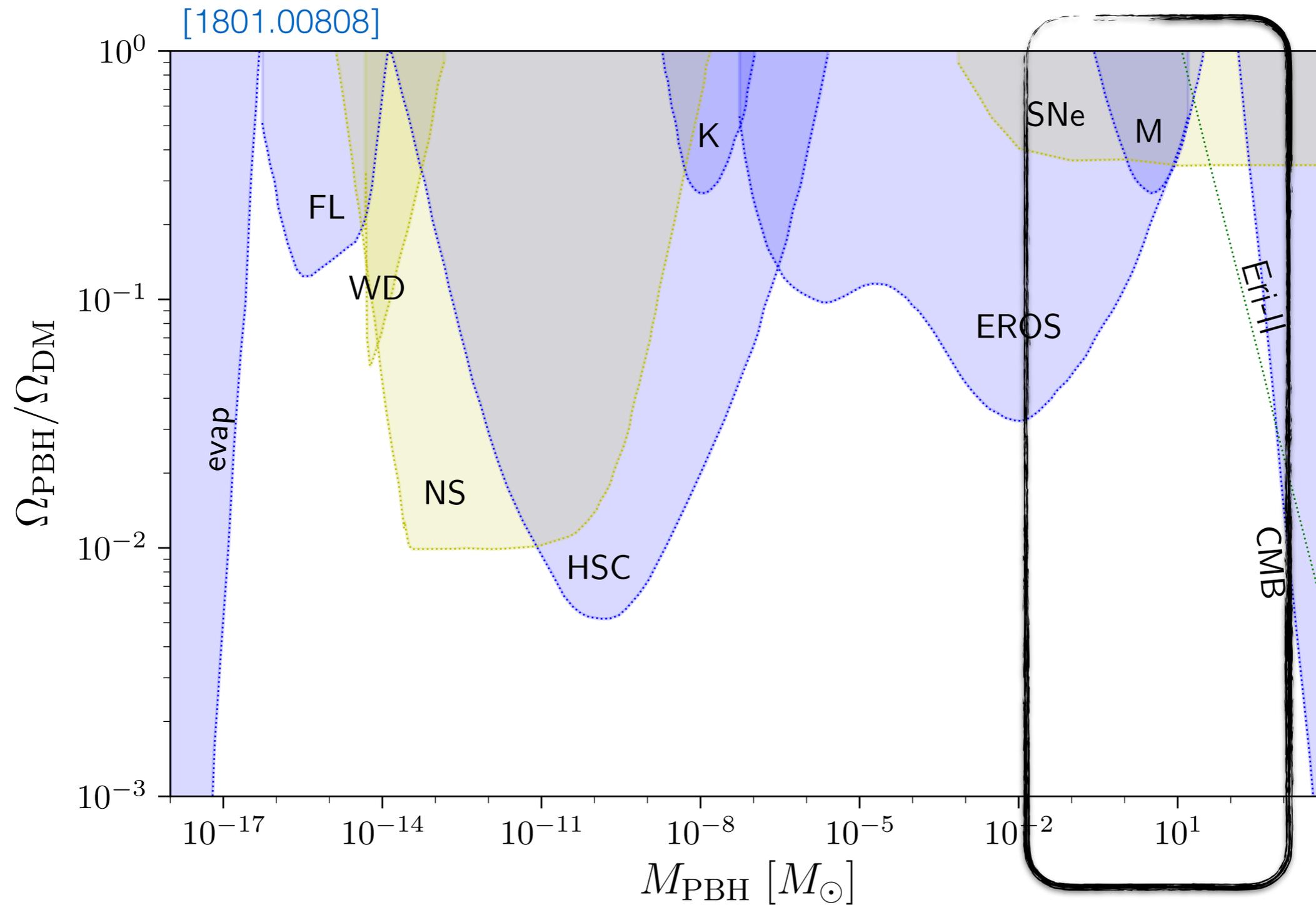
[[Green, 1403.1198](#); [Sasaki et al, 1801.05235](#)]

# PBHs as Dark Matter



[See 1607.06077, 1806.05195 and references therein]

# PBHs as Dark Matter



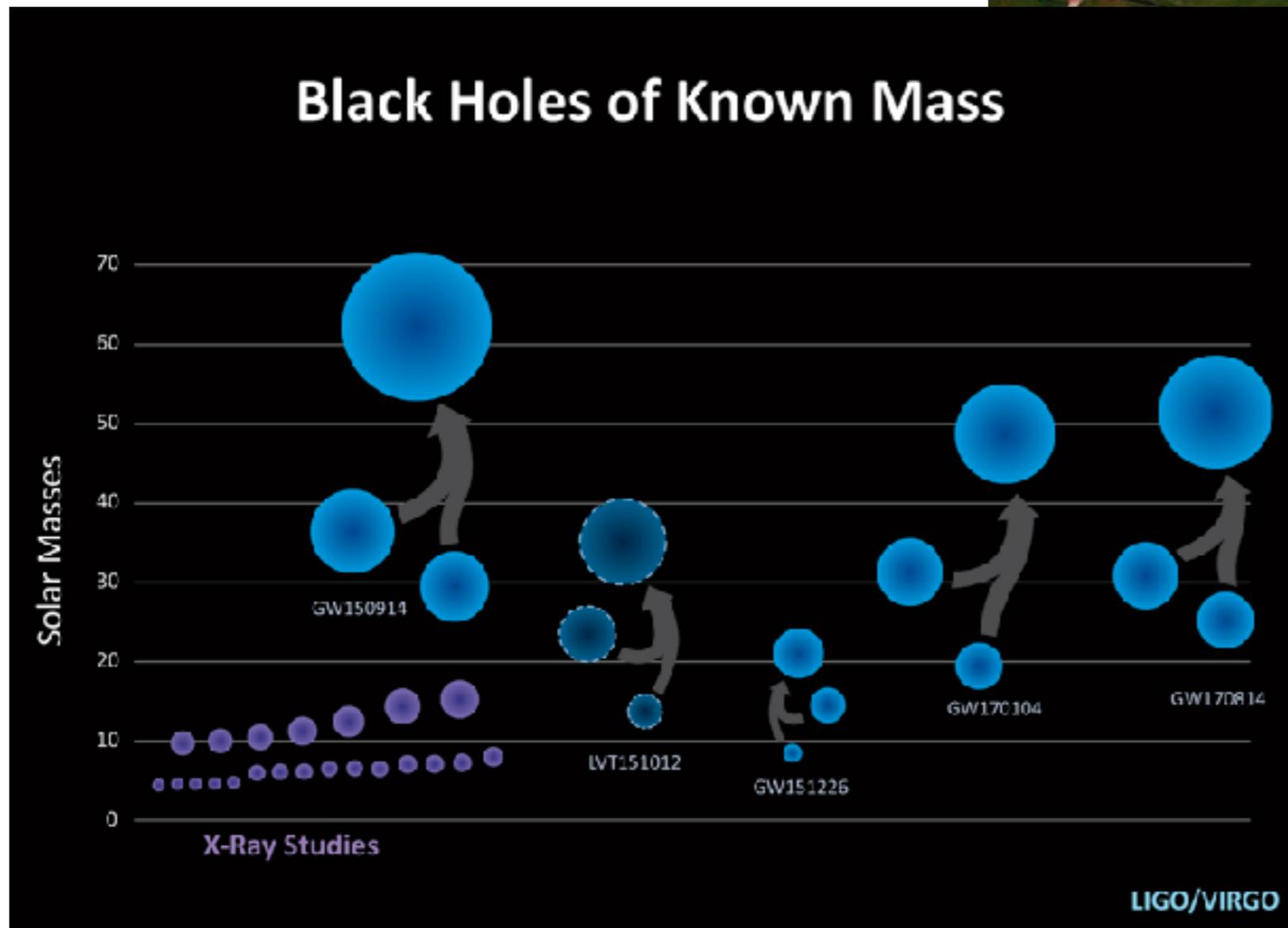
[See 1607.06077, 1806.05195 and references therein]

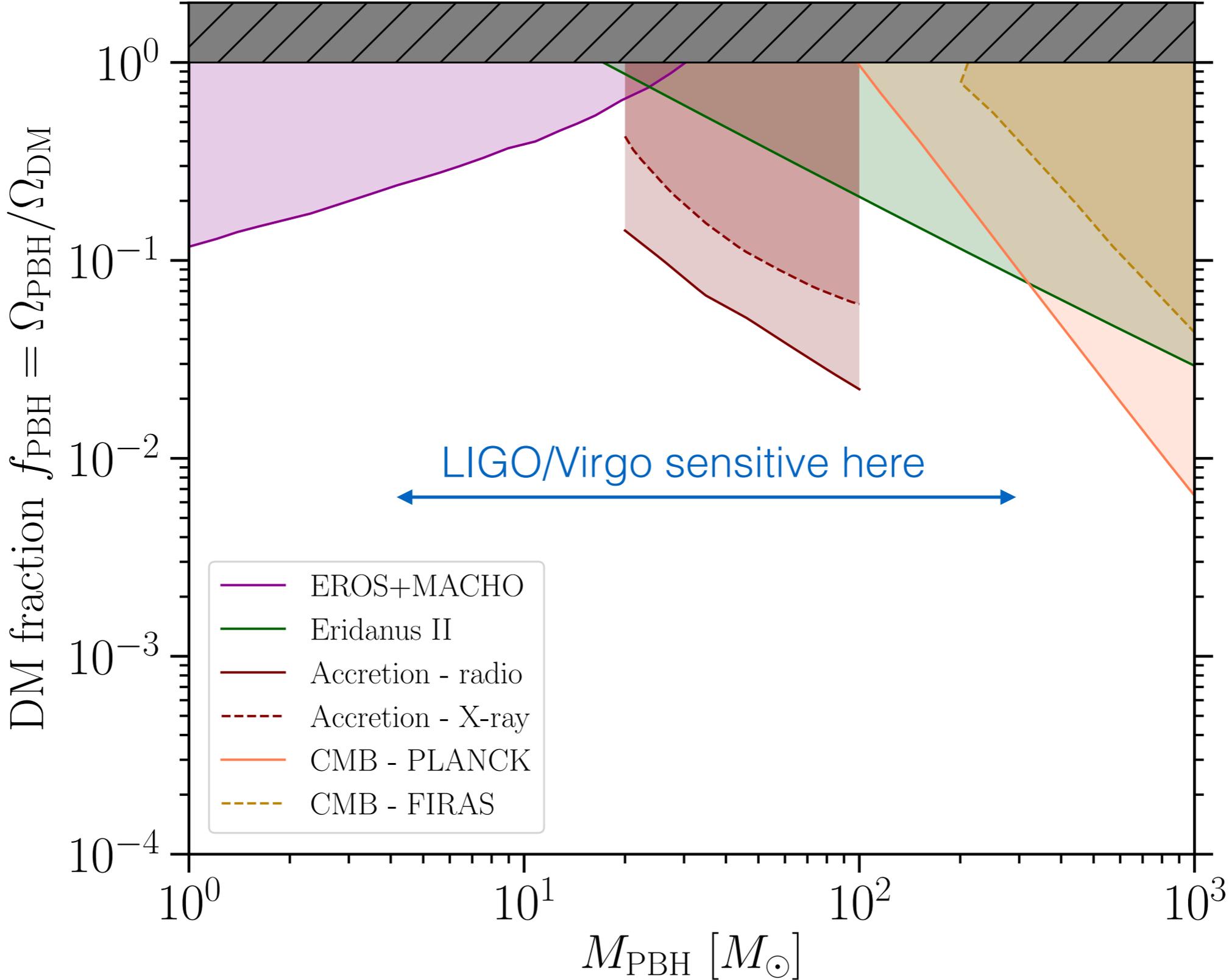
# LIGO/Virgo Mergers



LIGO/Caltech/Sonoma State (Aurore Simonnet)

The Virgo collaboration/CCO 1.0





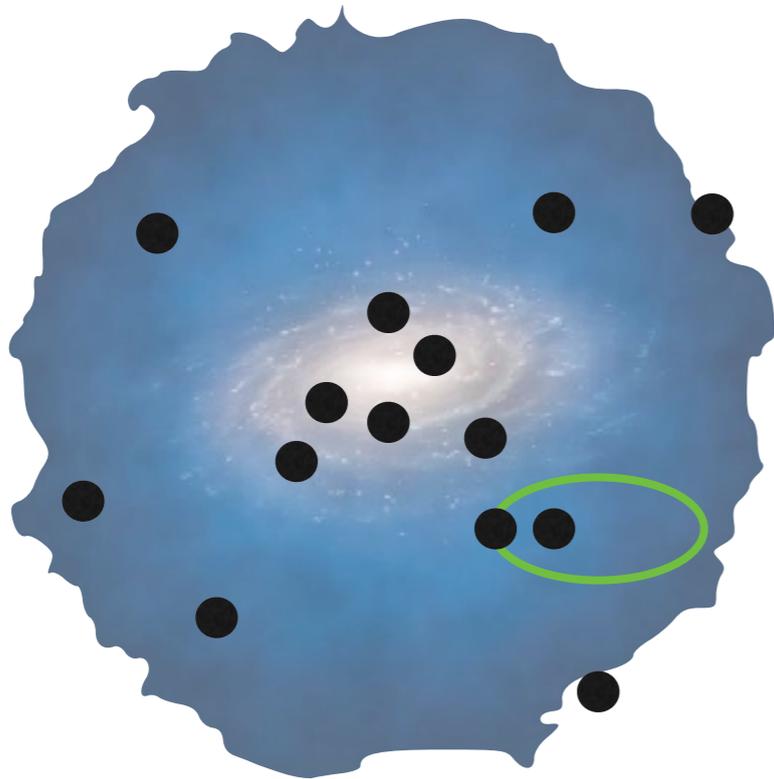
# Merger rates of PBHs

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# A tale of two binaries

A) Binaries formed after close encounters

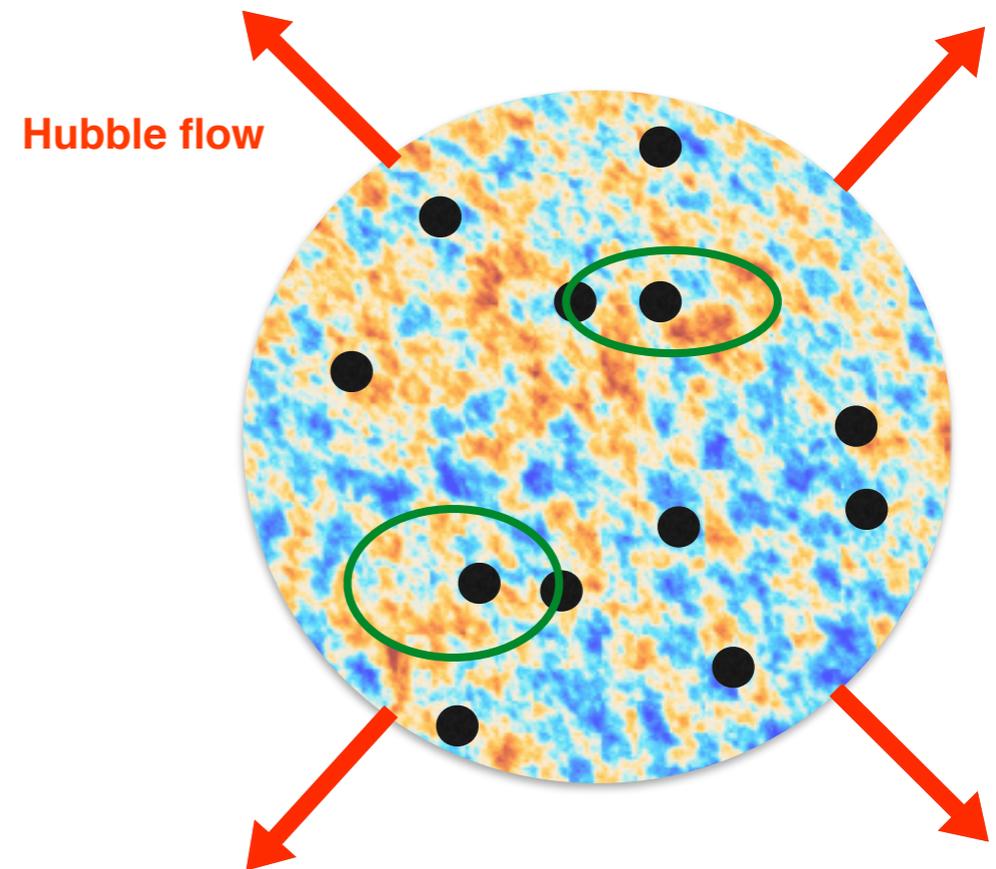
[Bird et al., 1603.00464]



$$\begin{aligned}\sigma &= \pi \left( \frac{85 \pi}{3} \right)^{2/7} R_s^2 \left( \frac{v_{\text{pbh}}}{c} \right)^{-18/7} \\ &= 1.37 \times 10^{-14} M_{30}^2 v_{\text{pbh}-200}^{-18/7} \text{pc}^2\end{aligned}$$

B) Binaries formed in the early Universe

[Sasaki et al, 1603.08338]



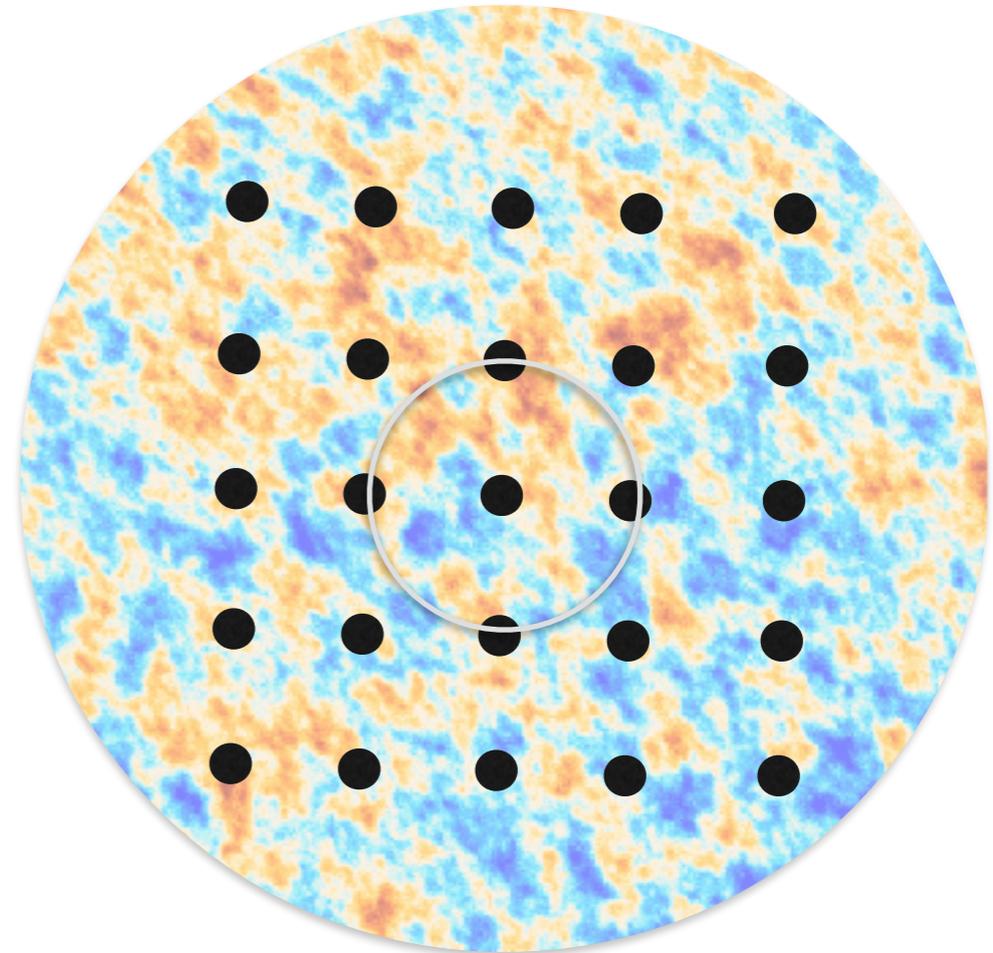
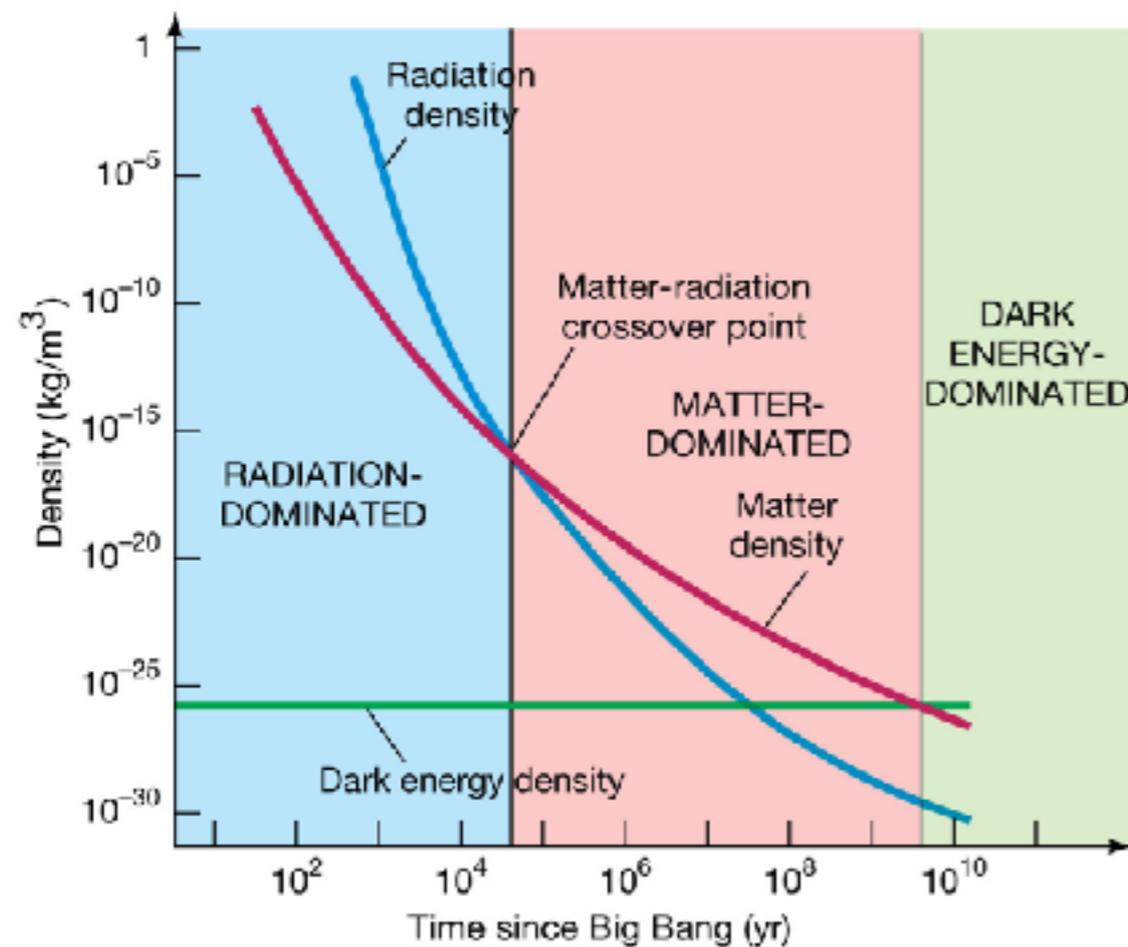
Require:

$$M_{\text{BH}} R^{-3} > \rho(z) \text{ before } z_{\text{eq}}$$

[Daniele Gaggero, UCI 20/02/2018]

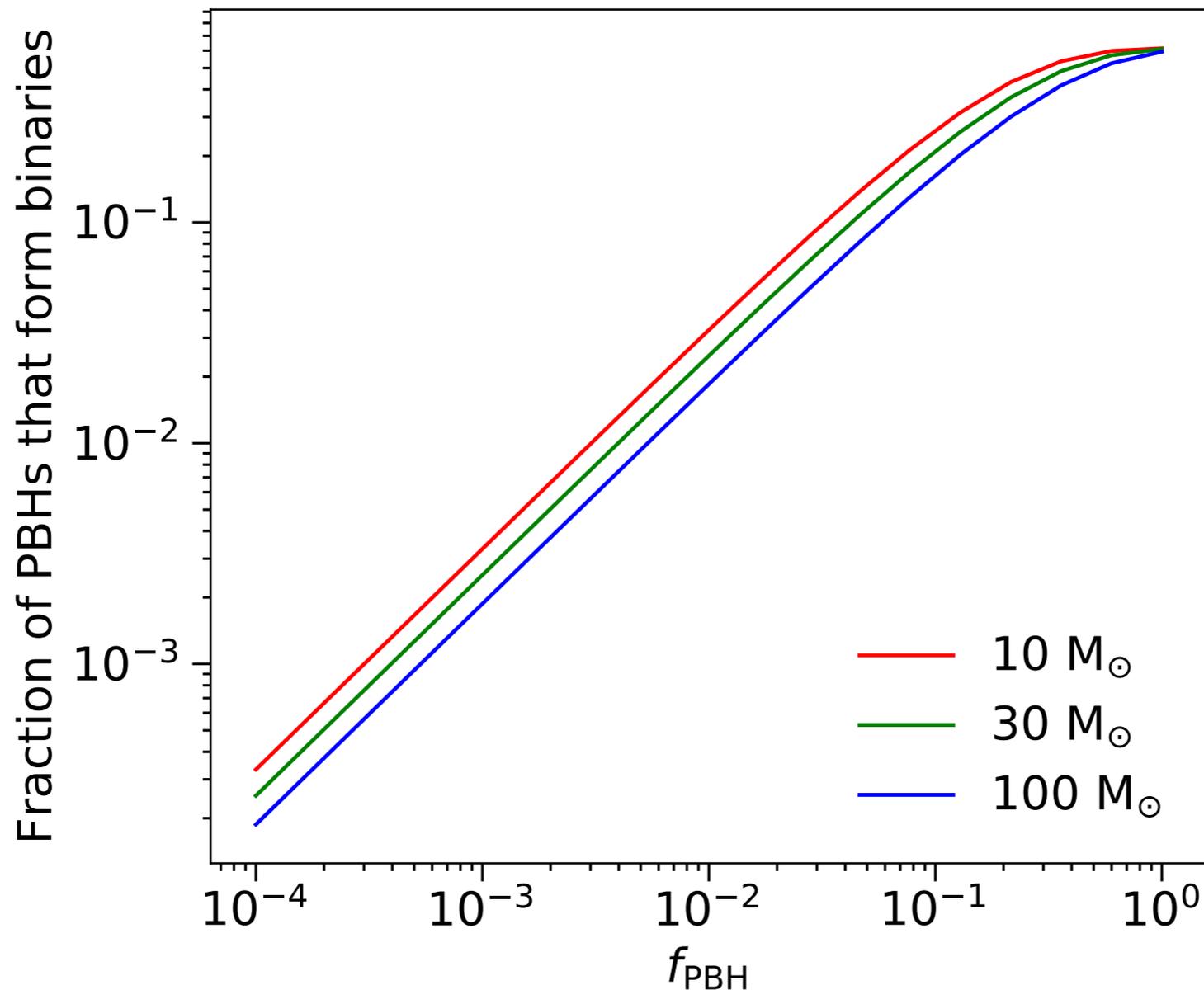
# Early Universe Binaries

If  $f \sim 1$ , the relative density of PBHs *equals* the background radiation density at matter-radiation equality. All PBHs form binaries...



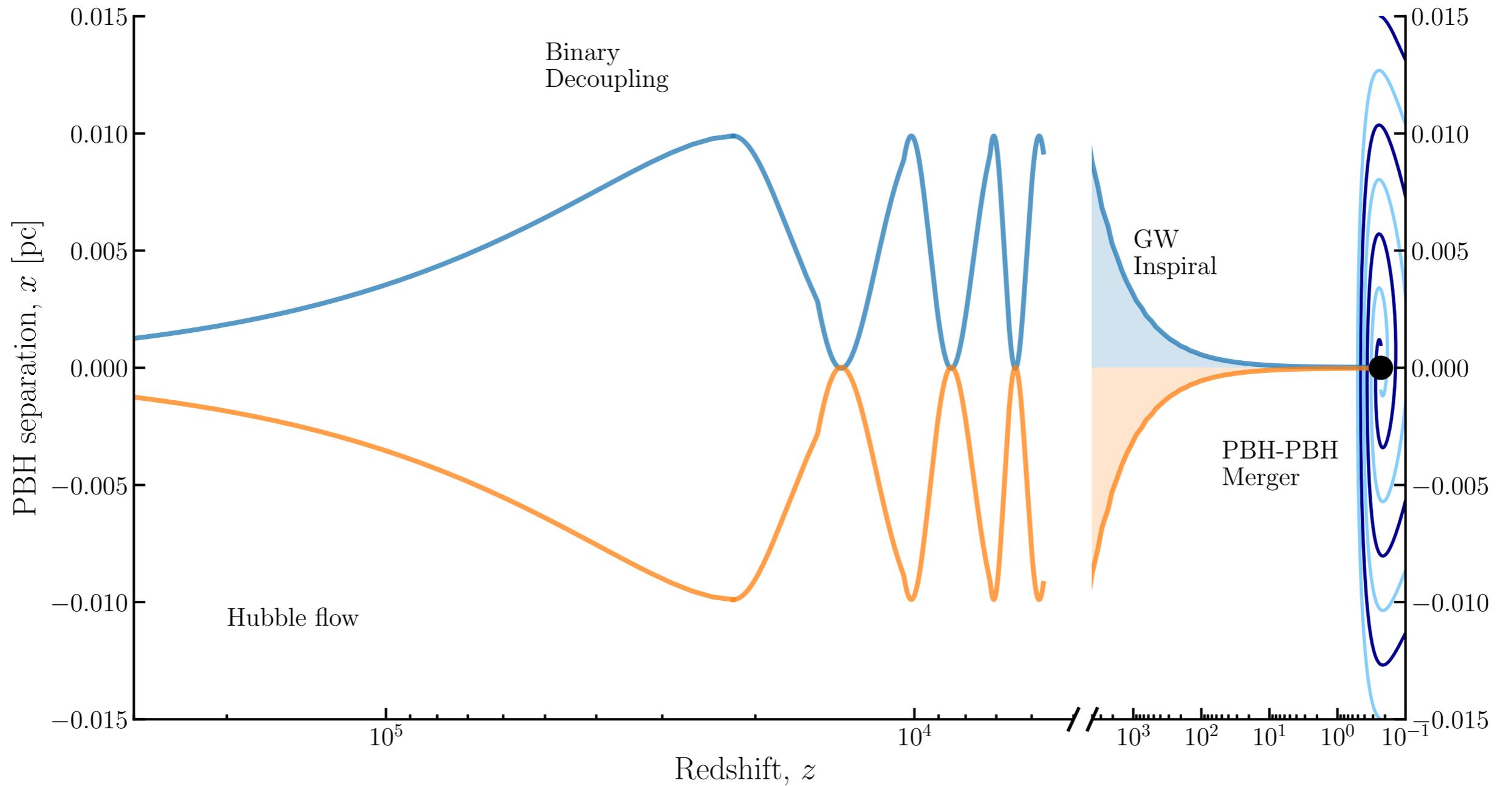
# Early Universe Binaries

If  $f \sim 1$ , the relative density of PBHs *equals* the background radiation density at matter-radiation equality. All PBHs form binaries...



As  $f$  decreases, only 'nearby' pairs form binaries.

# Life of a PBH binary



$$a = 0.01 \text{ pc}$$

$$e = 0.995$$

# PBH Binary Population

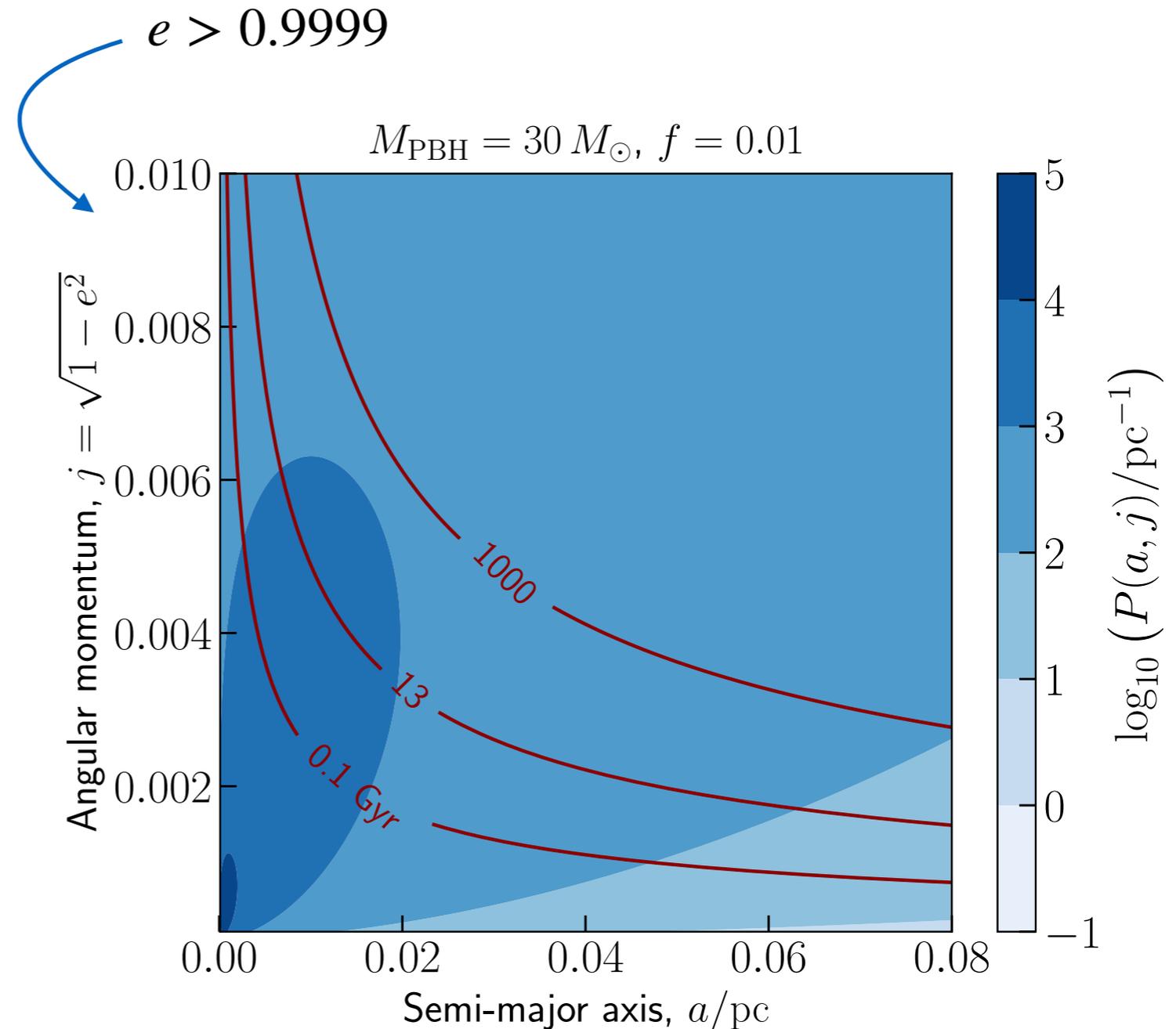
Randomly distributed  
(unclustered) PBHs

Angular momentum set by  
torques from smooth density  
perturbations and *all other PBHs*

Close, eccentric binaries  
merge today:

$$t_{\text{merge}} = \frac{3c^5}{170G_N^3} \frac{a^4 j^7}{M_{\text{PBH}}^3}$$

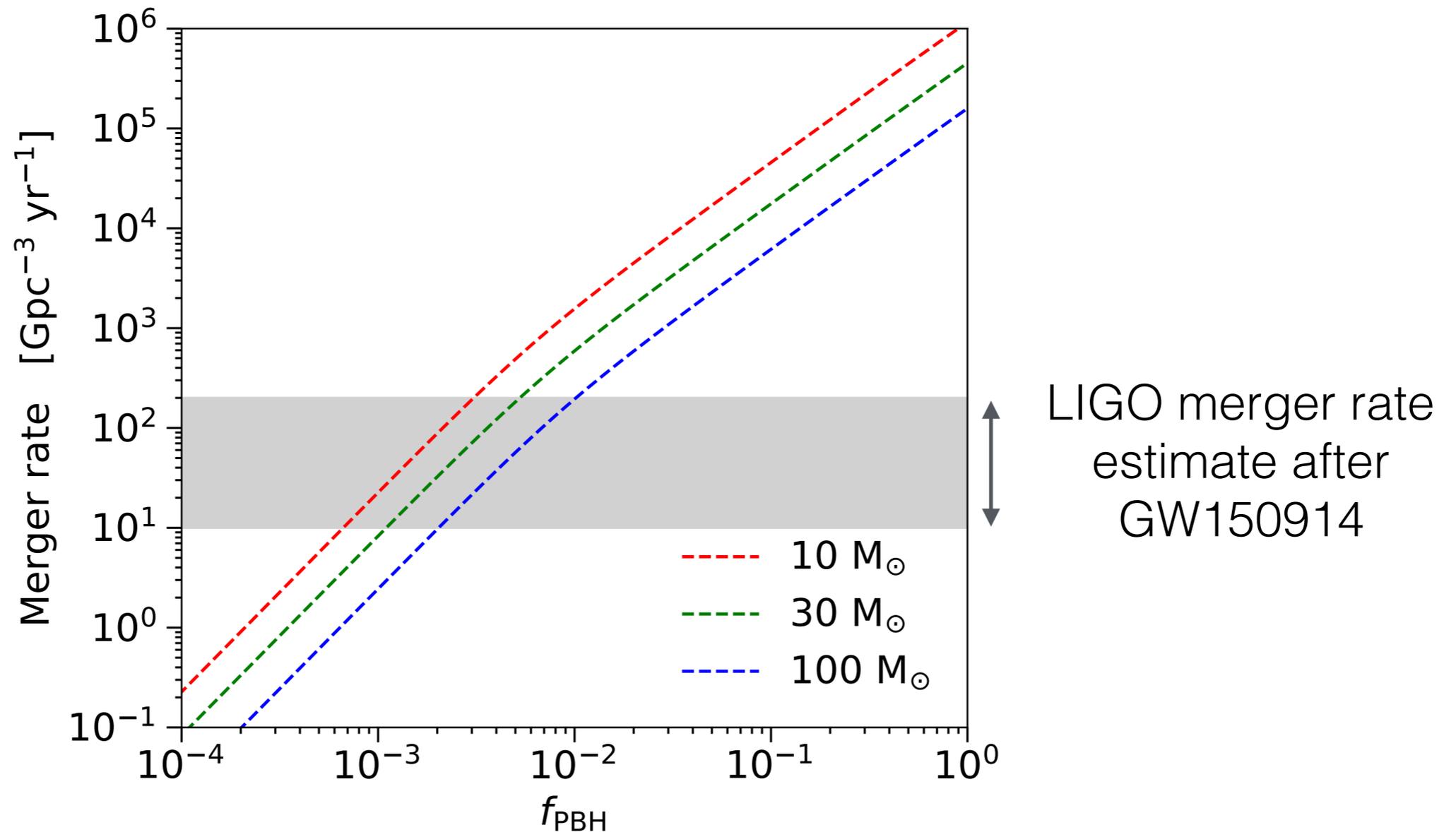
$$j = \sqrt{1 - e^2}$$



[Ali-Haïmoud et al., 1709.06576,  
**BJK**, Gaggero & Bertone, 1805.09034]

# Merger rate estimate

$$\mathcal{R}(t_{\text{merge}}) = \frac{1}{2} n_{\text{PBH}} P_{\text{binary}} P(t_{\text{merge}})$$



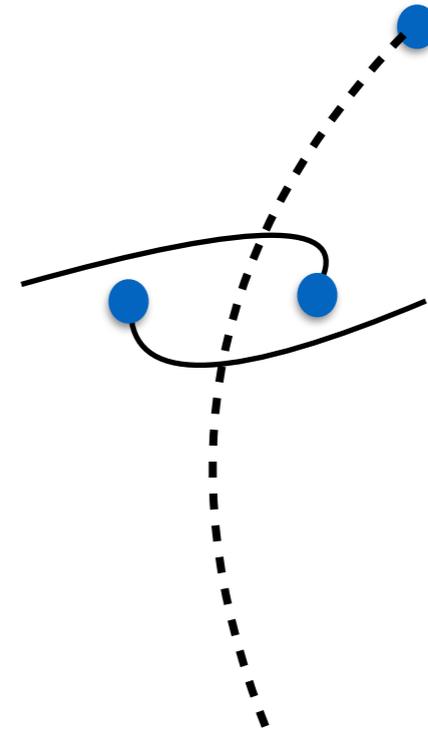
Solar mass PBHs should only be a sub-dominant (%-level) contribution to the DM density in the Universe

[Ali-Haïmoud et al., 1709.06576, **BJK**, Gaggero & Bertone, 1805.09034]

# Caveats

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- Survival
- Clustering
- Baryons
- Dark Matter



Do these binaries survive for the age of the Universe?

Smooth density perturbations and close encounters  
are unlikely to disrupt the binaries

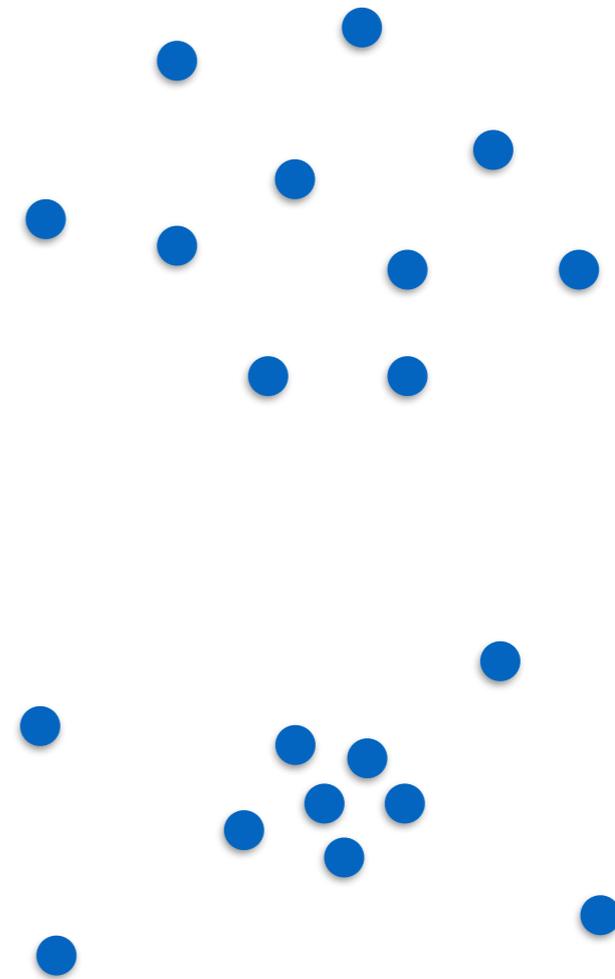
$$a \lesssim 10^{-2} \text{ pc}$$

[Ali-Haïmoud et al., 1709.06576]

# Caveats

---

- Survival
- Clustering
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- Dark Matter



How does the distribution of PBHs affect the merger rate?

Clustering could substantially enhance the merger rate ('cascade' mergers) but PBHs are unlikely to form in clusters...

[1808.05910]

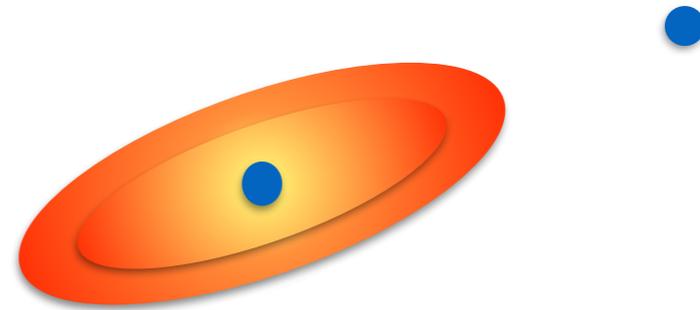
[1807.02084]

[See also 1805.05912, 1806.10414 and others]

# Caveats

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- Survival
- Clustering
- Baryons
- Dark Matter



Does baryonic accretion disrupt the binary?

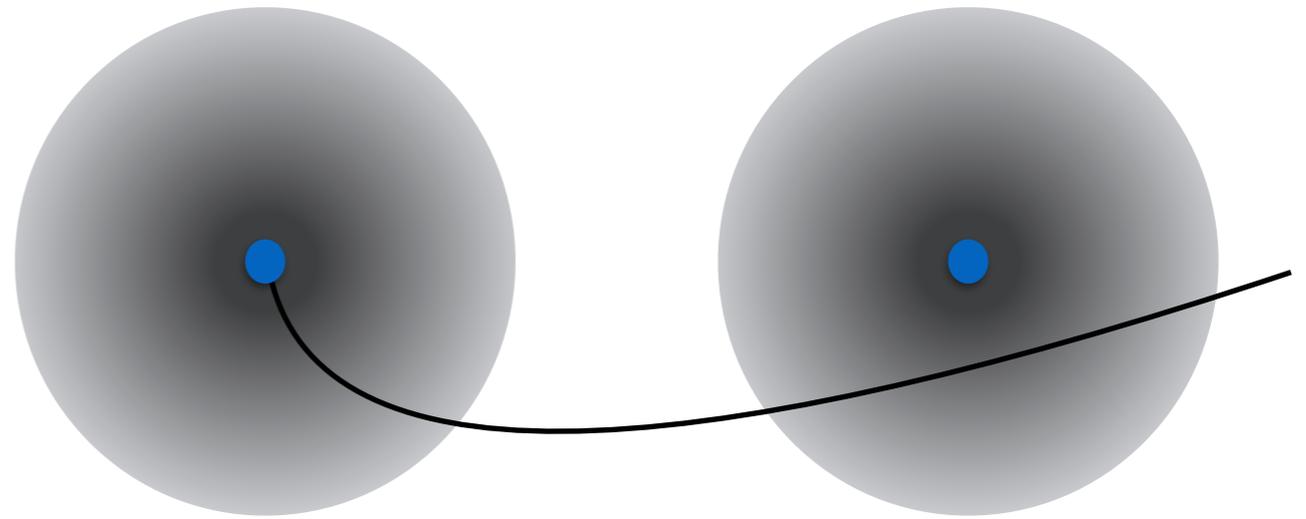
Some simulations have been performed, but the effects are still unclear (especially for highly eccentric binaries)

[[0909.1738](#), [0805.3408](#), [astro-ph/0607467](#), [1703.03913](#)]

# Caveats

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- Survival
- Clustering
- Baryons
- **Dark Matter**



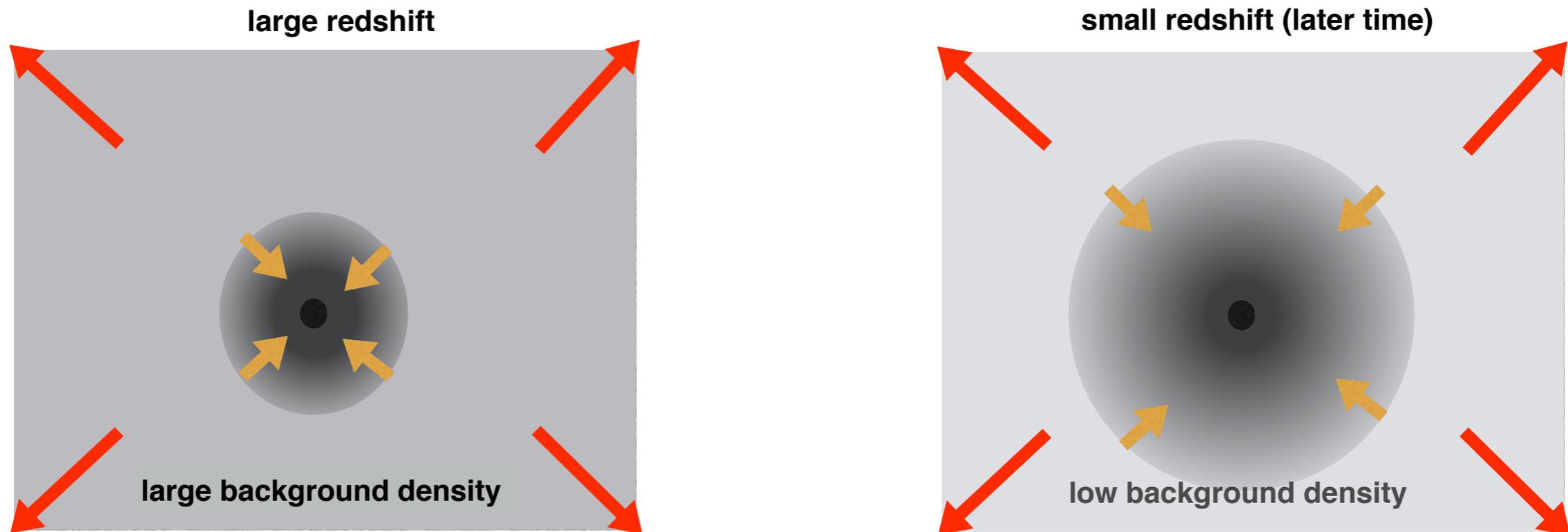
Do *local* Dark Matter halos disrupt PBH binaries?

# Local DM Halos



# Dark Dresses

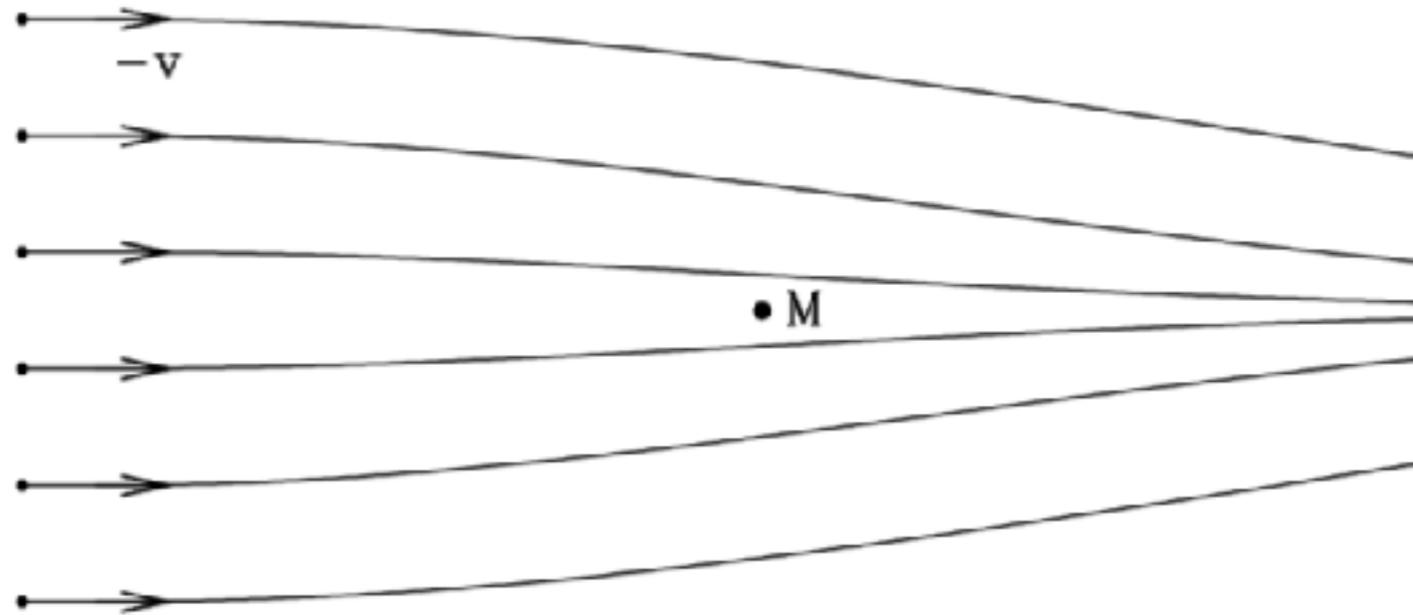
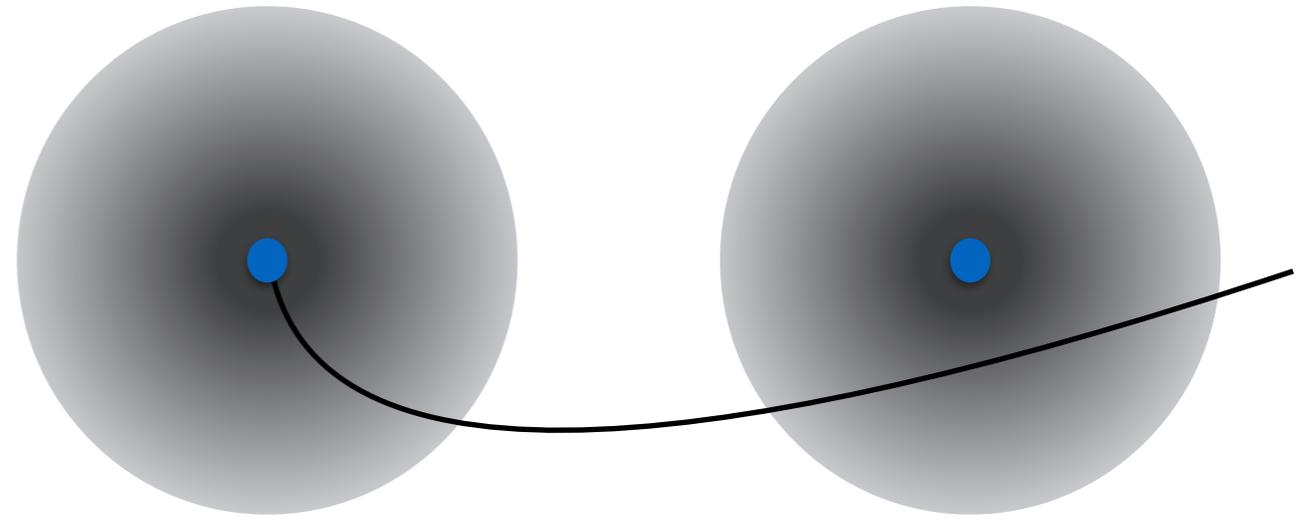
PBHs seed the formation of 'local' DM halos:



$$R_{\text{tr}}(z) = 0.0063 \left( \frac{M_{\text{PBH}}}{M_{\odot}} \right) \left( \frac{1 + z_{\text{eq}}}{1 + z} \right) \text{pc} \quad \rho(r) \propto r^{-3/2}$$

By matter-radiation equality,  $M_{\text{halo}} \sim M_{\text{PBH}}$

# Dynamical Friction

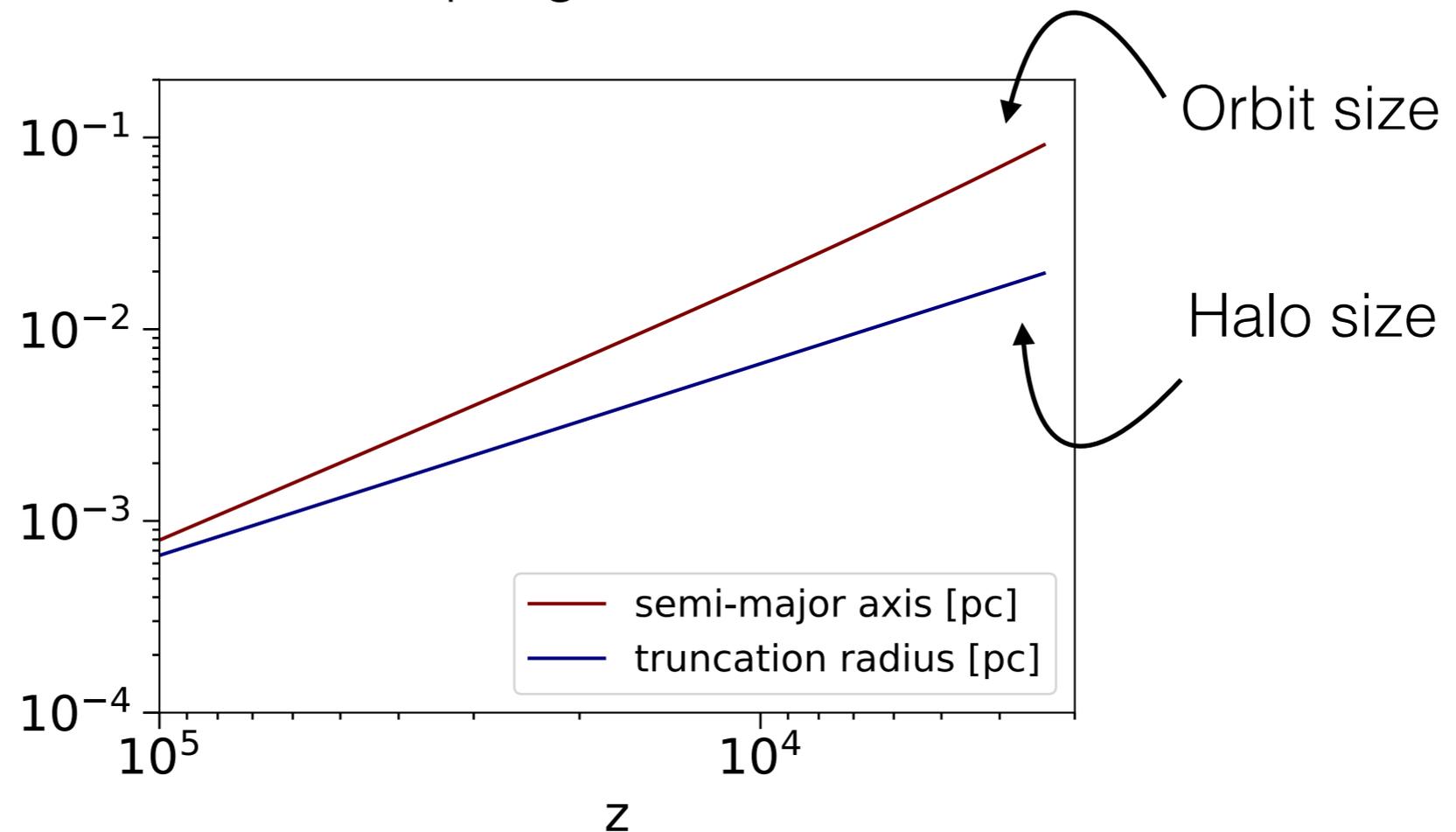


$$\vec{F}_{\text{DF}} = -\frac{\vec{v}}{v^3} 4\pi\rho (GM_{\text{BH}})^2 \ln \Lambda$$

# Simulations

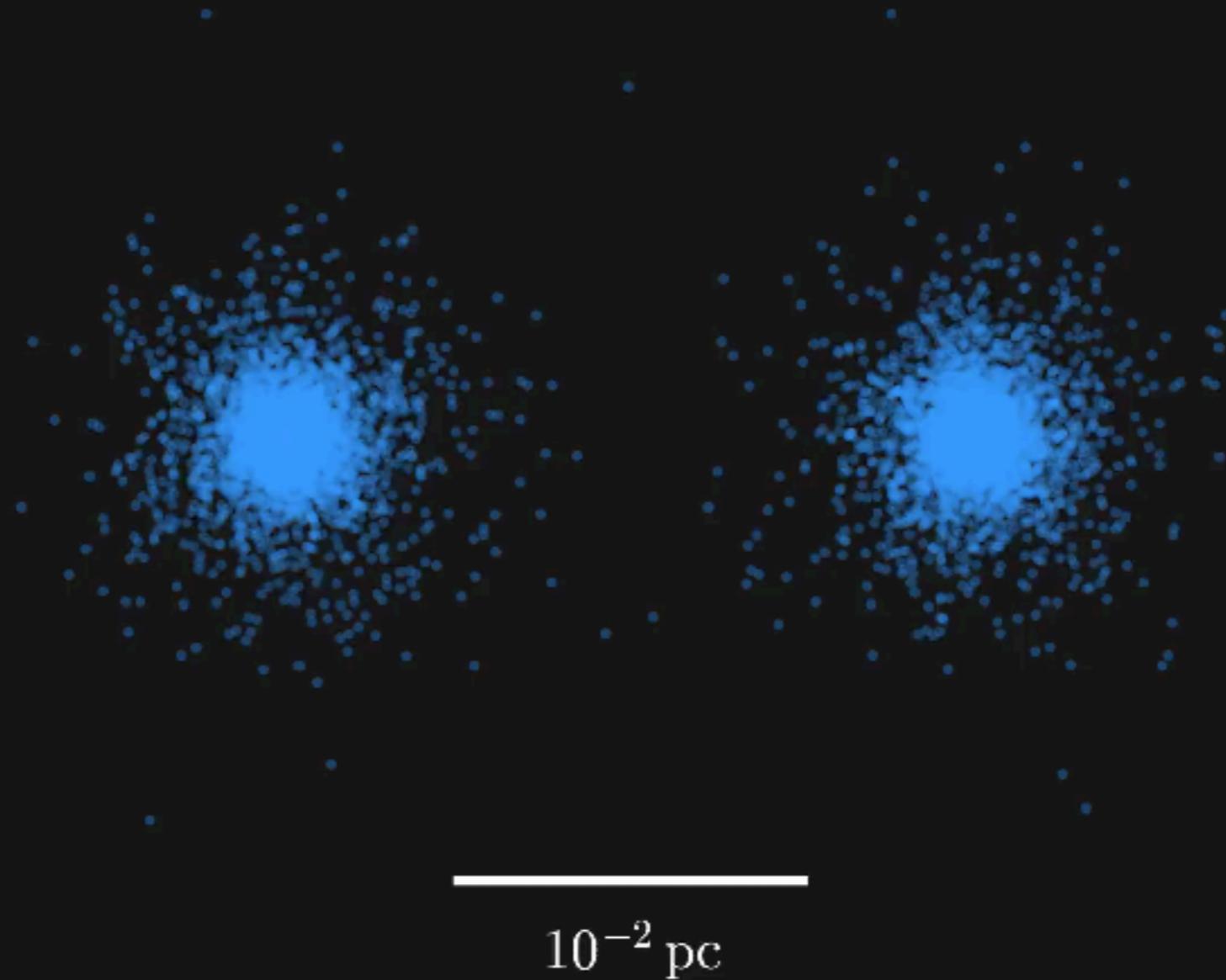
Use GADGET-2 as a pure N-body solver [Springel, astro-ph/0505010]

Initialise the PBHs self-consistently, with DM halos of the correct size, depending on the redshift of decoupling...

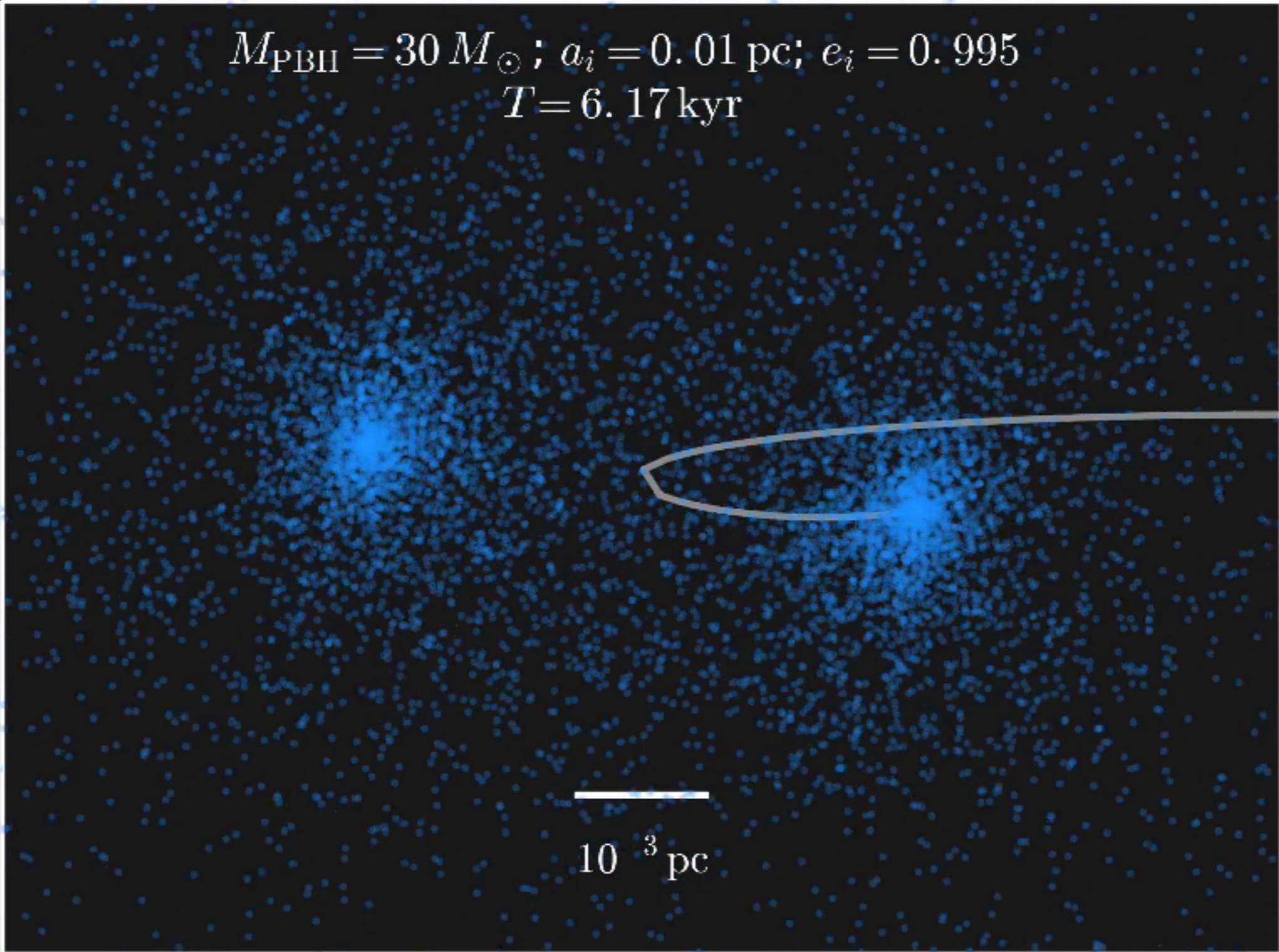


...then follow the evolution of the PBH binary...

$$M_{\text{PBH}} = 30 M_{\odot}; a_i = 0.01 \text{ pc}; e_i = 0.995$$
$$T = 0.00 \text{ kyr}$$

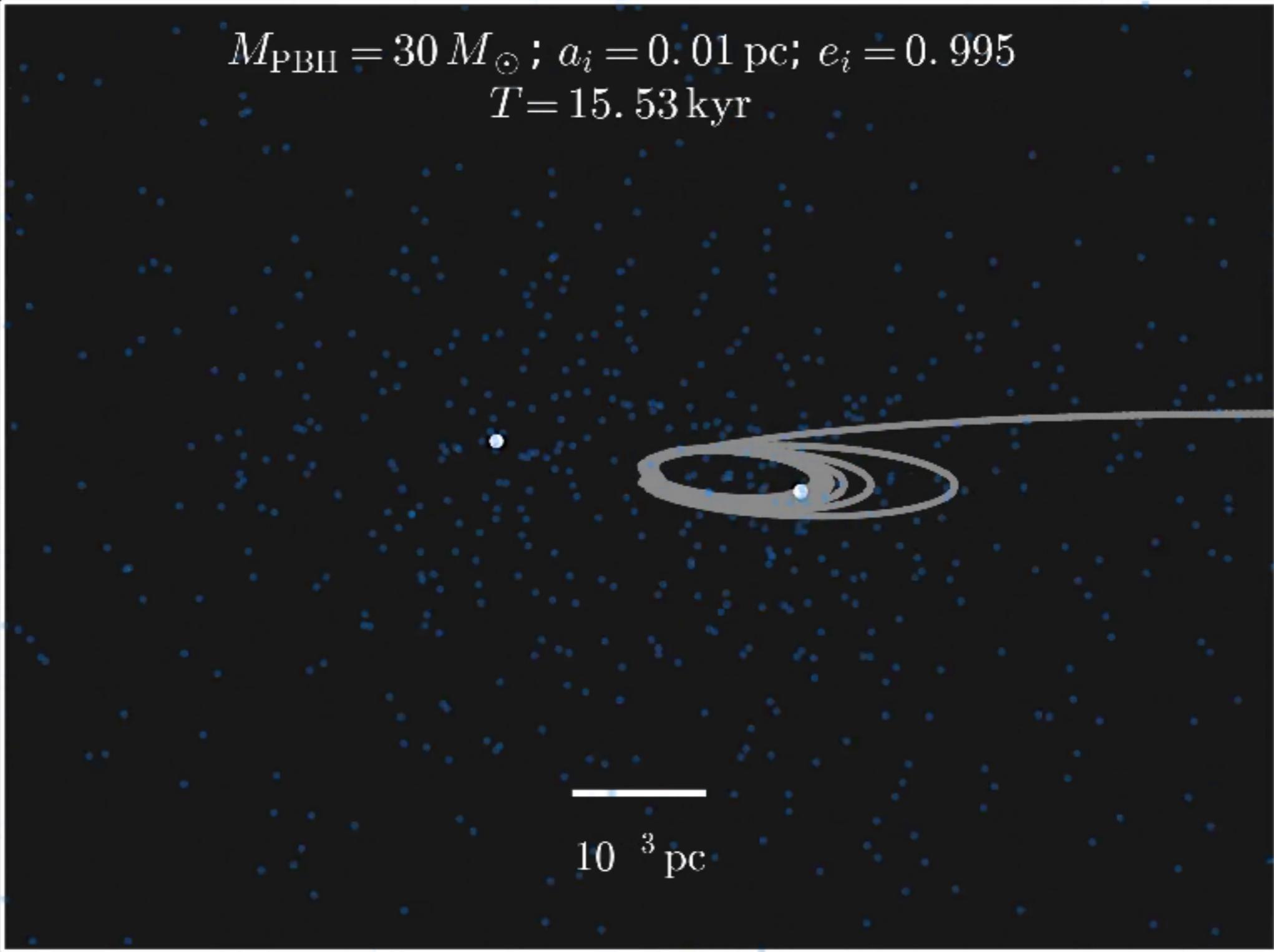


$$M_{\text{PBH}} = 30 M_{\odot}; a_i = 0.01 \text{ pc}; e_i = 0.995$$
$$T = 6.17 \text{ kyr}$$



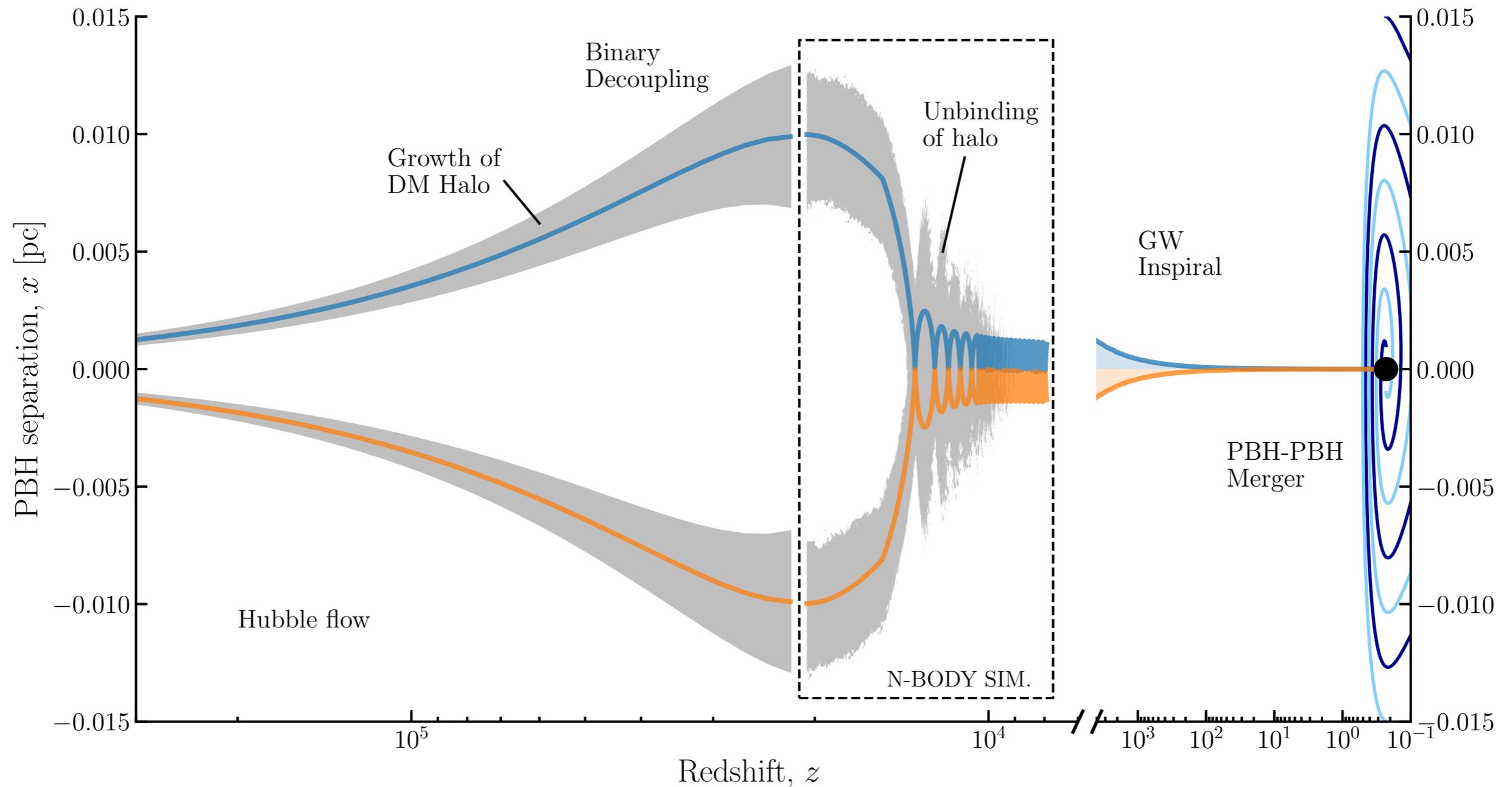
$10^3 \text{ pc}$

$$M_{\text{PBH}} = 30 M_{\odot}; a_i = 0.01 \text{ pc}; e_i = 0.995$$
$$T = 15.53 \text{ kyr}$$



$10^3 \text{ pc}$

# Life of a *dressed* PBH binary

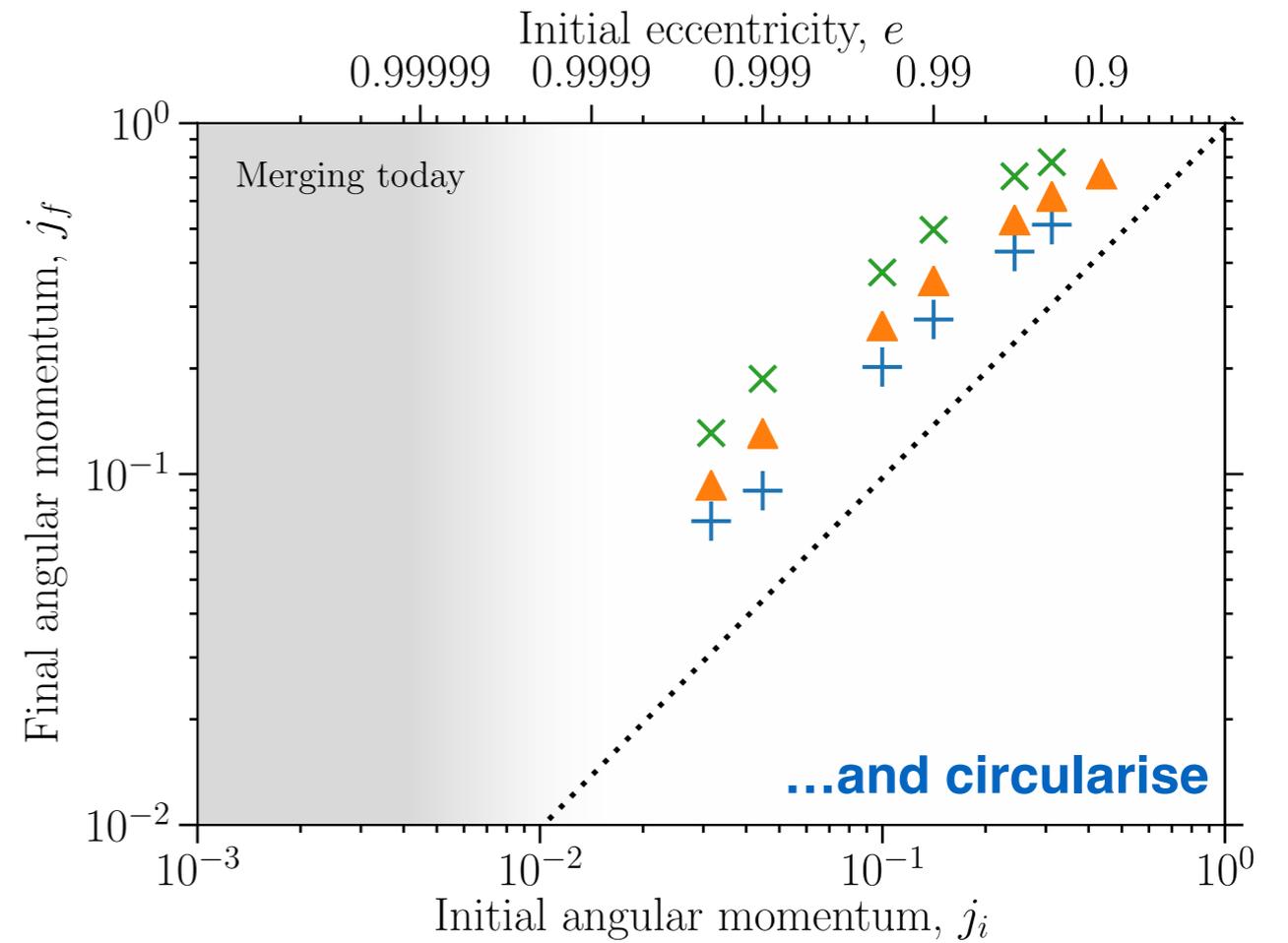
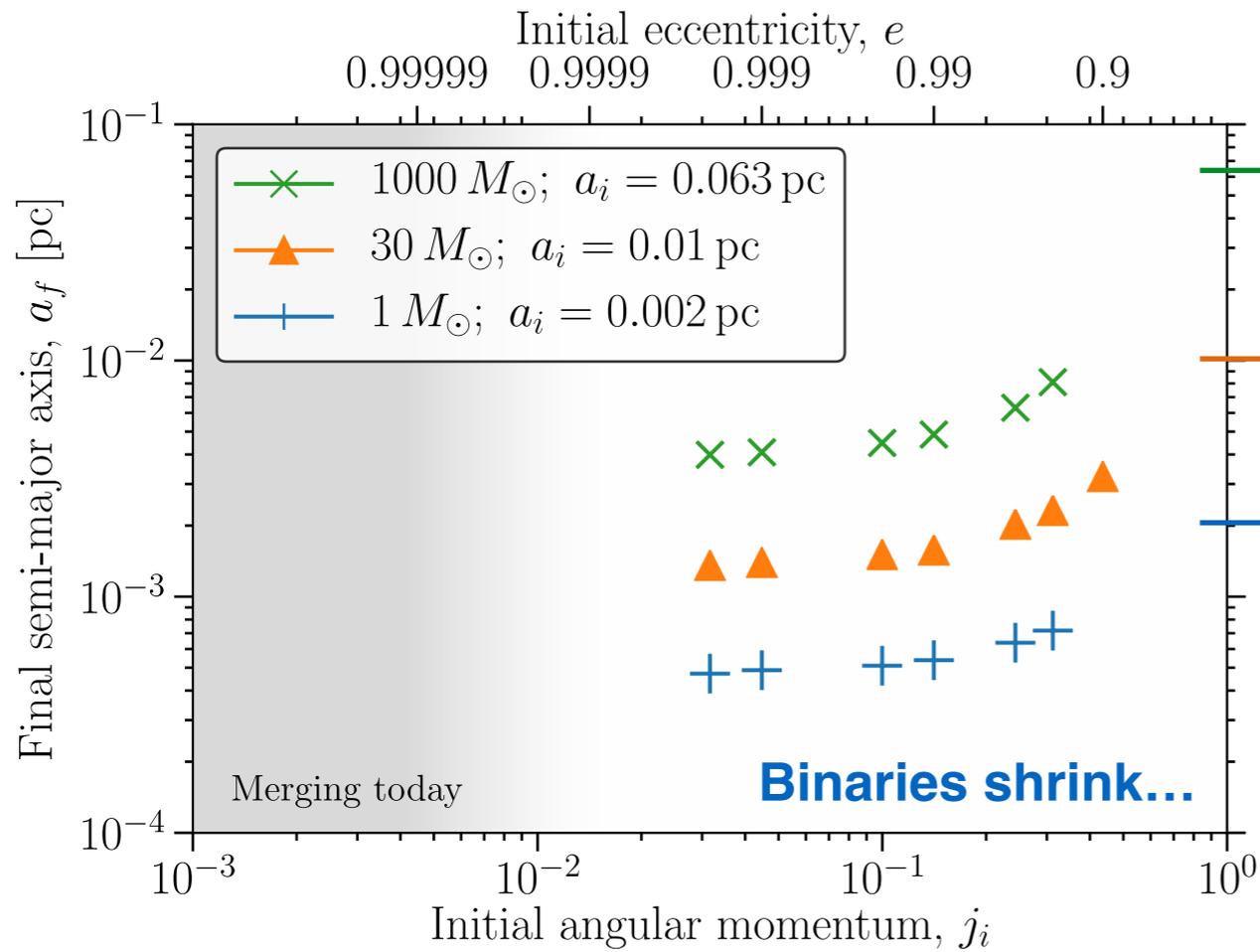


$$a_i = 0.01 \text{ pc}$$

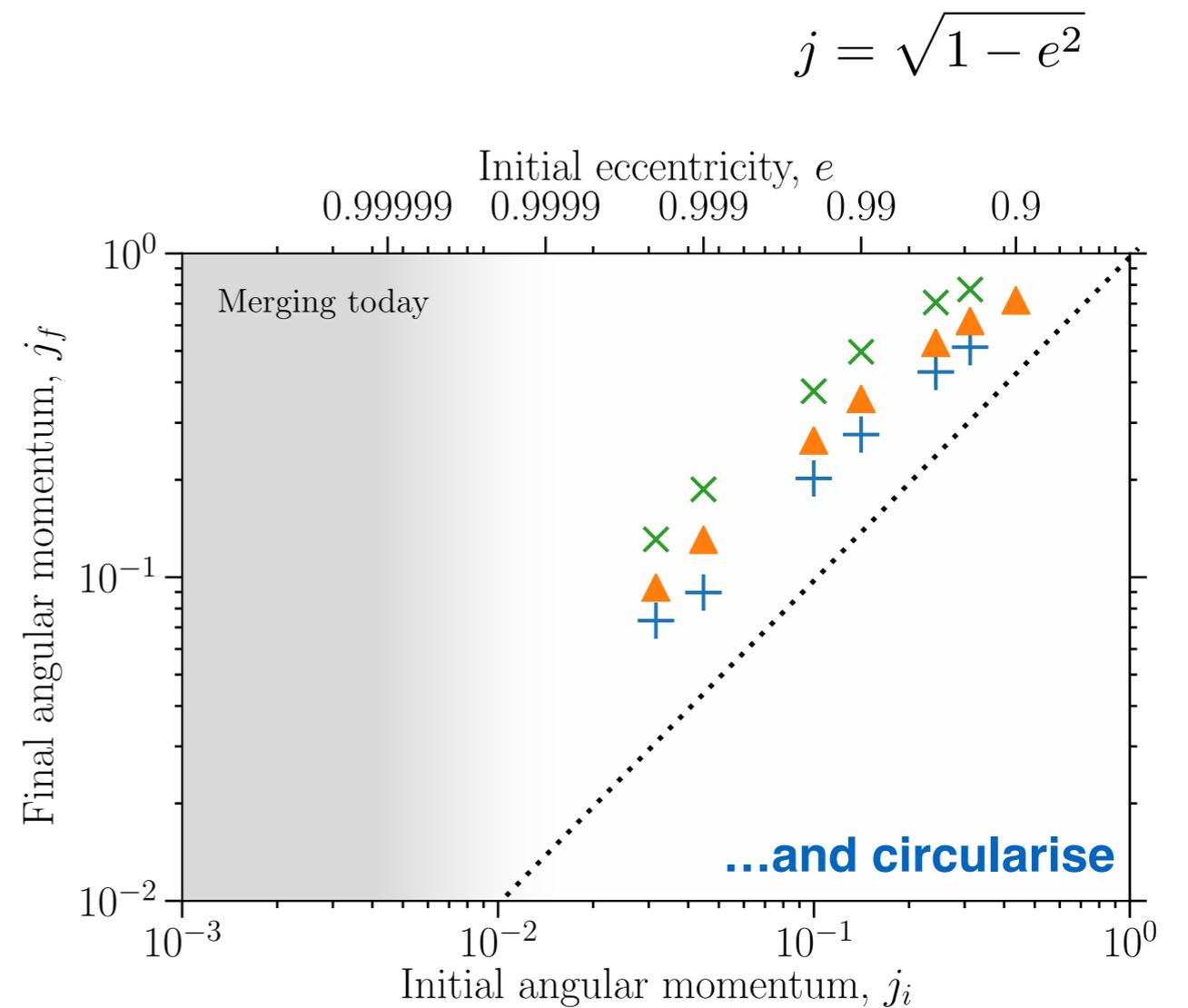
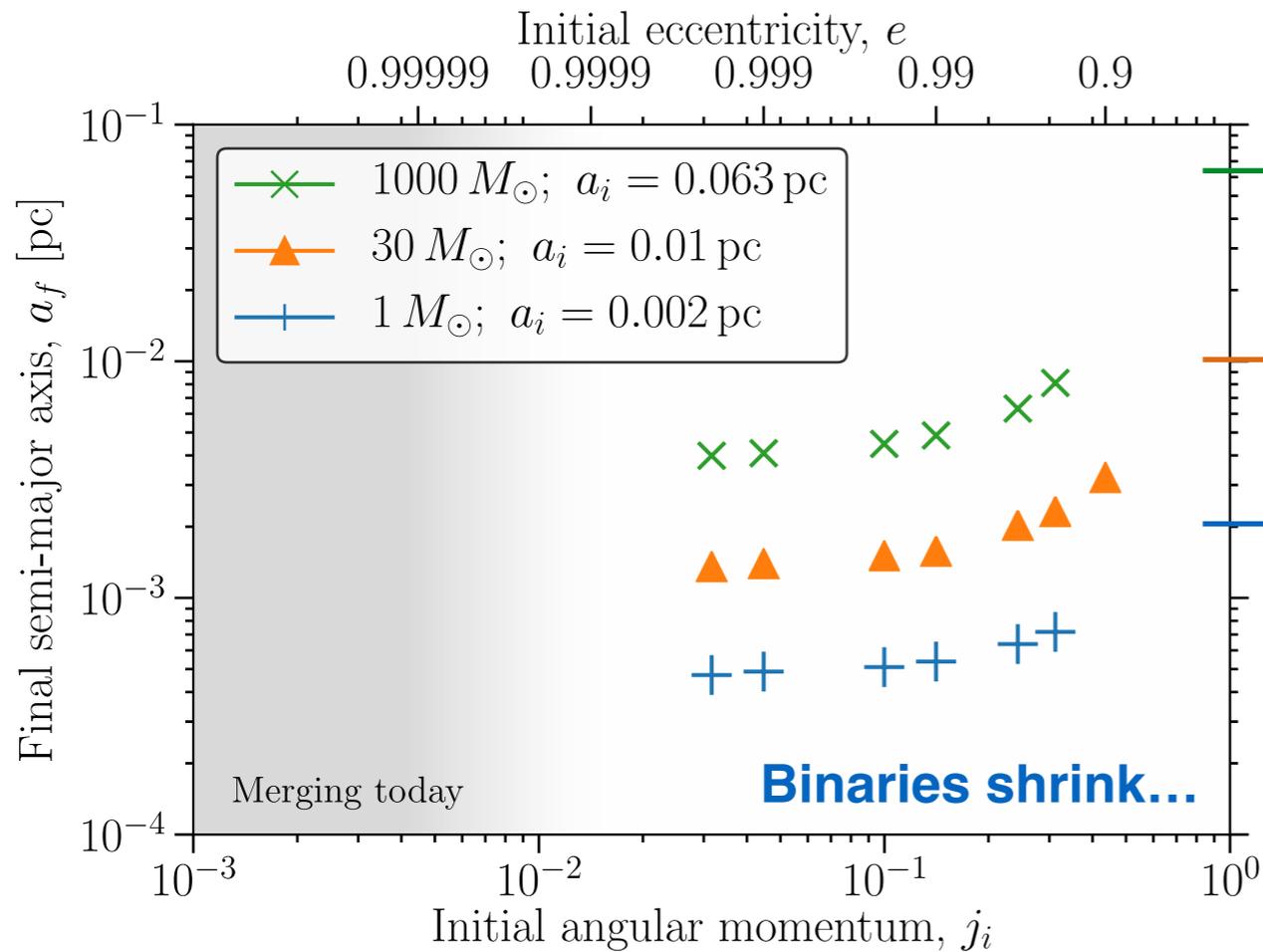
$$e_i = 0.995$$

# Simulation Results

$$j = \sqrt{1 - e^2}$$

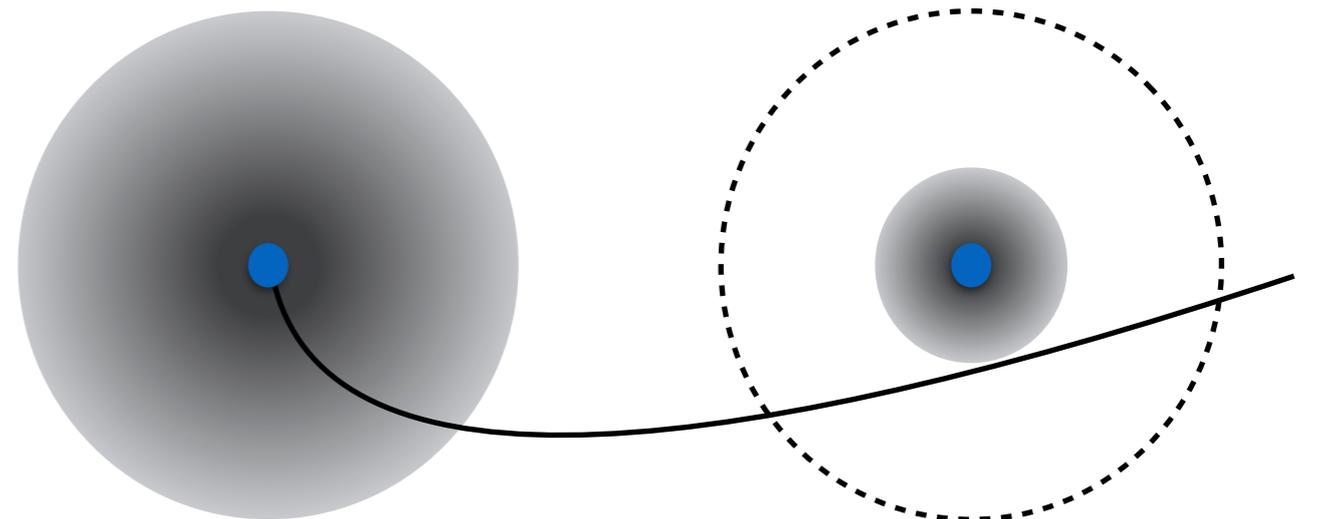


# Results: Semi-major Axis



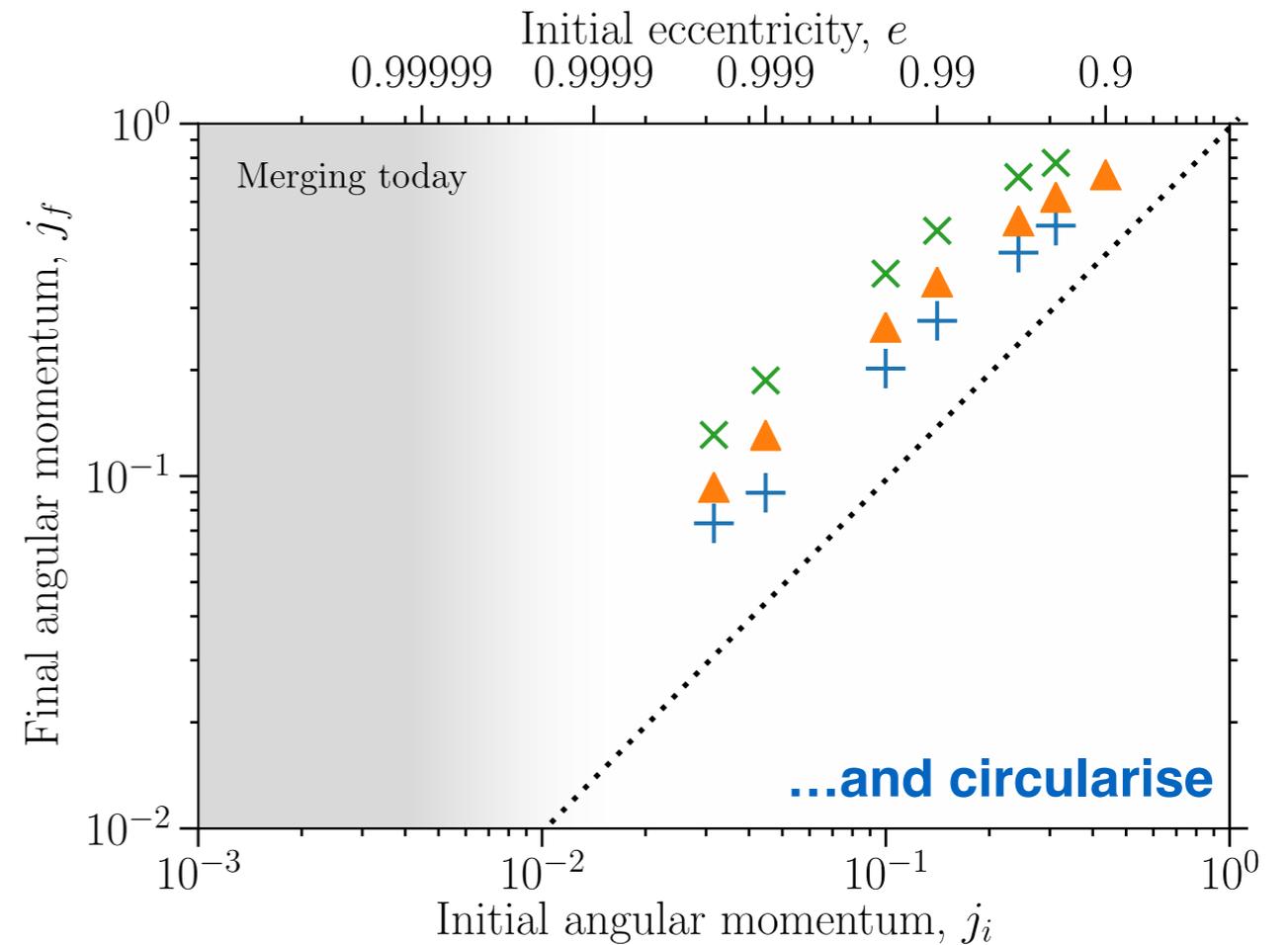
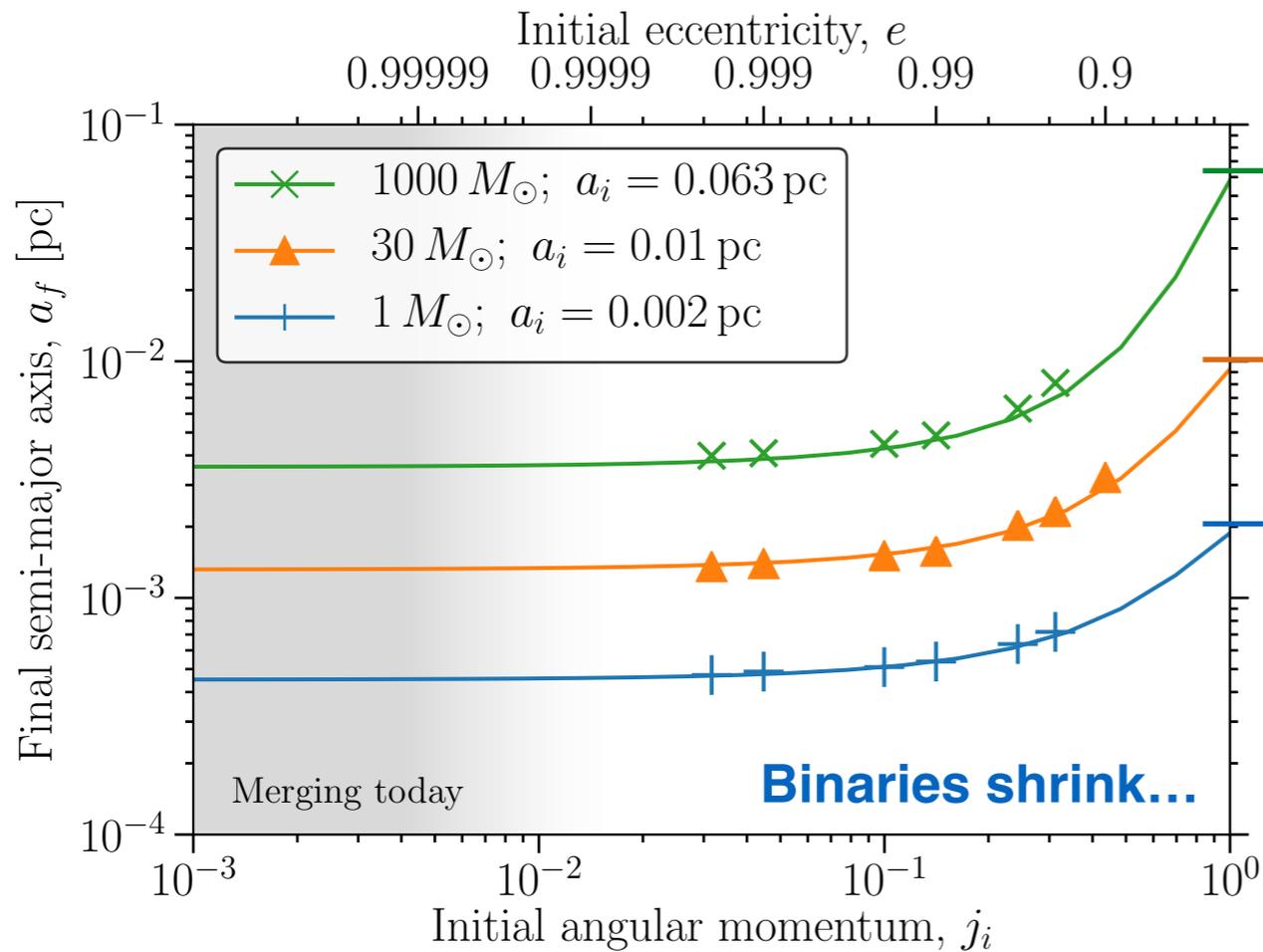
## Conservation of energy

$$E_i^{\text{orb}} + 2U^{\text{bind}} = E_f^{\text{orb}}$$



# Results: Semi-major Axis

$$j = \sqrt{1 - e^2}$$

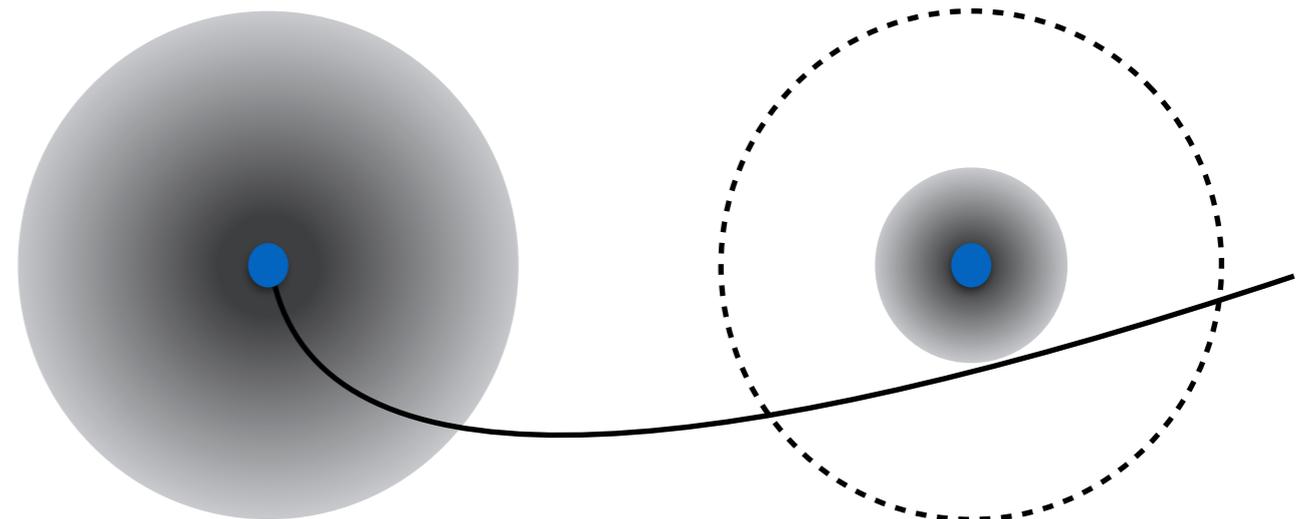


Conservation of energy

$$E_i^{\text{orb}} + 2U^{\text{bind}} = E_f^{\text{orb}}$$

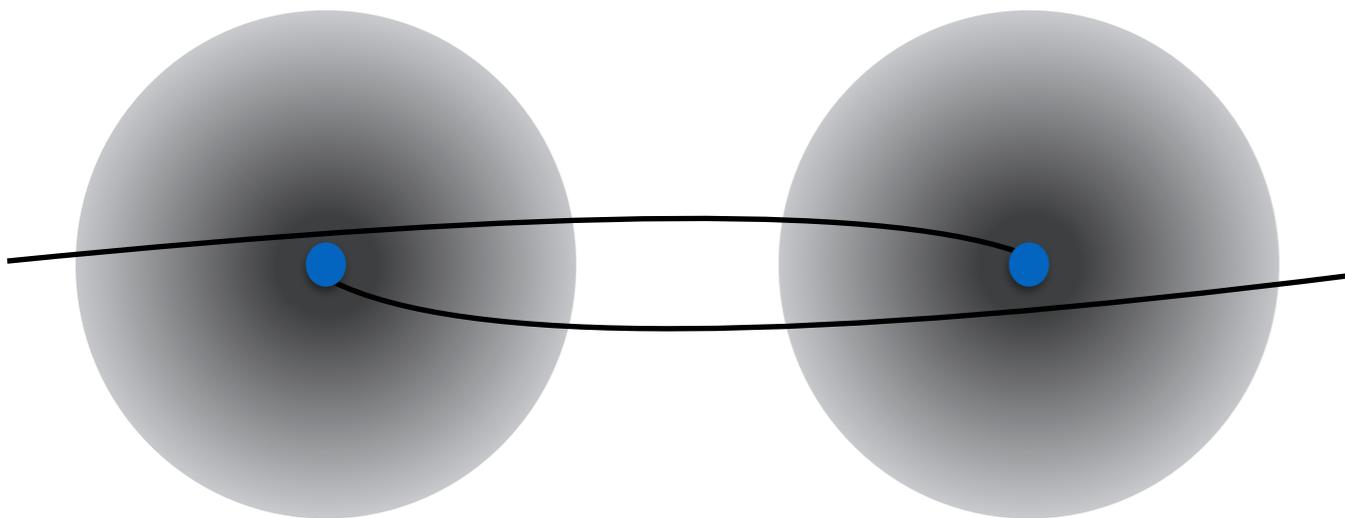
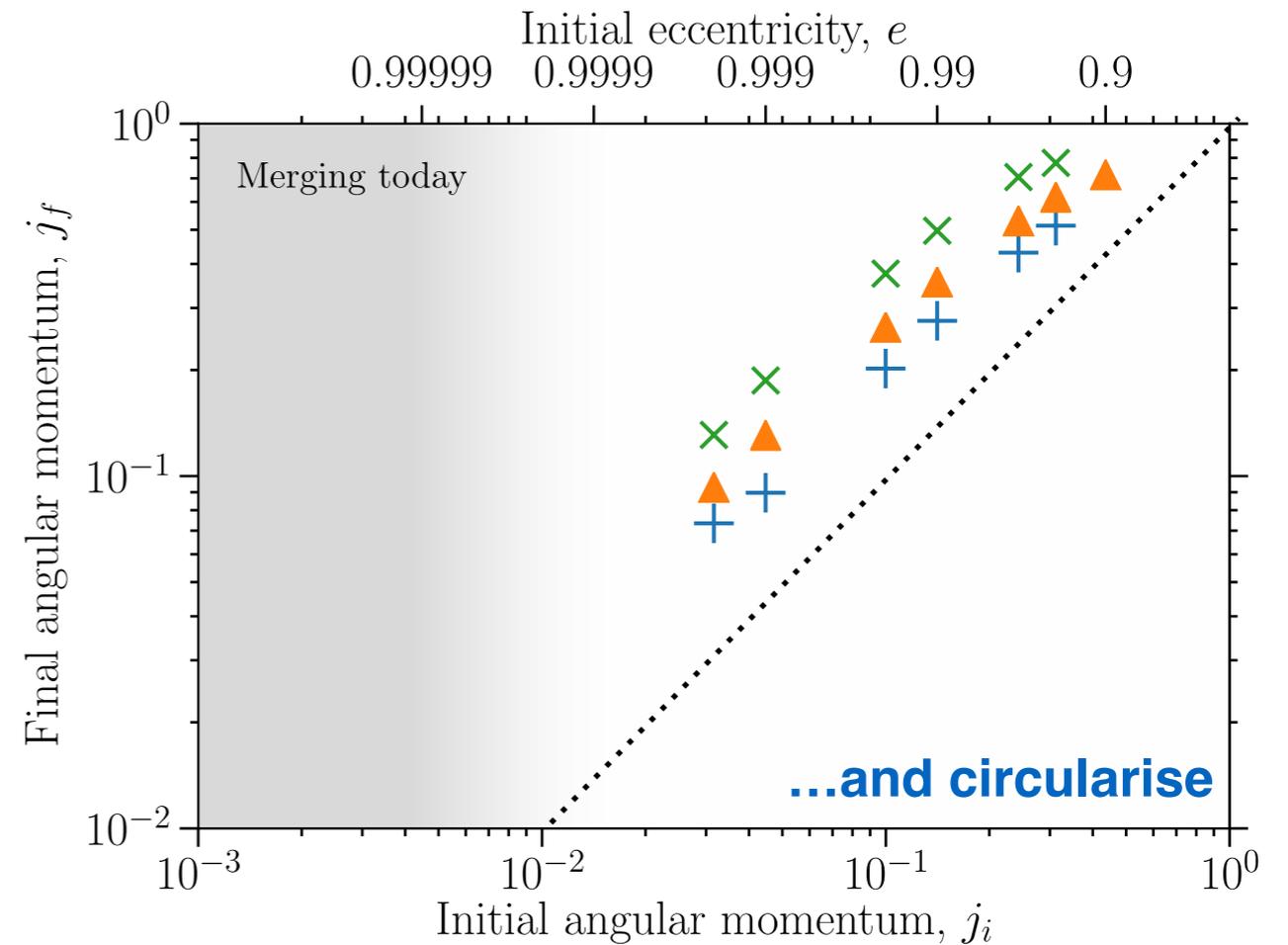
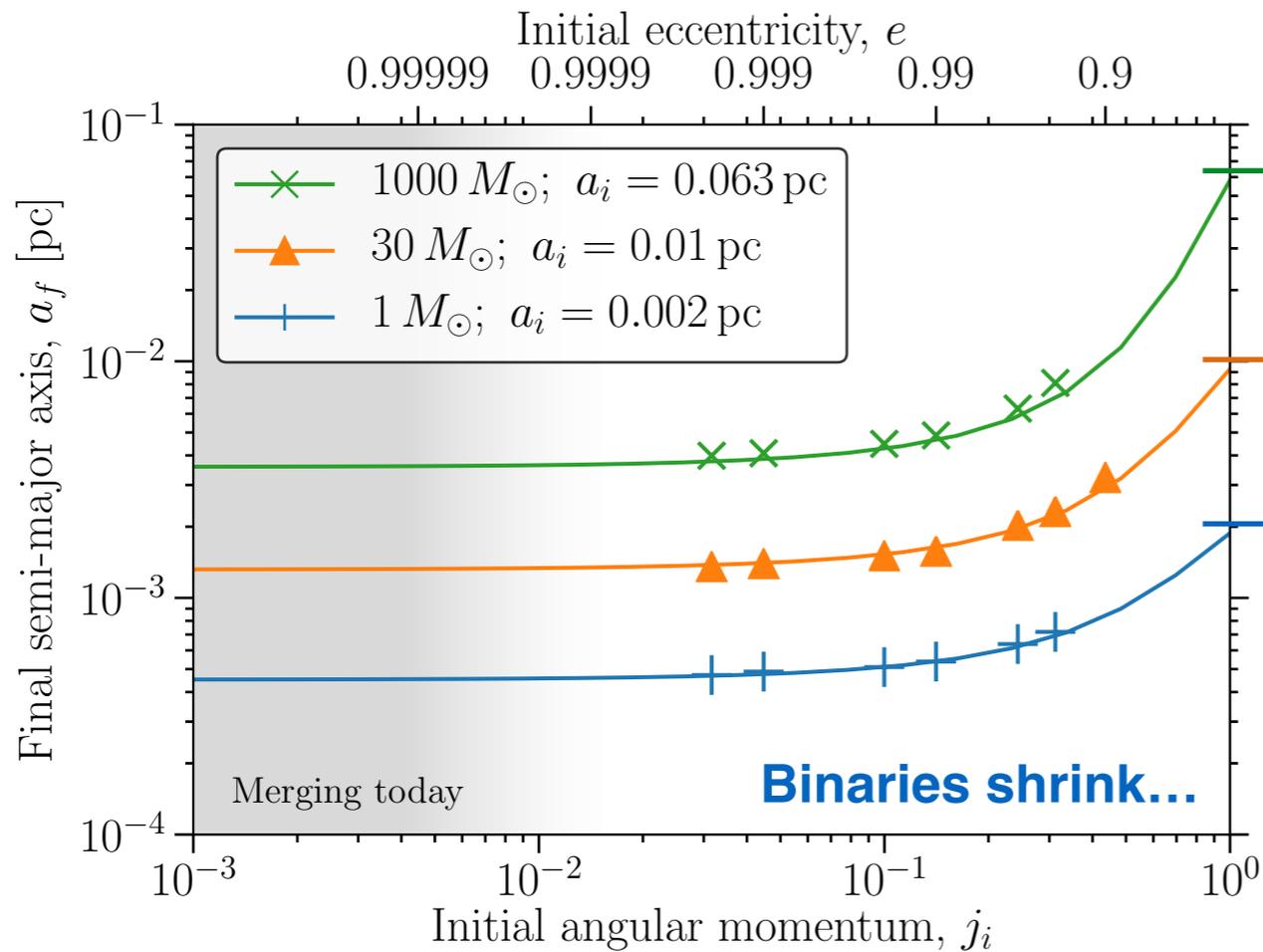


fixes semi-major axis,  $a$



# Results: Angular Momentum

$$j = \sqrt{1 - e^2}$$



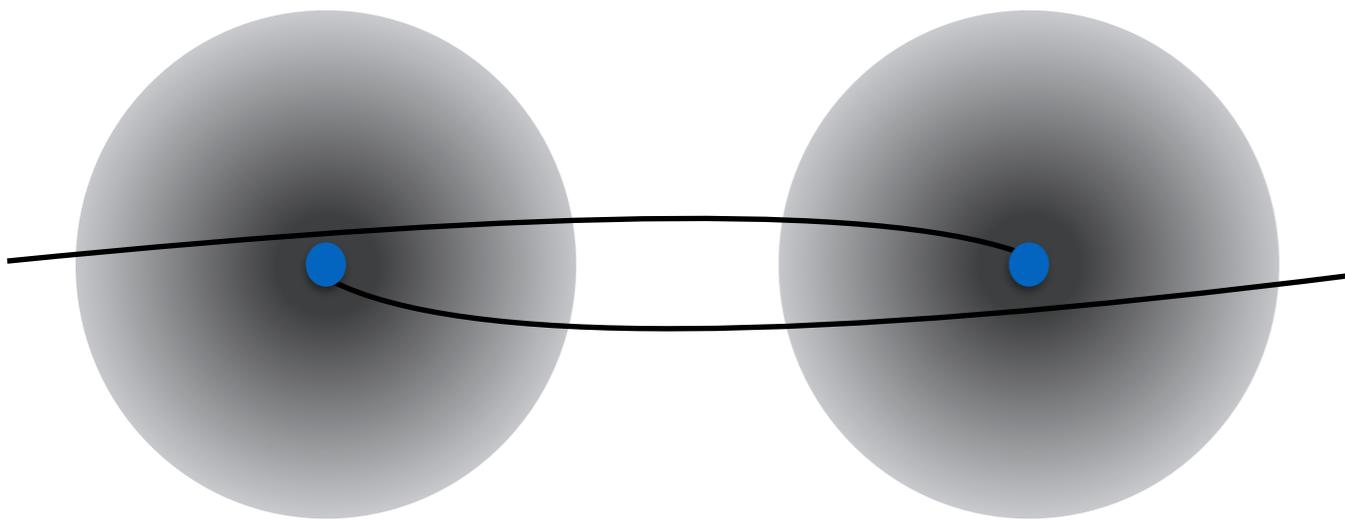
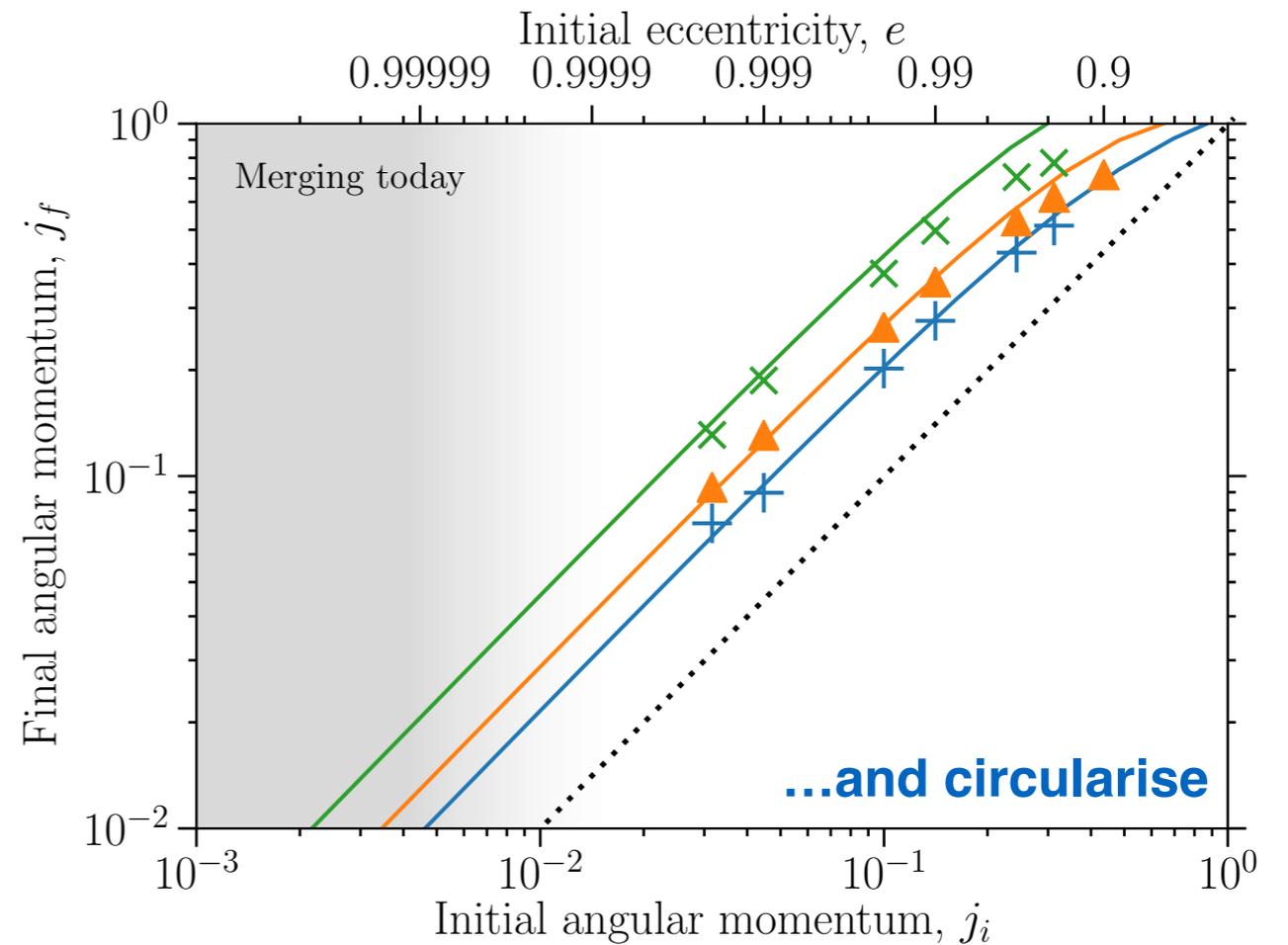
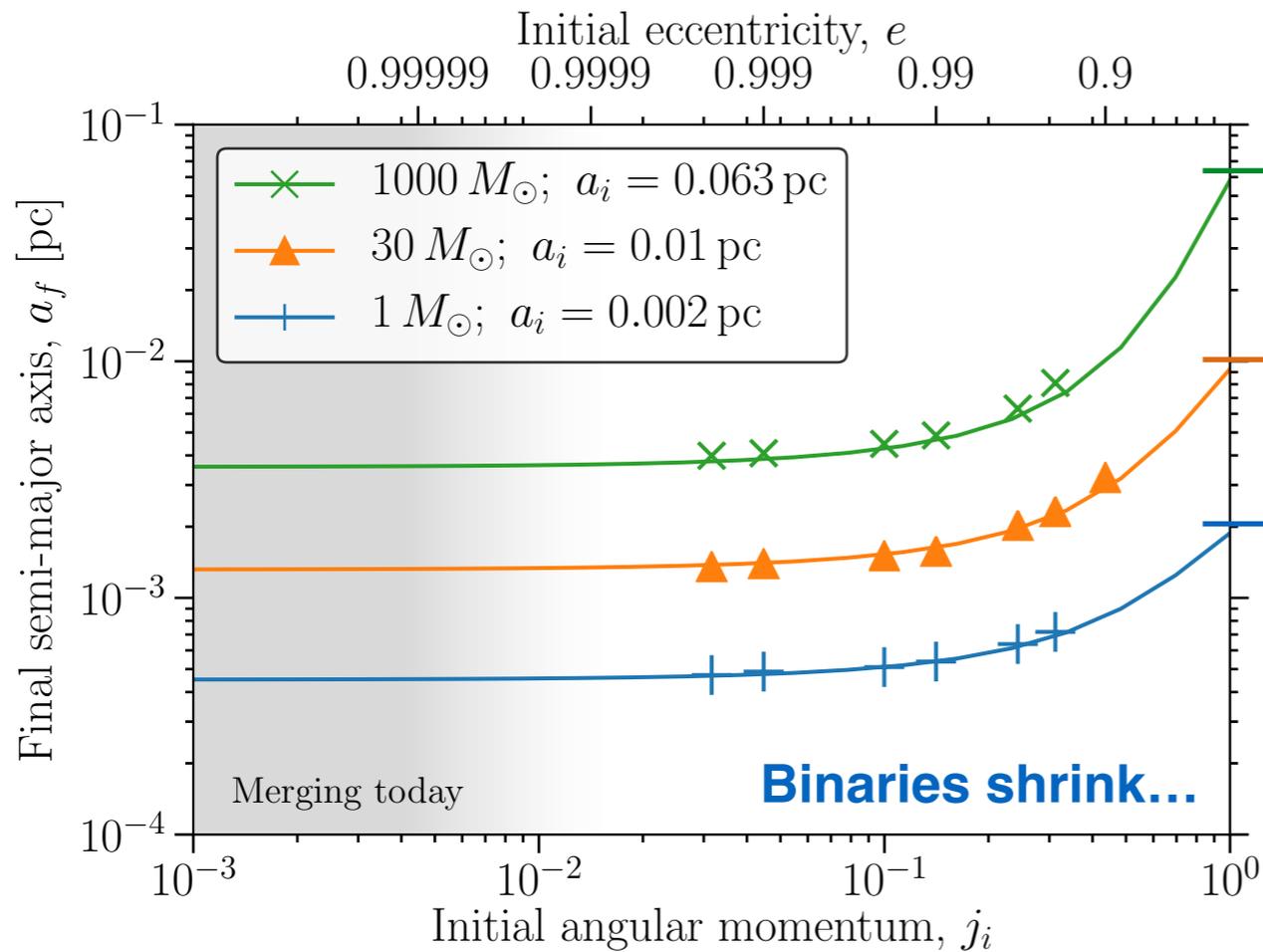
## Conservation of angular momentum

$$L_i^{\text{PBH}} = L_f^{\text{PBH}}$$

$$L_i^{\text{halo}} = L_f^{\text{halo}}$$

# Results: Angular Momentum

$$j = \sqrt{1 - e^2}$$



Conservation of angular momentum

$$L_i^{\text{PBH}} = L_f^{\text{PBH}}$$

$$L_i^{\text{halo}} = L_f^{\text{halo}}$$

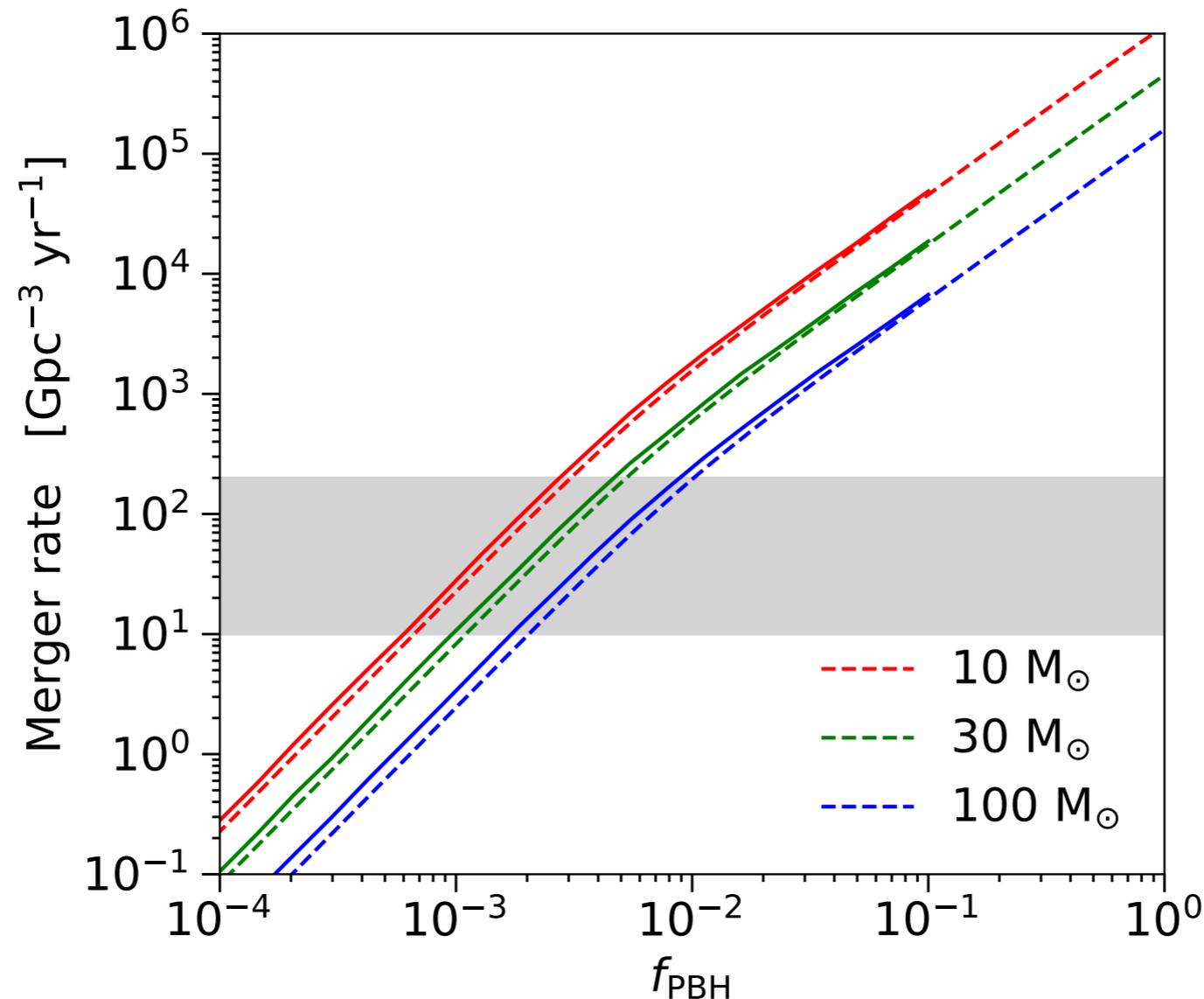
fixes  $j_f = j_i \sqrt{a_i/a_f}$

# Calculating the final merger rate

$$j = \sqrt{1 - e^2}$$

Draw PBH binaries from the distribution of  $(a, e)$

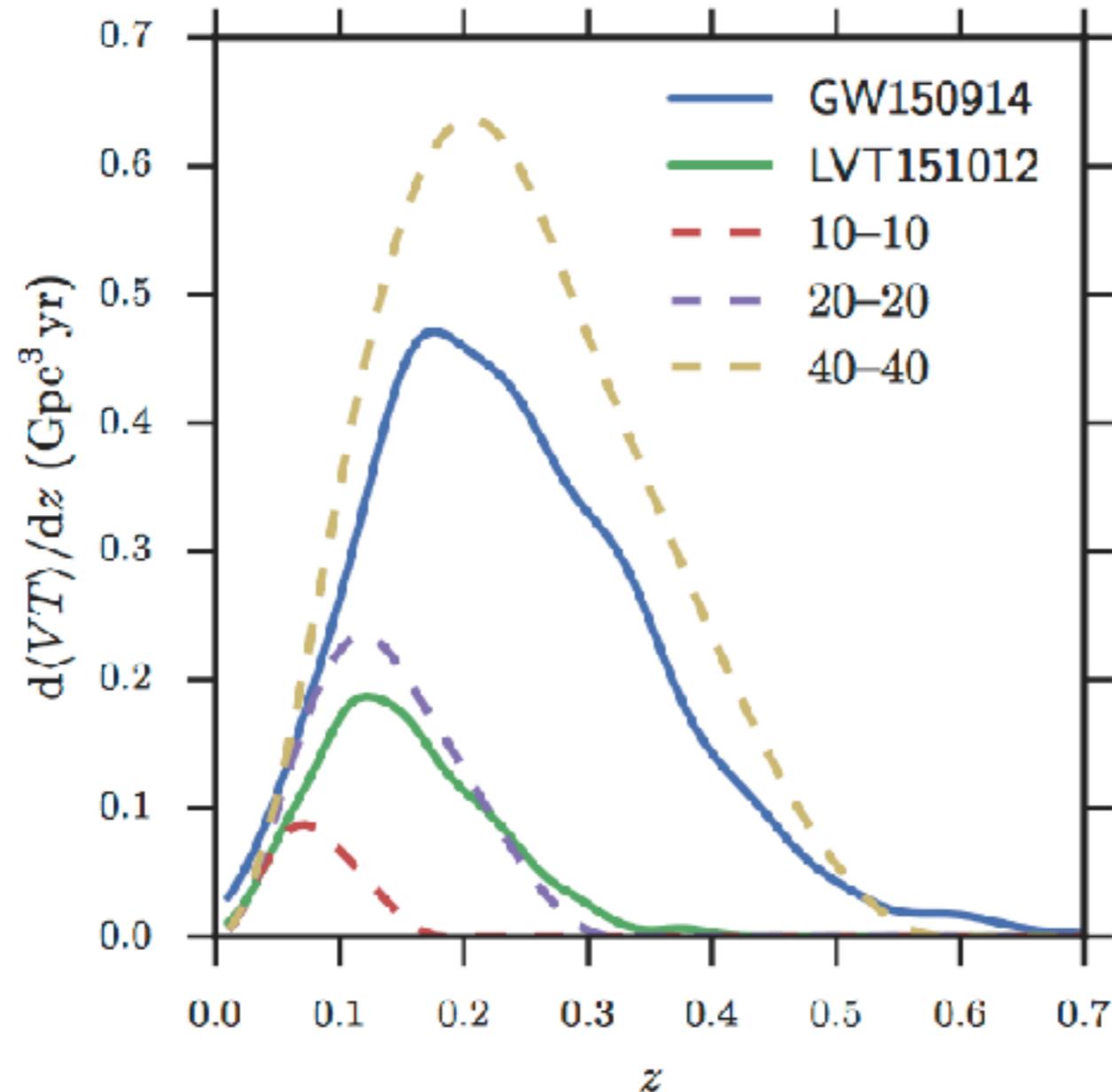
Guided by the simulations, map  $(a_i, e_i) \rightarrow (a_f, e_f)$



Merger time  $t_{\text{merge}} = \frac{3 c^5}{170 G_N^3} \frac{a^4 j^7}{M_{\text{PBH}}^3}$  is almost conserved:  $t_f = \sqrt{\frac{a_i}{a_f}} t_i$

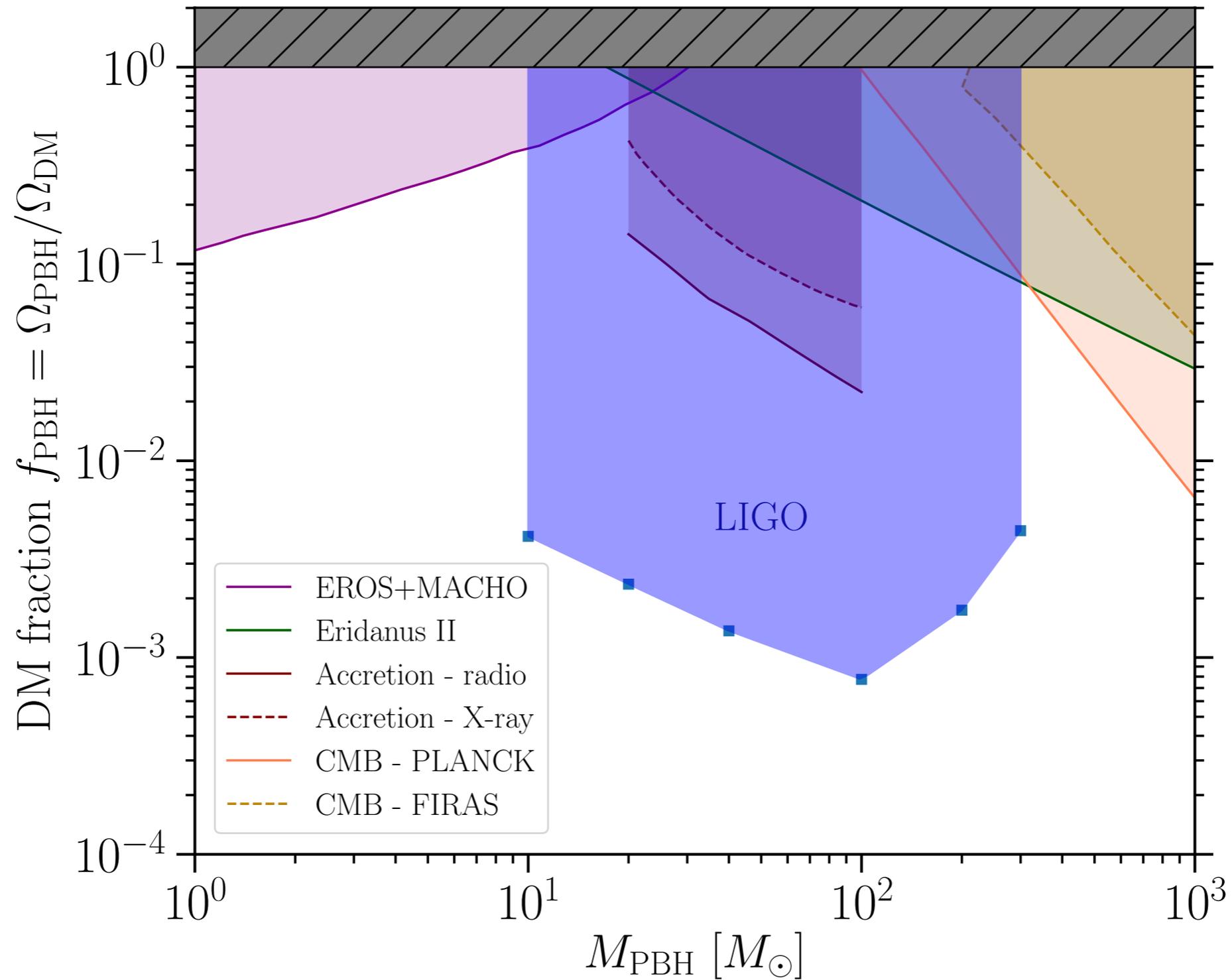
$$\mathcal{R}_{\text{LIGO}} = \frac{1}{2} n_{\text{PBH}} P_{\text{binary}} \frac{\int S(z) P(t[z]) dz}{\int S(z) dz}$$

$$S(z) = d\langle VT \rangle / dz$$



Compare expected LIGO-Virgo rate with reported 90% upper limit...

[1606.03939, 1704.04628, recently extended to sub-solar mass binaries in 1808.04771]



*Local DM halos strengthen constraints by around a factor of 2.*

# Caveats (once again)

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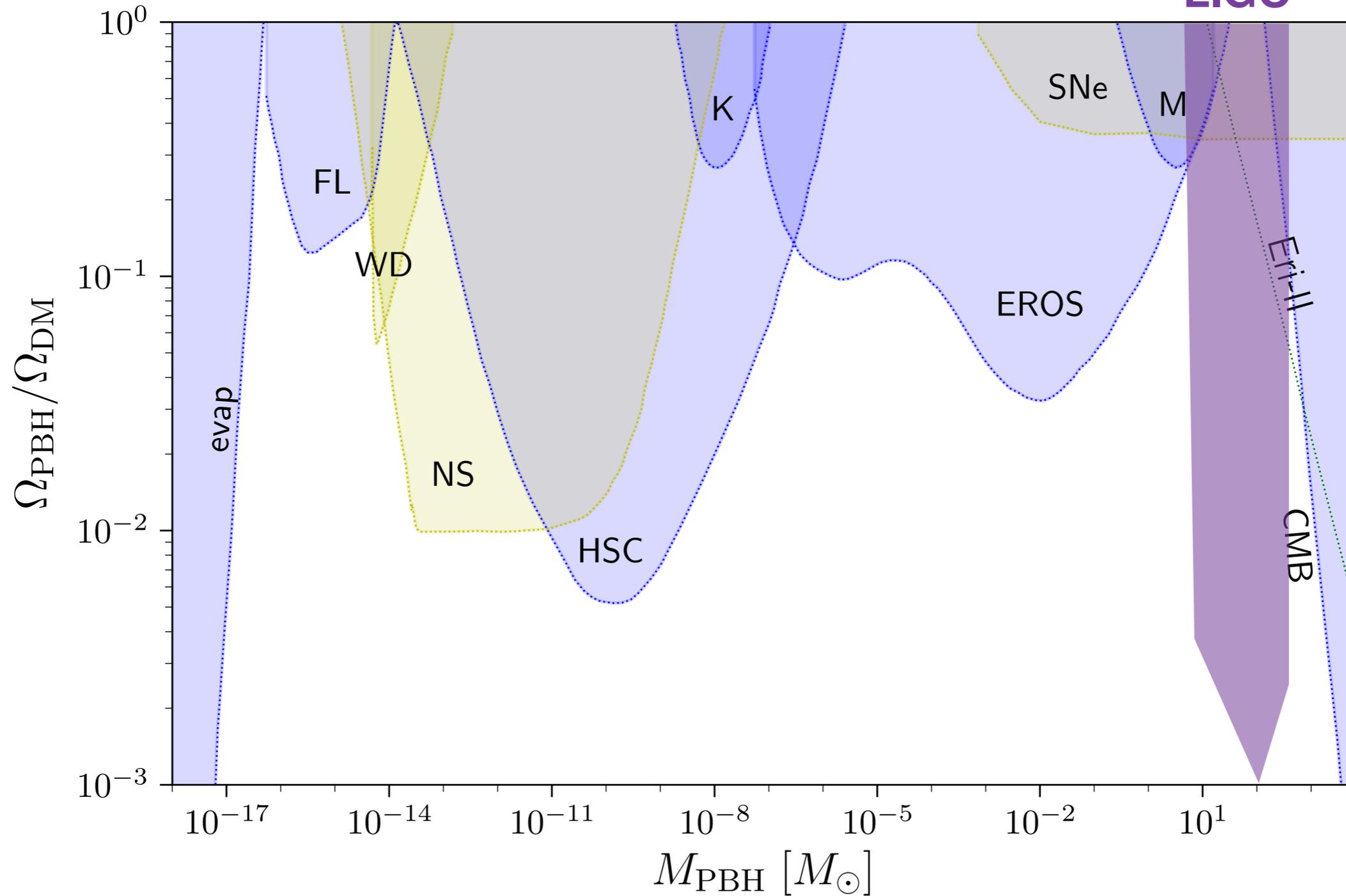
- Survival ✓ [Ali-Haïmoud et al., 1709.06576]
- Clustering ✓ [1807.02084, 1808.05910]
- Baryons ? [More (tough) simulation needed]
- Dark Matter ✓ [**BJK**, Gaggero & Bertone, 1805.09034]

Bounds from merging PBHs are being placed on  
a more and more solid footing!

# PBHs and their DM Halos

[LIGO Bound from  
**BJK**, Gaggero & Bertone, 1805.09034]

[Selected bounds from 1801.00808]



*Where else can DM halos have an influence?*

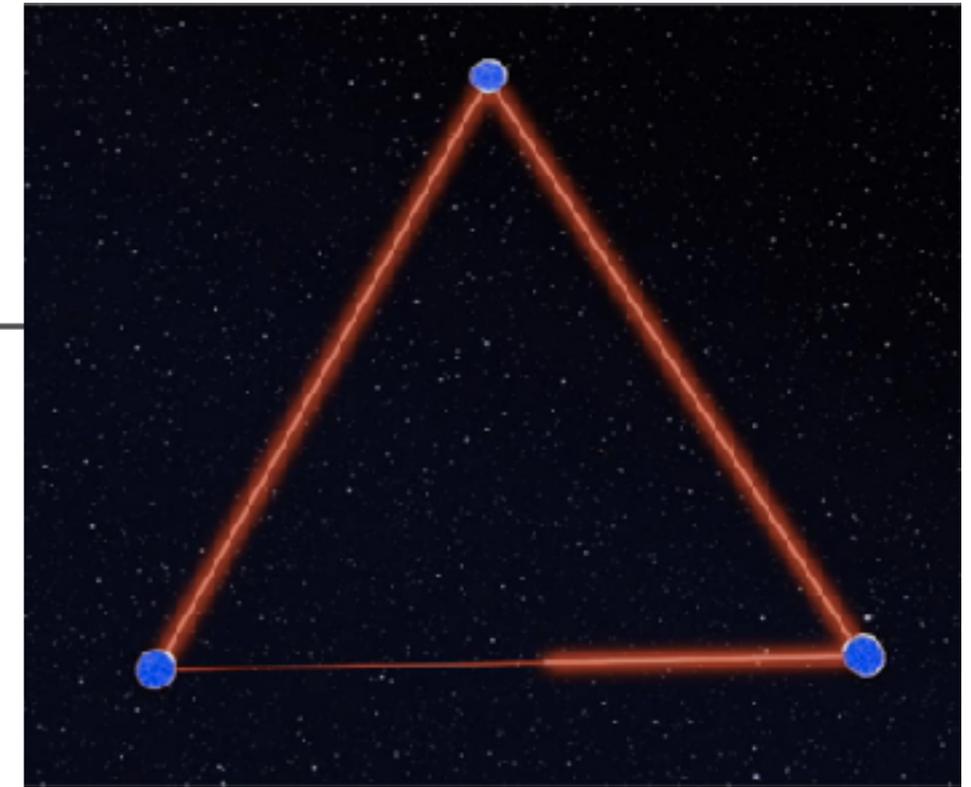
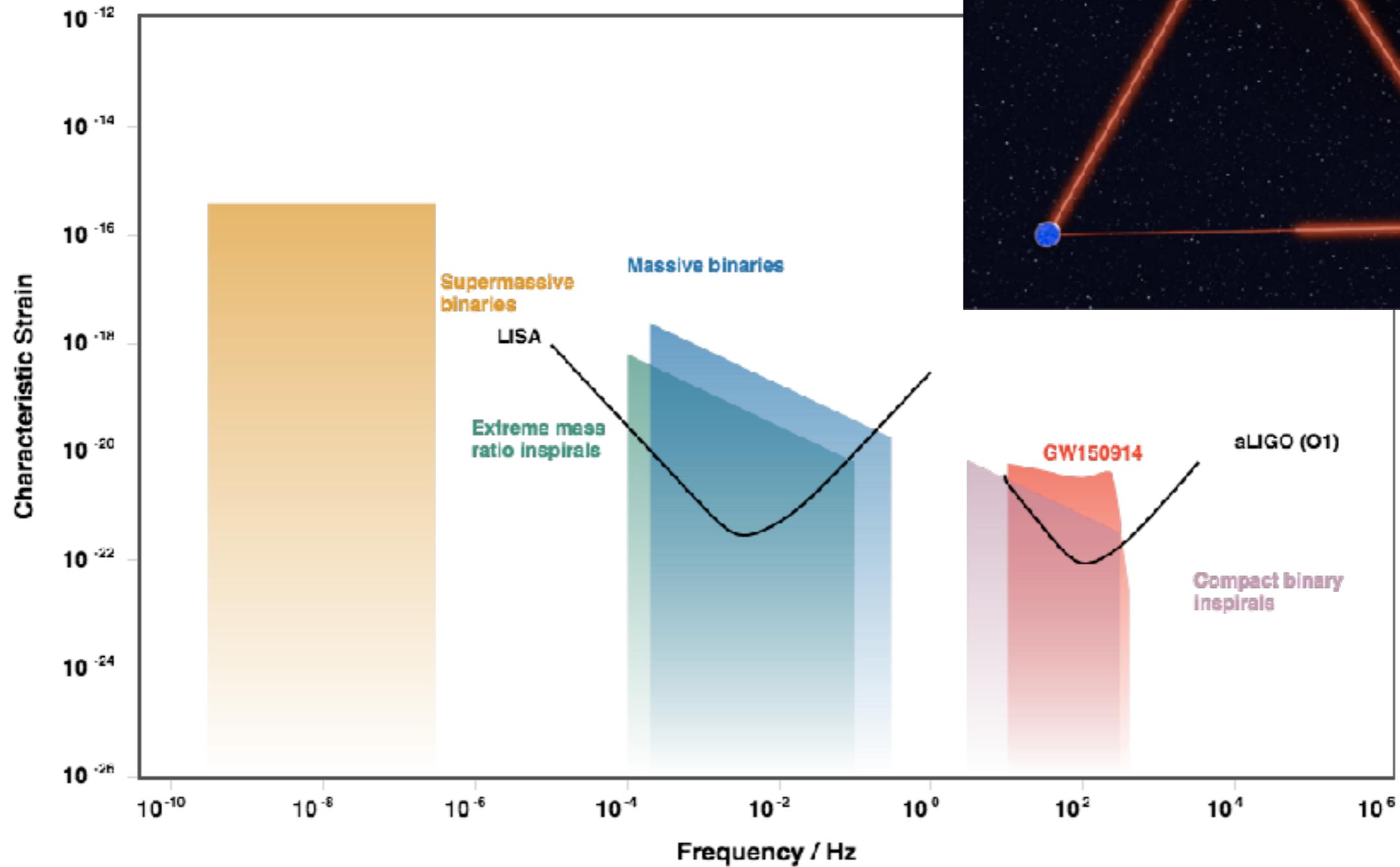
# Dark Dresses at LISA

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[Preliminary work with Daniele Gaggero,  
David Nichols and Gianfranco Bertone]

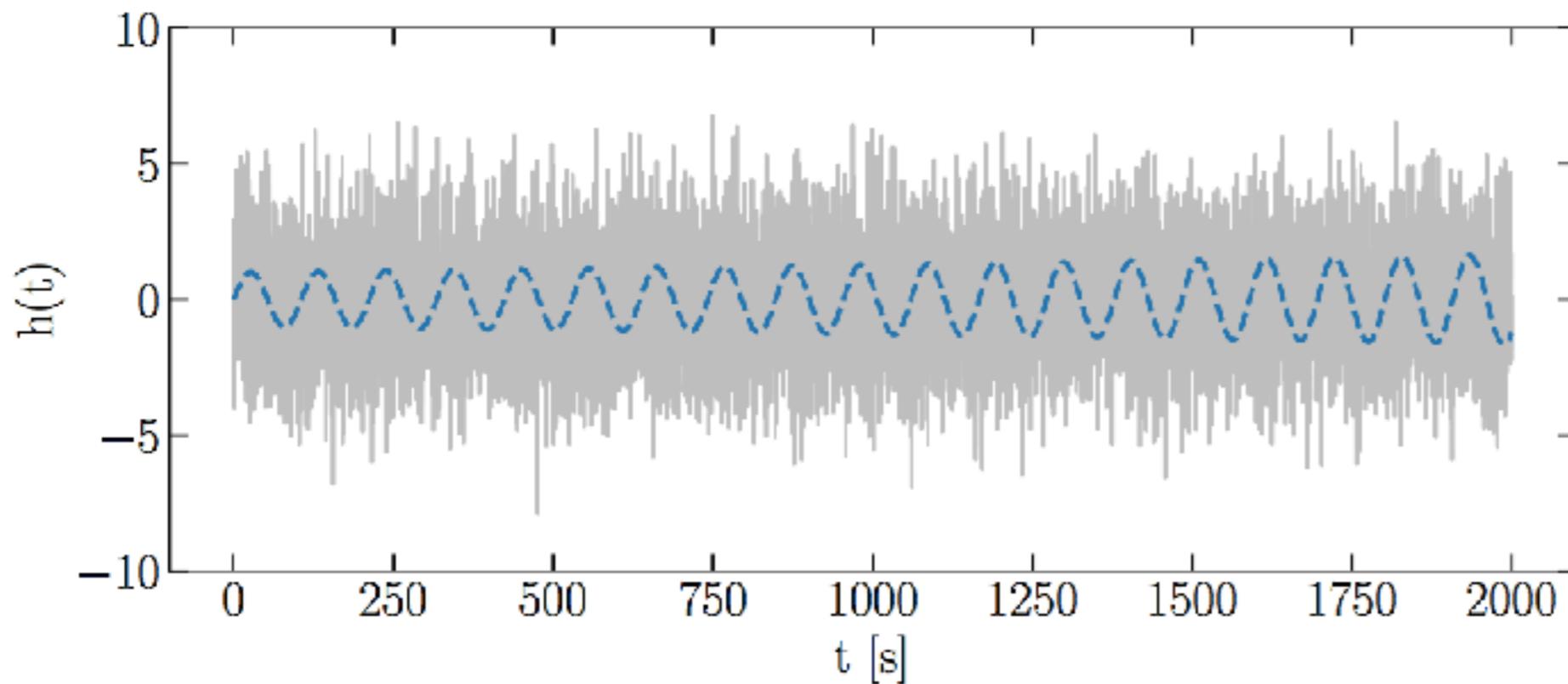
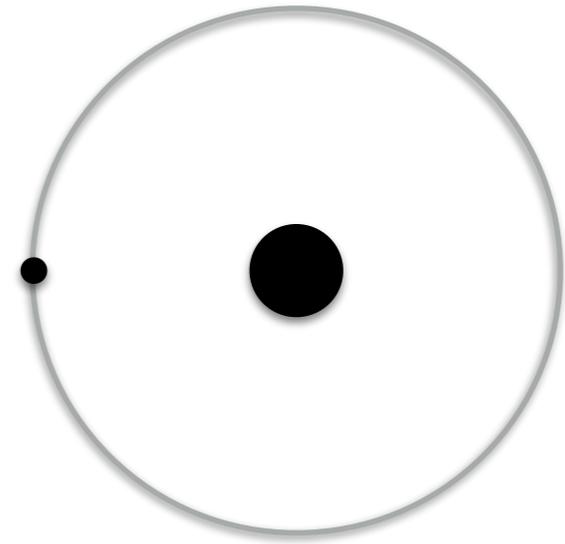
# LISA: GWs in Space

© AEI / MM / exozet



[[gwplotter.com](http://gwplotter.com)]

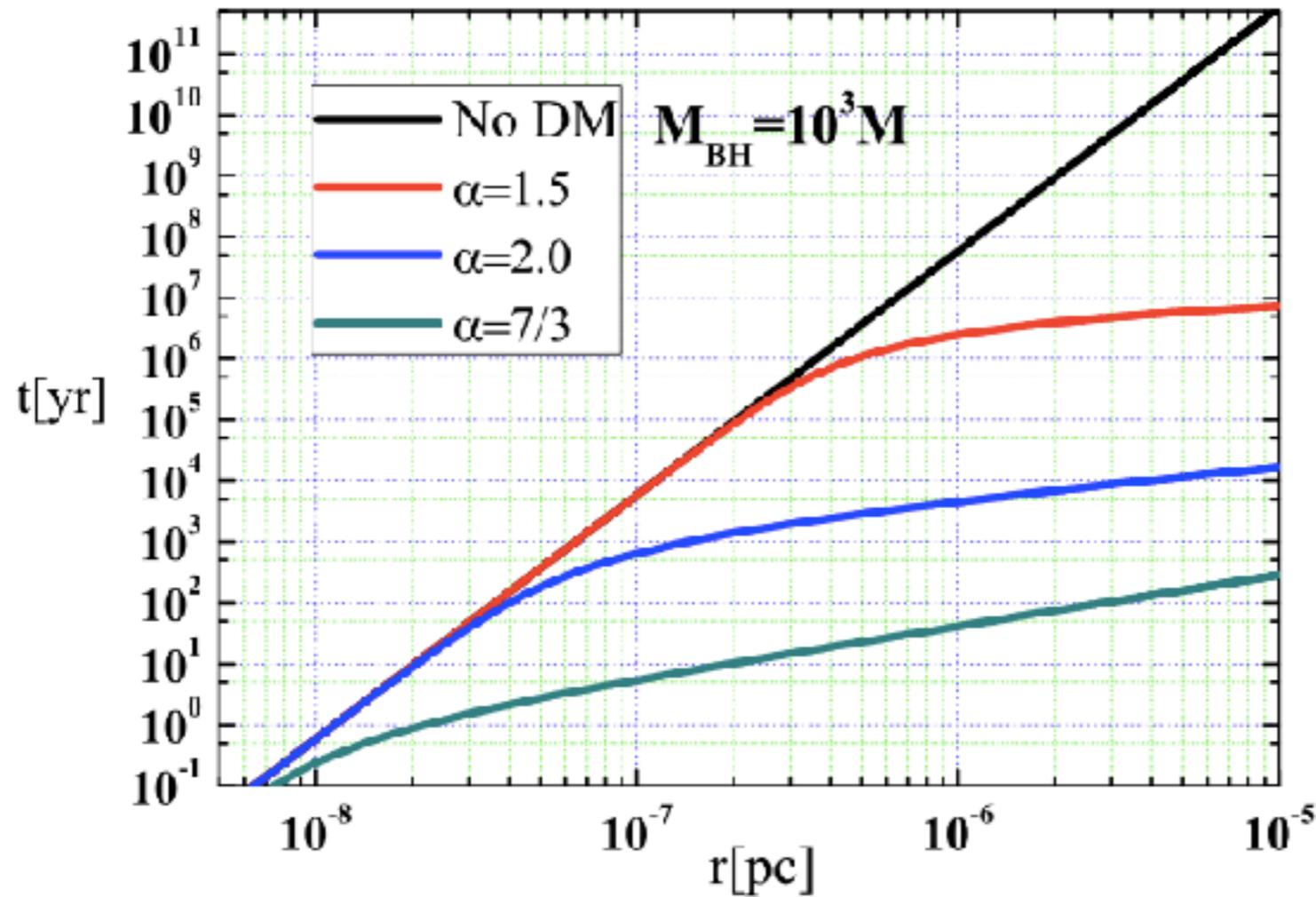
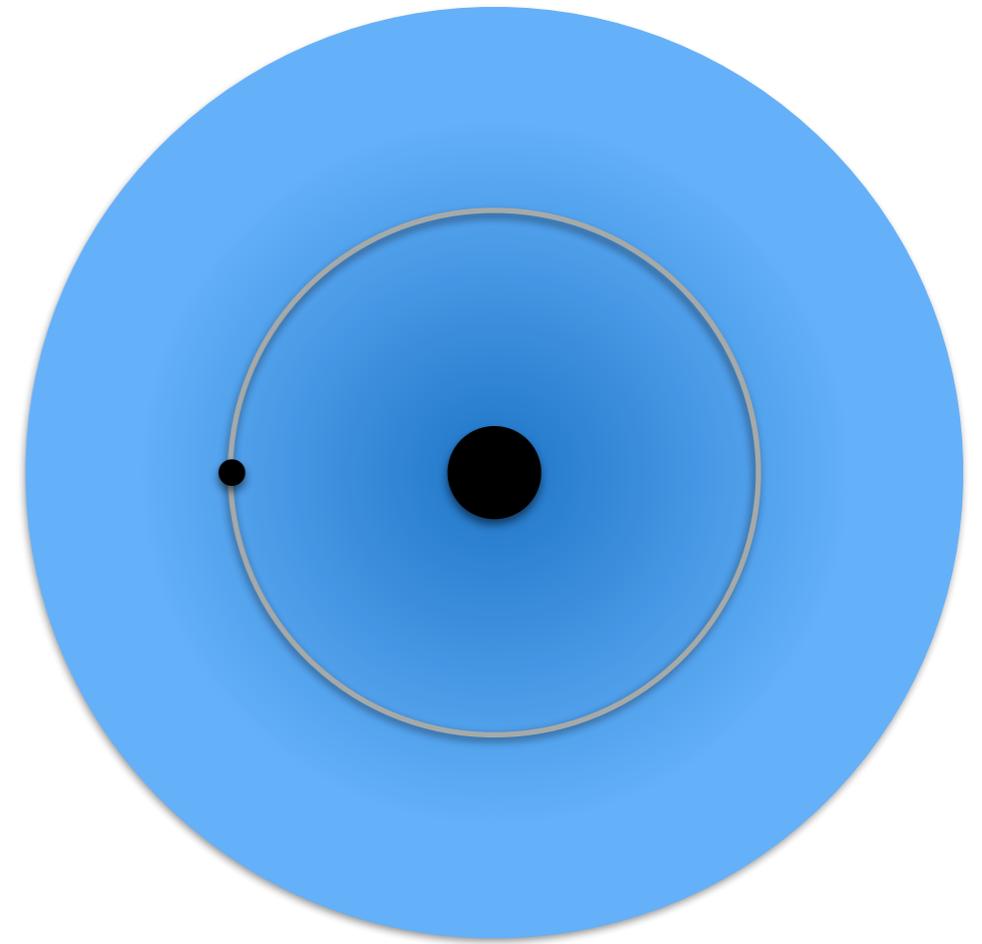
# Intermediate Ratio Mass Inspirals (IMRIs)



$$M_{\text{IMBH}} = 1000 M_{\odot}$$
$$\mu = 1 M_{\odot}$$

[1705.09421, 1807.03824]

# Speeding up IMRIs

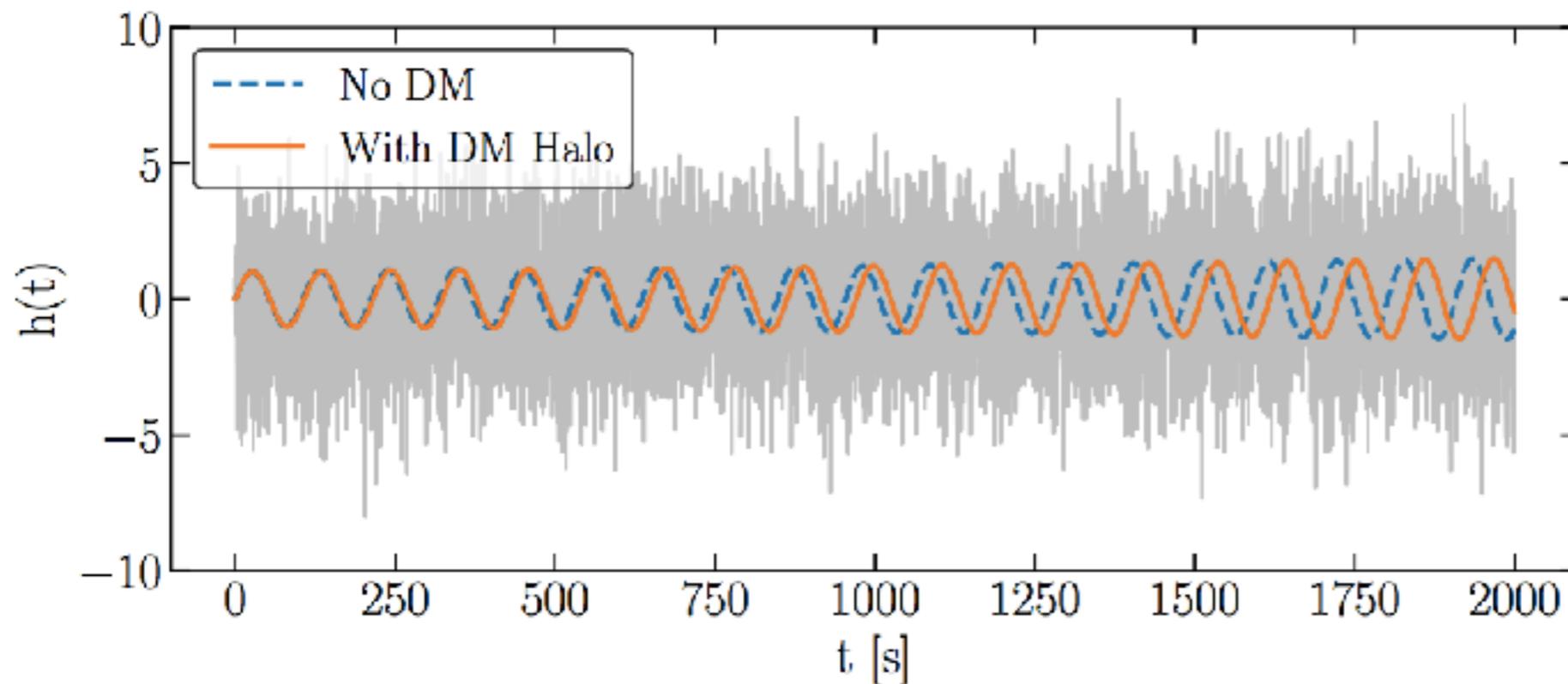
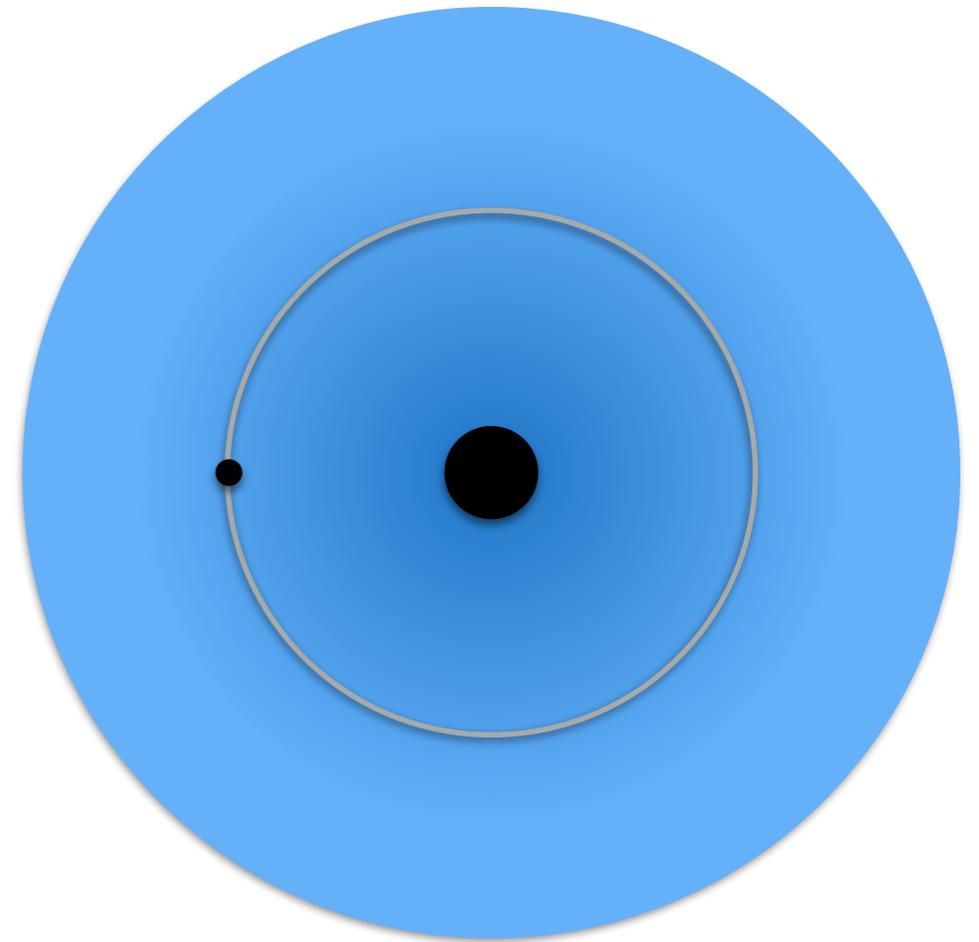
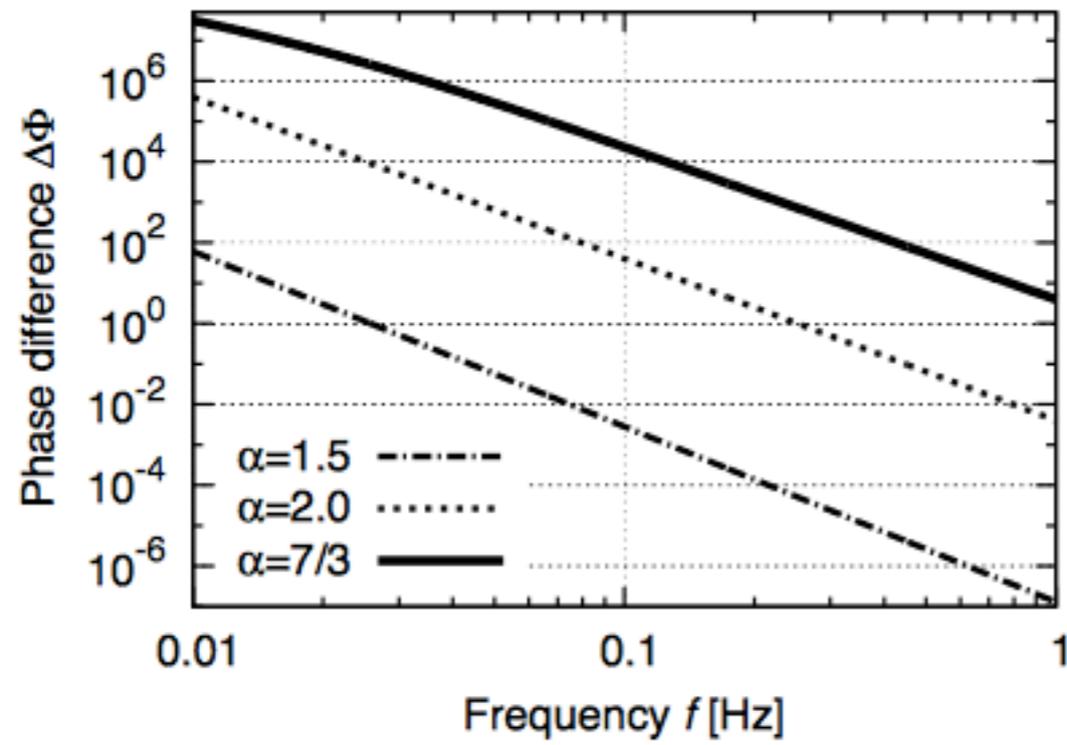


$$M_{\text{IMBH}} = 1000 M_{\odot}$$

$$\mu = 1 M_{\odot}$$

[Yue & Han, 1711.09706, 1802.03739]

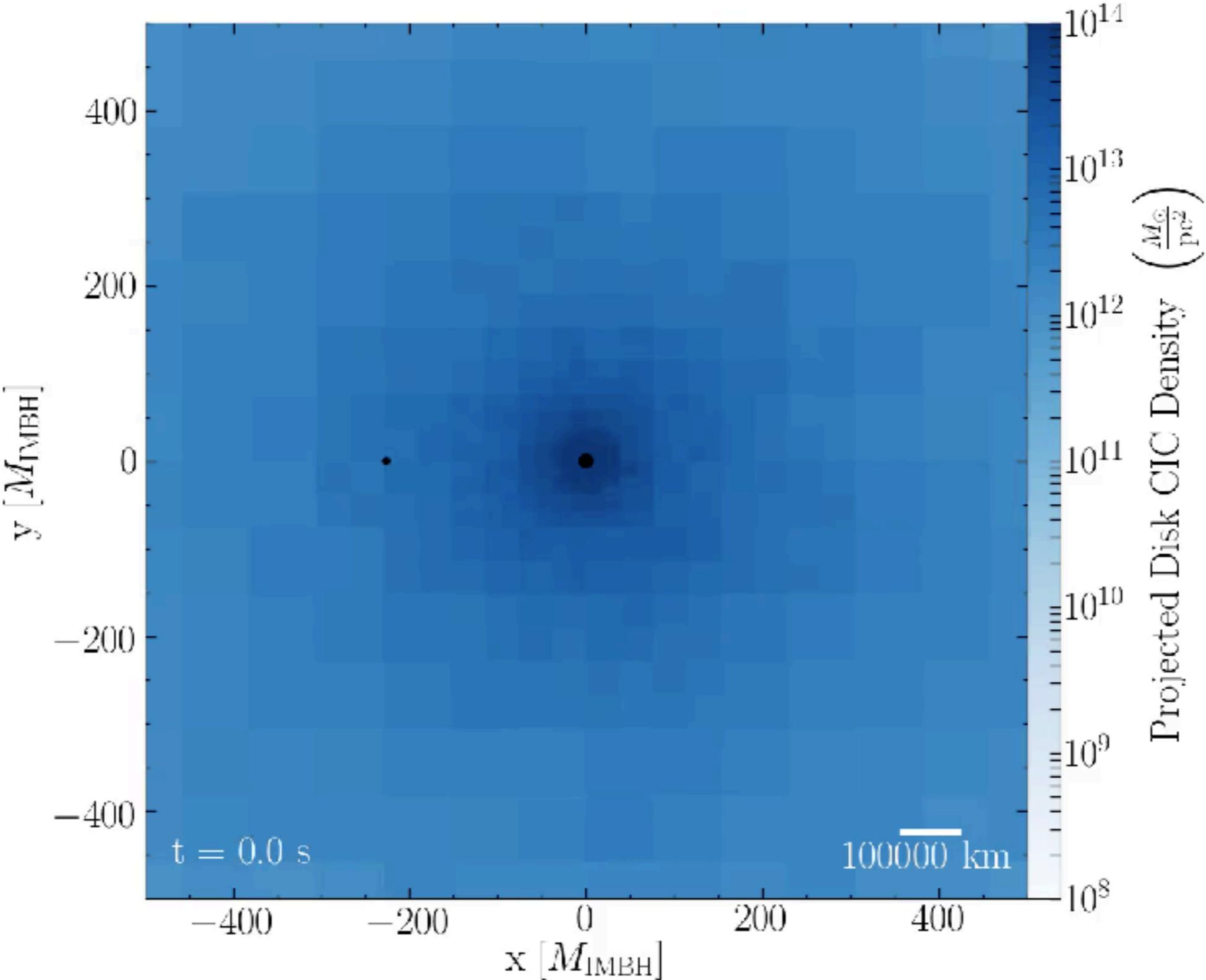
# Dark Matter de-phasing



$$M_{\text{IMBH}} = 1000 M_{\odot}$$

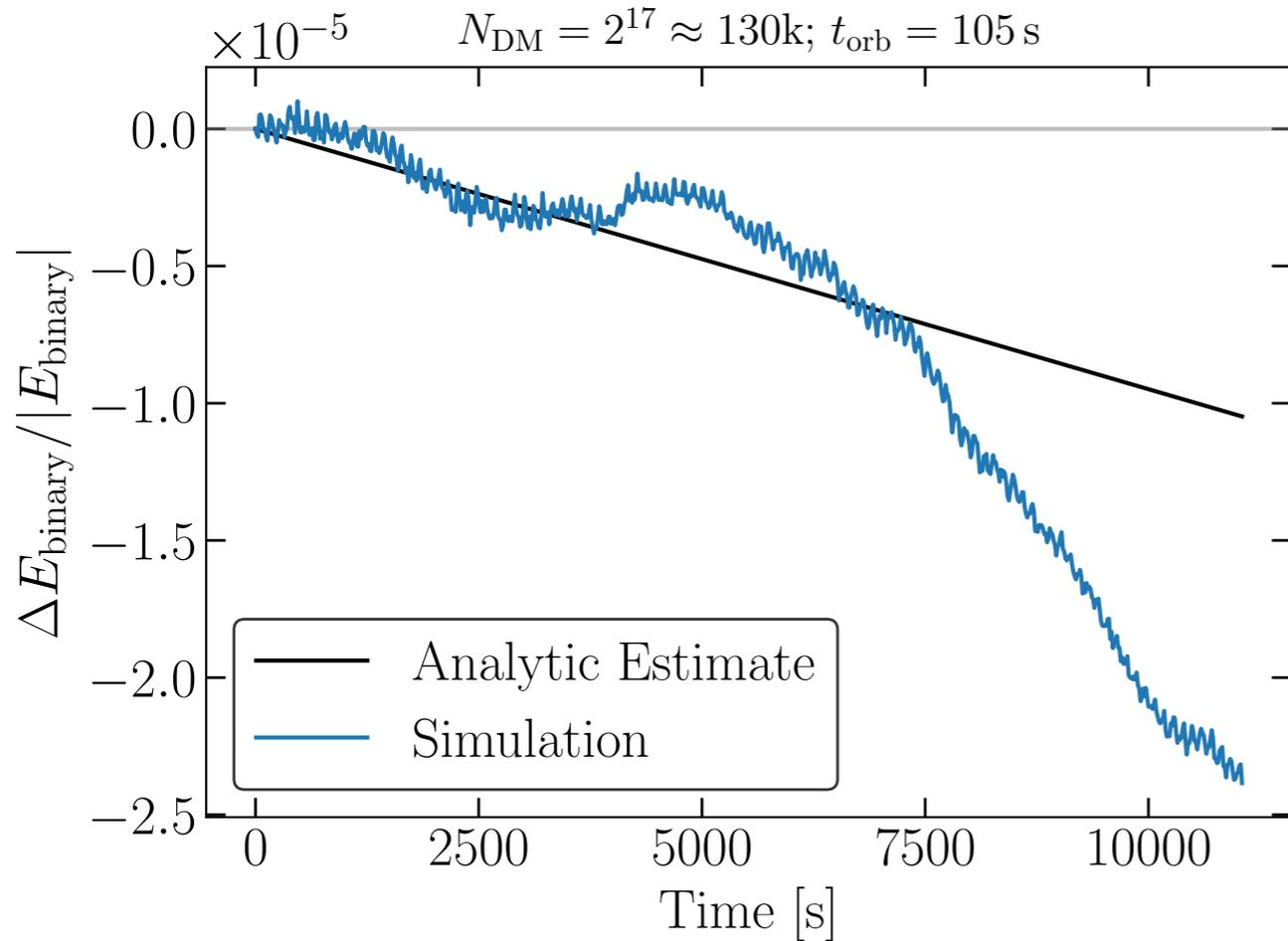
$$\mu = 1 M_{\odot}$$

[Eda et al., 1408.3534]

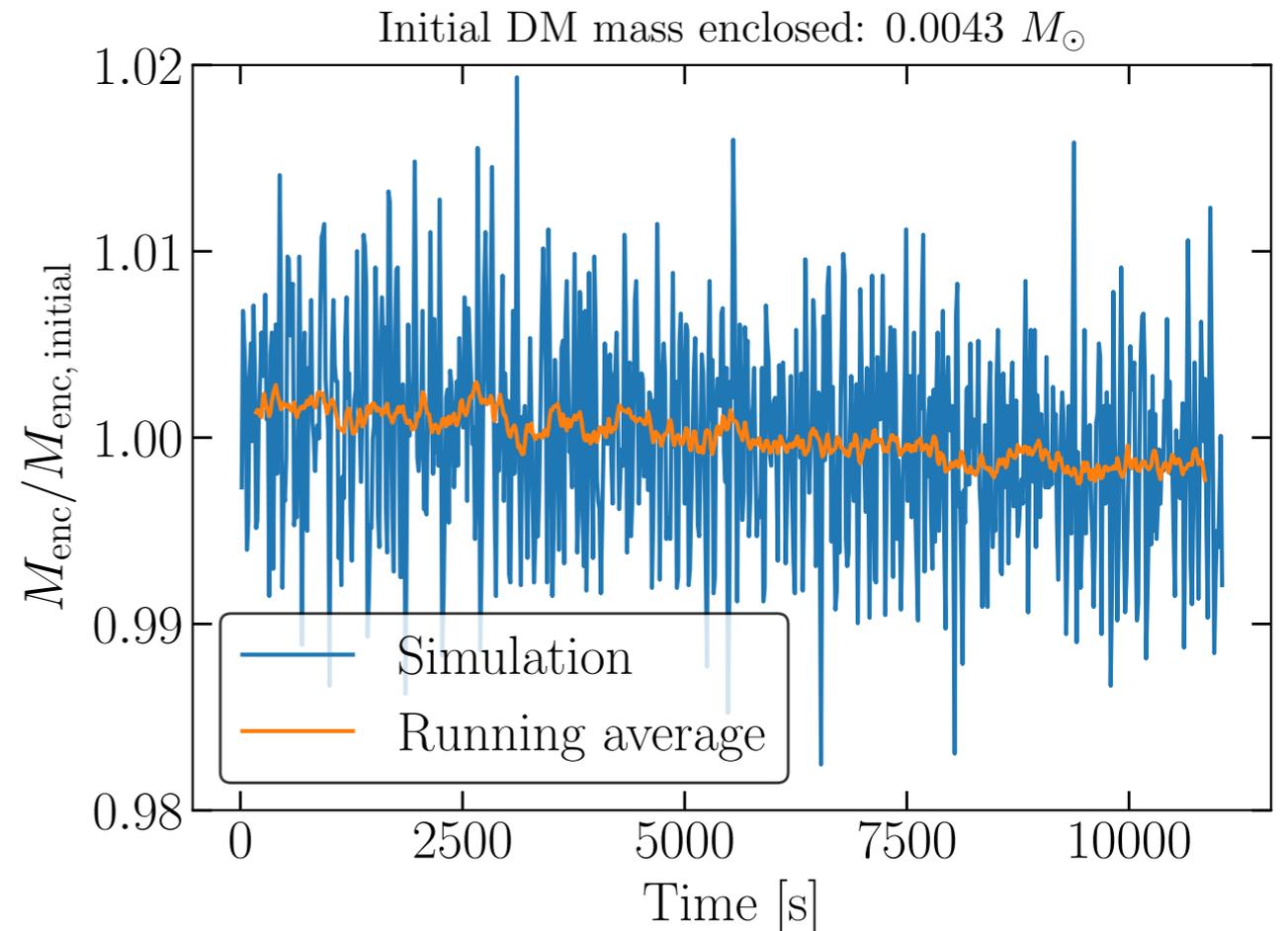


# Preliminary Results

Examine change in binary energy and disruption of DM halo:

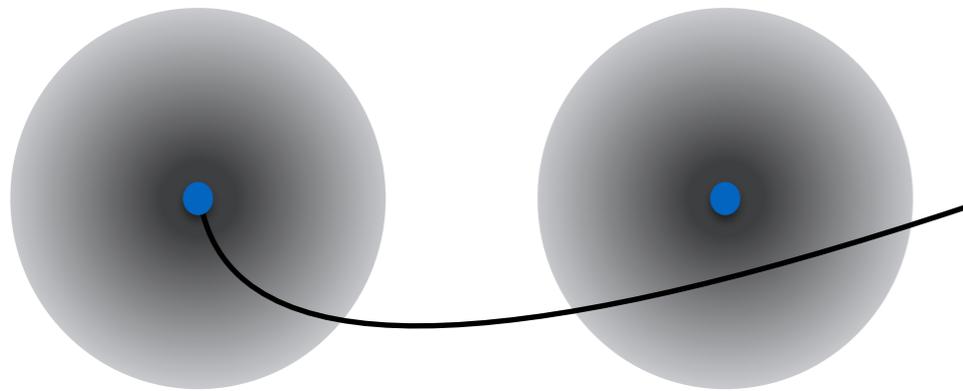


Can we measure these tiny effects over hundreds of thousands of orbits?



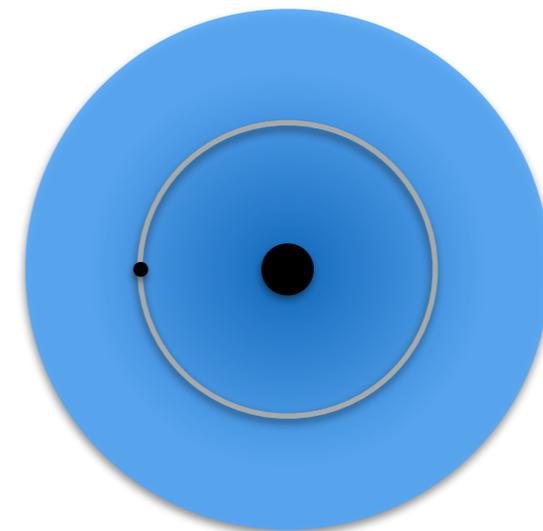
Can we extract the DM halo properties?

Local DM halos affect the size and shape of PBH binaries but (surprisingly) only increase merger rate by a factor of 2



LIGO bounds set the strongest constraints on 10 - 300 Solar Mass PBHs, at the sub-percent level

Can Dark Matter influence other systems and their GW signals?



Movies and code available at [github.com/bradkav/BlackHolesDarkDress](https://github.com/bradkav/BlackHolesDarkDress)

# Backup Slides

# First Remapping

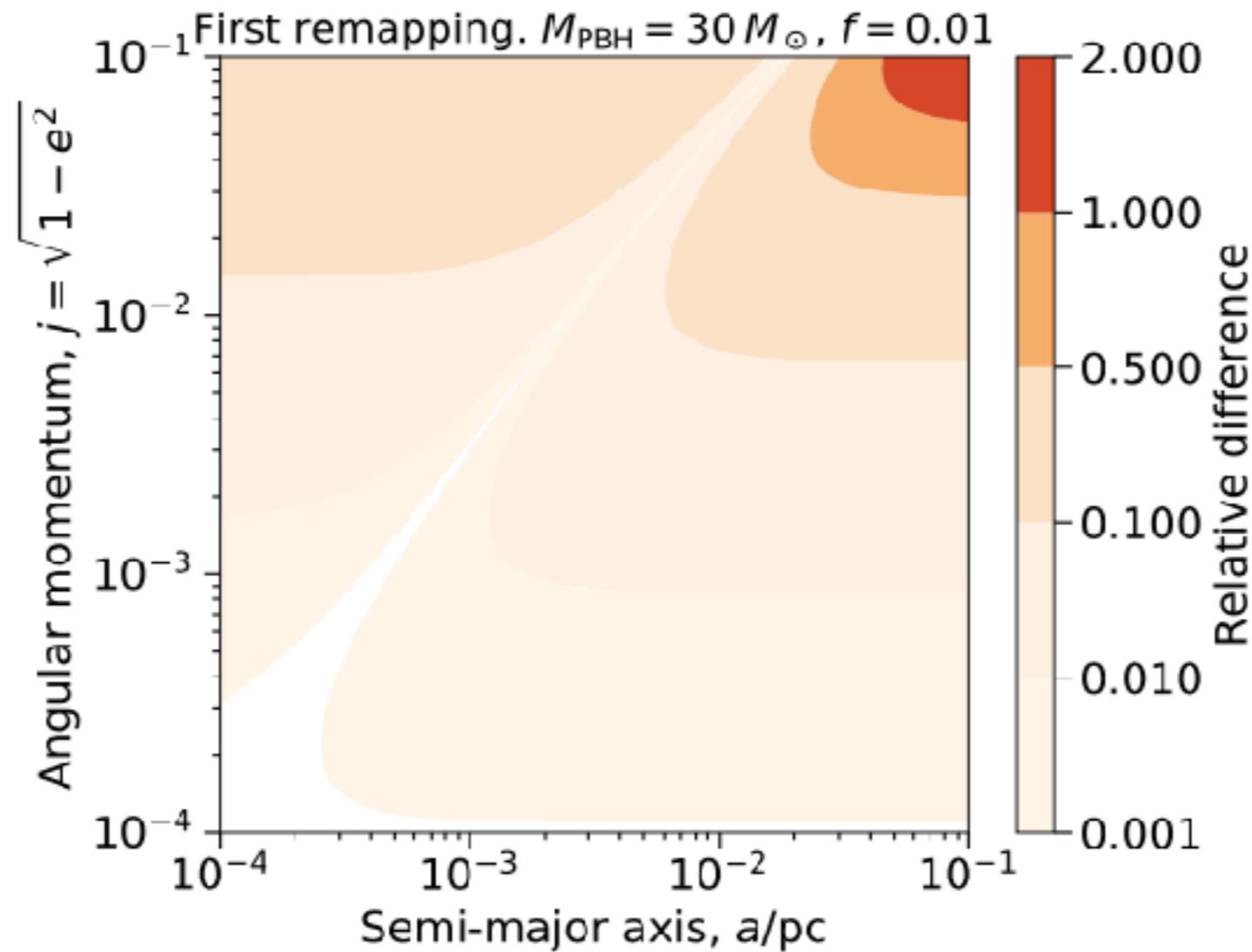
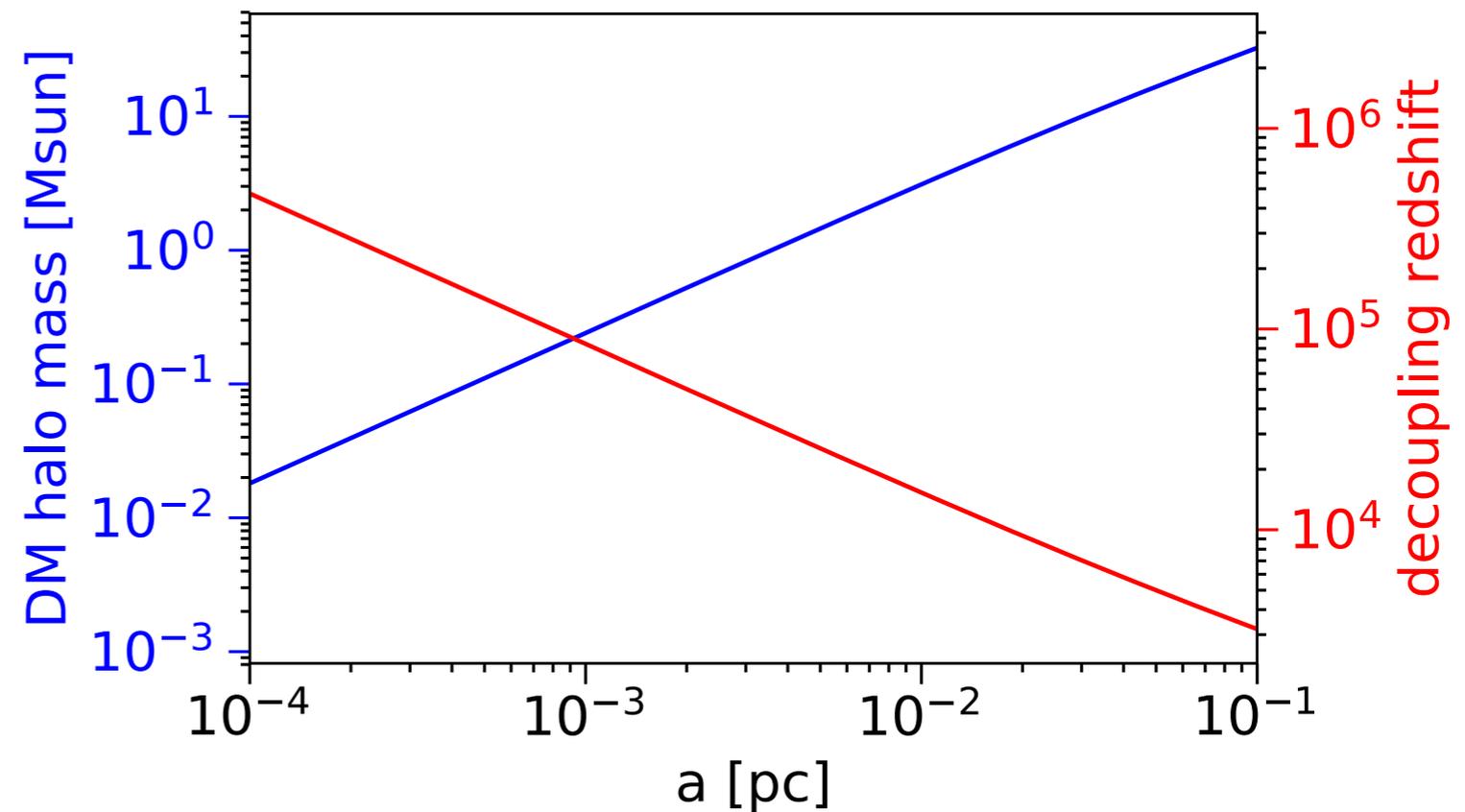
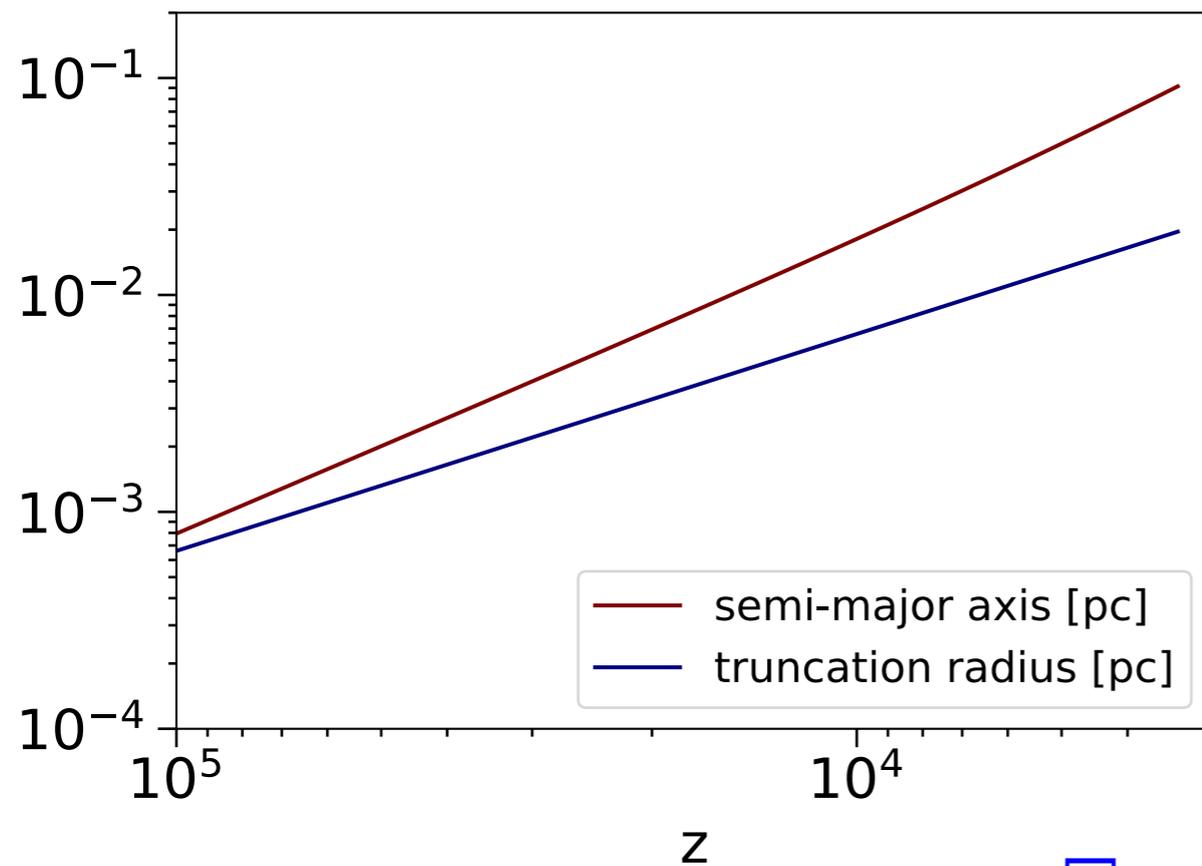


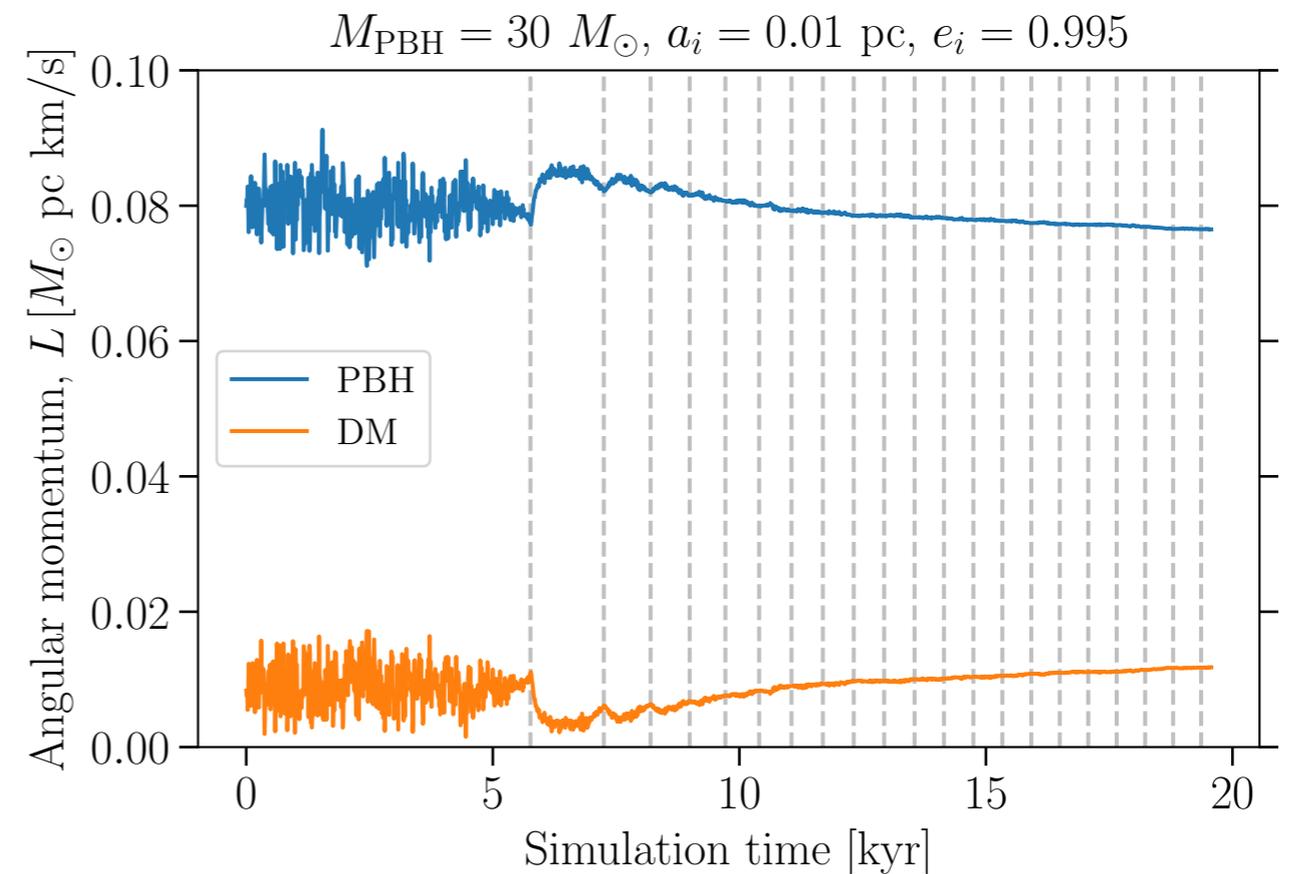
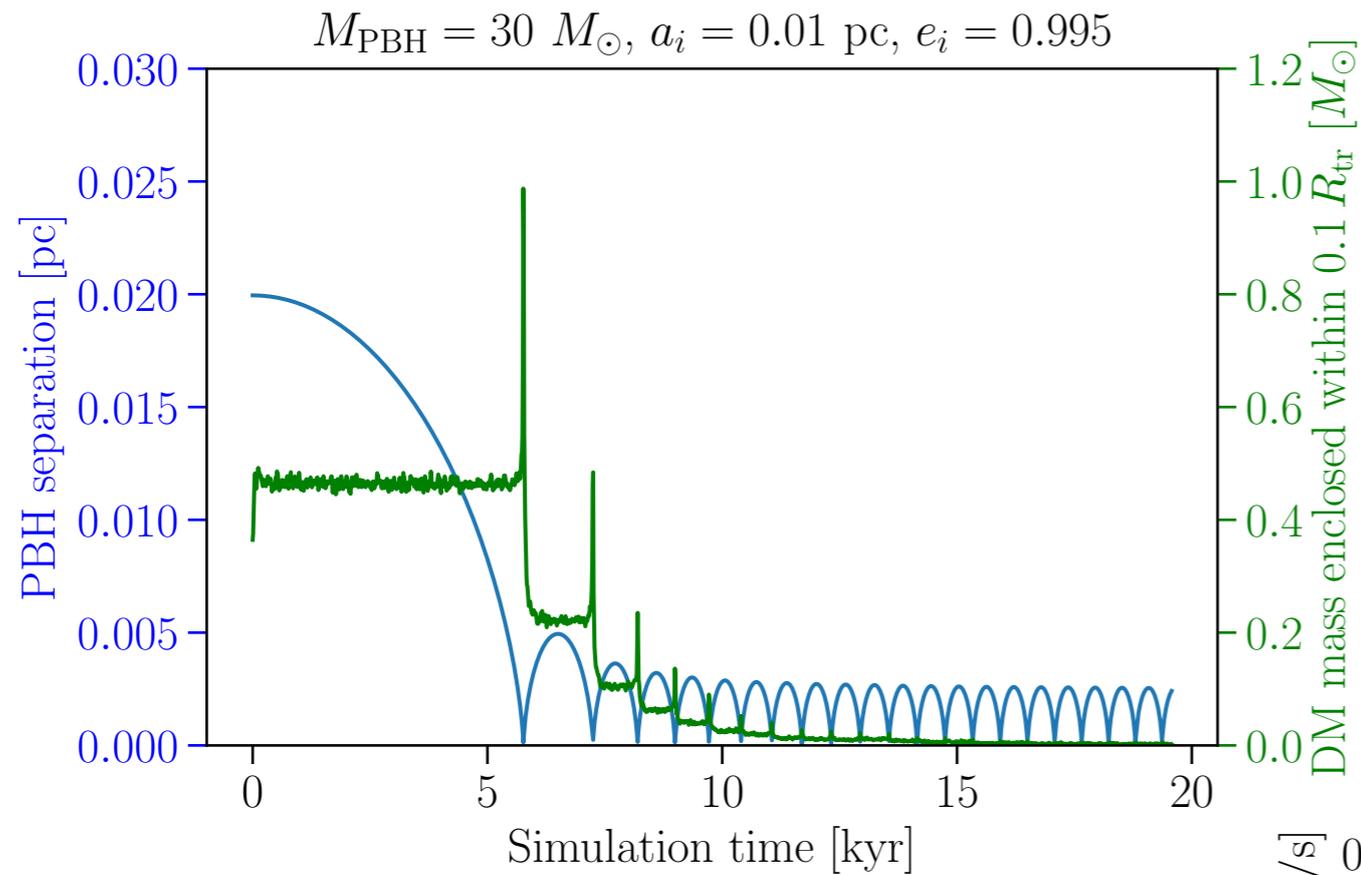
FIG. 5. **First remapping.** We represent the relative difference between the original PDF in the  $(a, j)$  parameter space and the one which takes into account the presence of the mini-halos (as described in Sec. II C). We remark that this initial remapping is mainly based on a rescaling of the mass parameter, and does not take into account the impact of the halos on the BBH orbits, which will be addressed in the next section.

# Decoupling and Halo Mass

$$M_{\text{PBH}} = 30 M_{\odot}$$

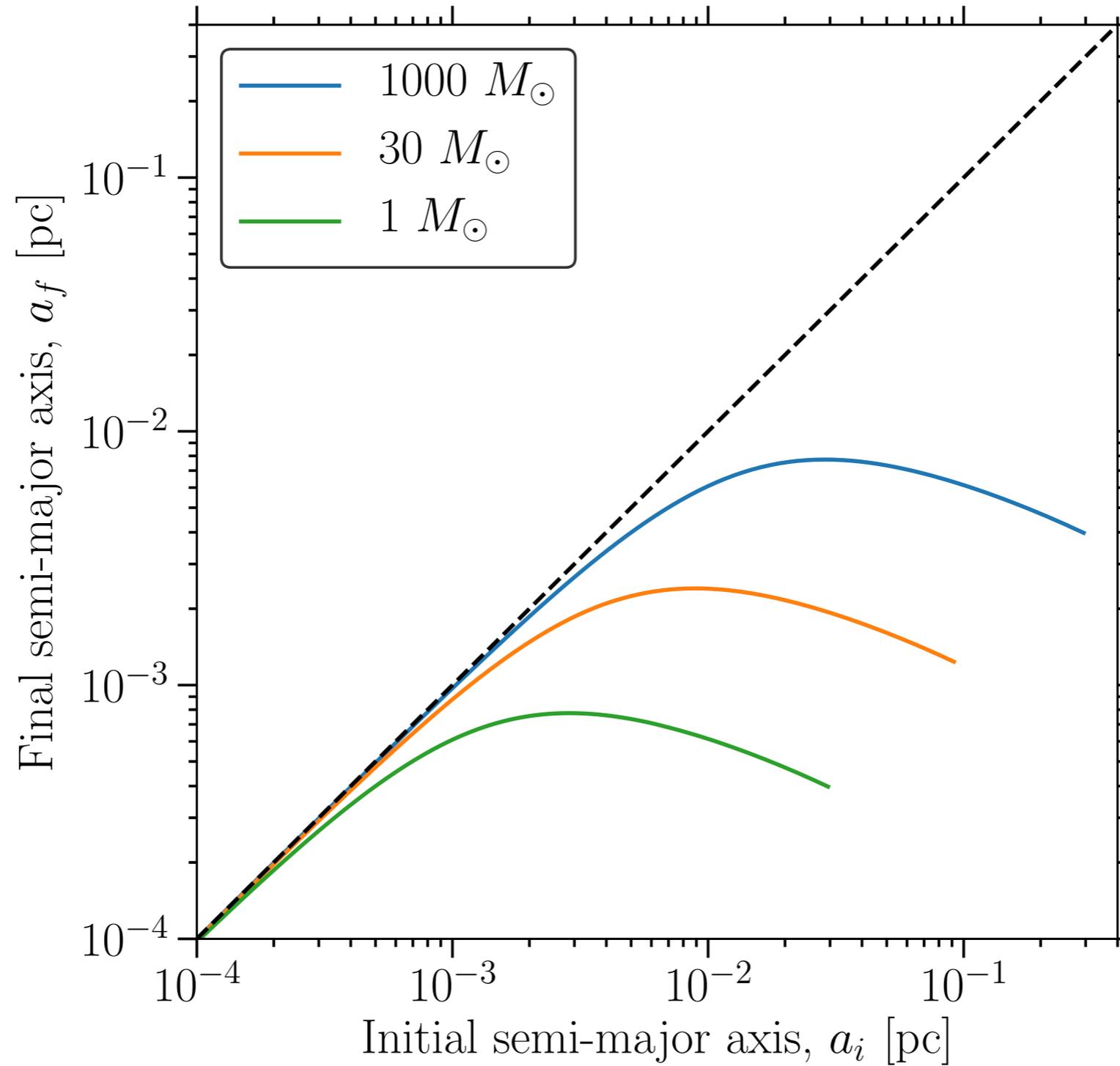


# Binary Evolution



$$j_f = \sqrt{\frac{a_i}{a_f}} j_i \quad \text{for } j \ll 1$$

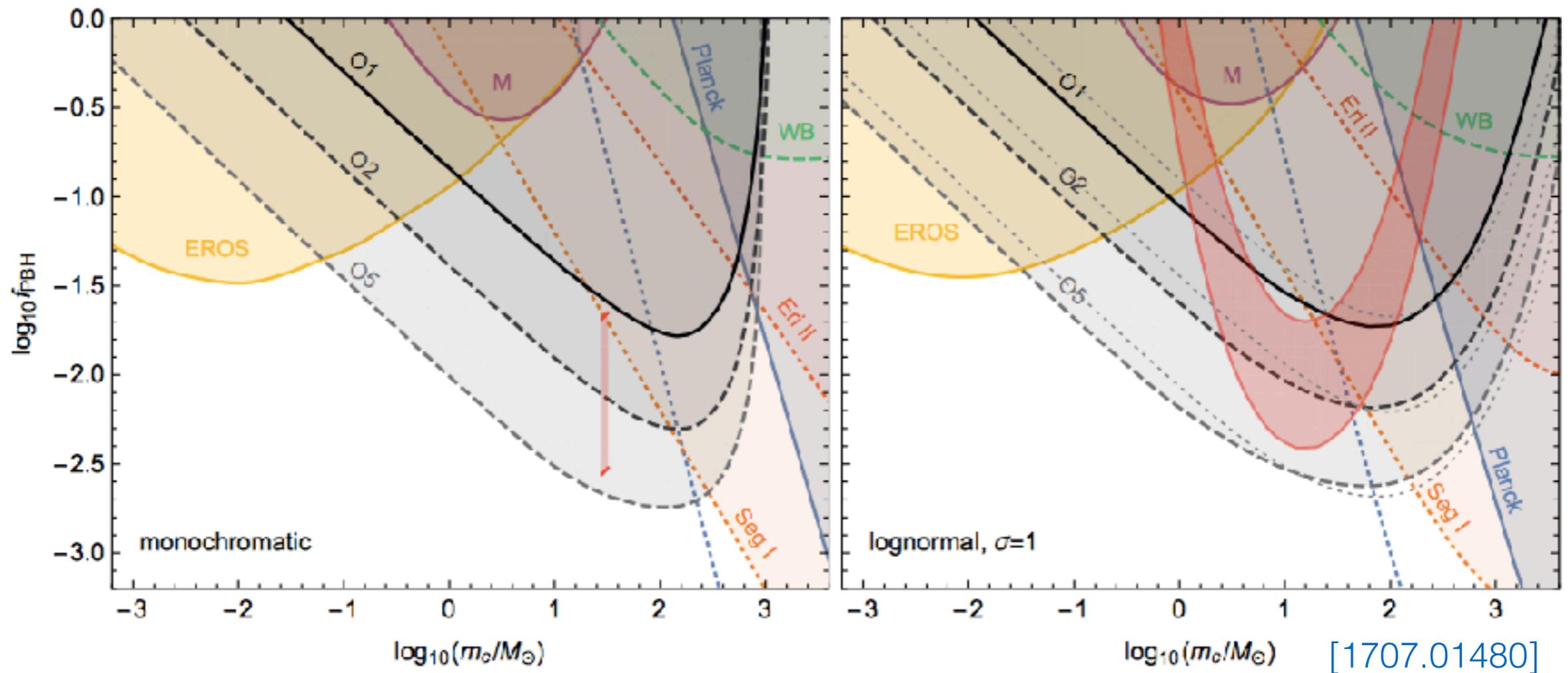
# Remapping the semi-major axis



$$t_f = \sqrt{\frac{a_i}{a_f}} t_i$$

# Extended mass functions

LIGO O1 Limit 



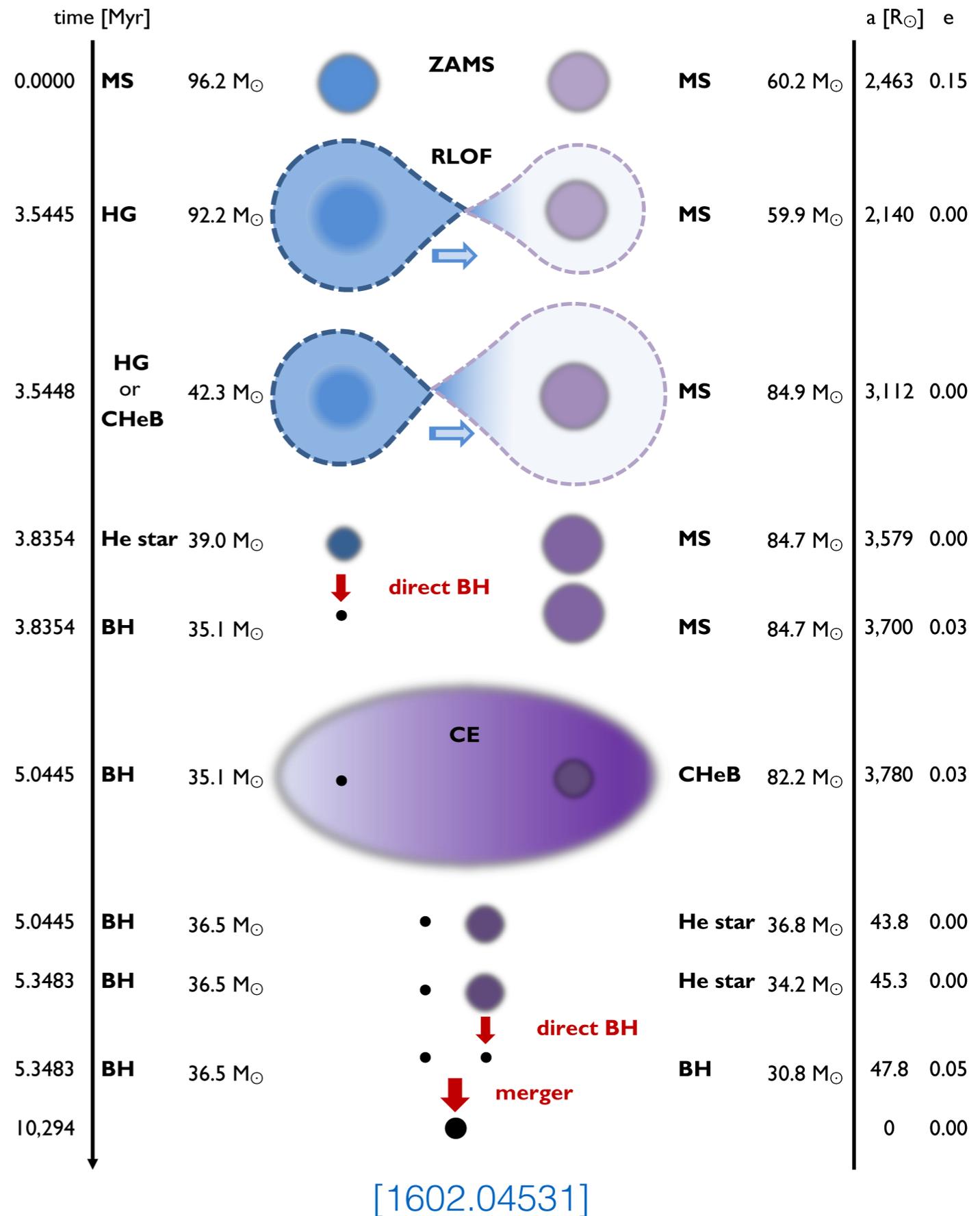
“Old” merger rate calculation *à la* Sasaki et al.,  
but picture doesn’t change too much...

[See also 1801.10327]

# Astrophysical BH binaries

Astrophysical BH binaries could be formed dynamically, or through e.g. common envelope evolution:

[Banerjee, 1611.09357,  
LIGO-Virgo, 1602.03846,  
Elbert et al., 1703.02551,  
Stevenson et al., 1704.01352,  
and many others...]



# Simulation Details

|  |                    |                |                    |
|--|--------------------|----------------|--------------------|
| <code>ErrTolForceAcc</code>              | $10^{-5}$          |                |                    |
| <code>ErrTolIntAccuracy</code>           | $10^{-3}$          |                |                    |
| <code>MaxTimestep</code> [yr]            | $10^{-2}$          |                |                    |
| $\ell_{\text{soft}}$ (PBH) [pc]          | $10^{-7}$          |                |                    |
| $M_{\text{PBH}} =$                       | $1 M_{\odot}$      | $30 M_{\odot}$ | $1000 M_{\odot}$   |
| $\ell_{\text{soft}}$ (DM, low-res) [pc]  | $2 \times 10^{-6}$ | $10^{-5}$      | $5 \times 10^{-5}$ |
| $\ell_{\text{soft}}$ (DM, high-res) [pc] | $2 \times 10^{-7}$ | $10^{-6}$      | $5 \times 10^{-6}$ |

TABLE I. **Summary of Gadget-2 parameters.** The parameters `ErrTolForceAcc` and `ErrTolIntAccuracy` control the accuracy of force calculation and time integration respectively. We also specify the softening lengths  $\ell_{\text{soft}}$  of the simulations, as described in the text. Low-resolution simulations contain roughly  $10^4$  DM particles per halo, while high-resolution simulations use a multi-mass scheme with roughly  $4 \times 10^4$  DM particles in total per halo.

For ‘high-resolution’ simulations, we use a multi-mass scheme in which the DM halo is composed of 4 different masses of pseudo-particles.

Each simulation takes  $\sim 3000$  CPU-hours, with very poor scaling with  $N_{\text{CPU}}$