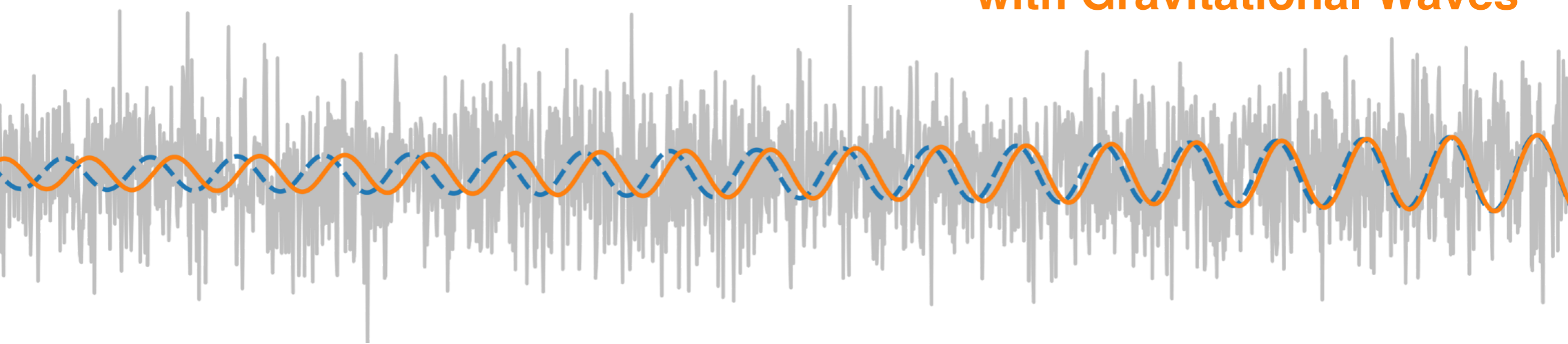


# Black Holes' Dark Dress

Detecting Particle Dark Matter around Primordial Black Holes  
with Gravitational Waves



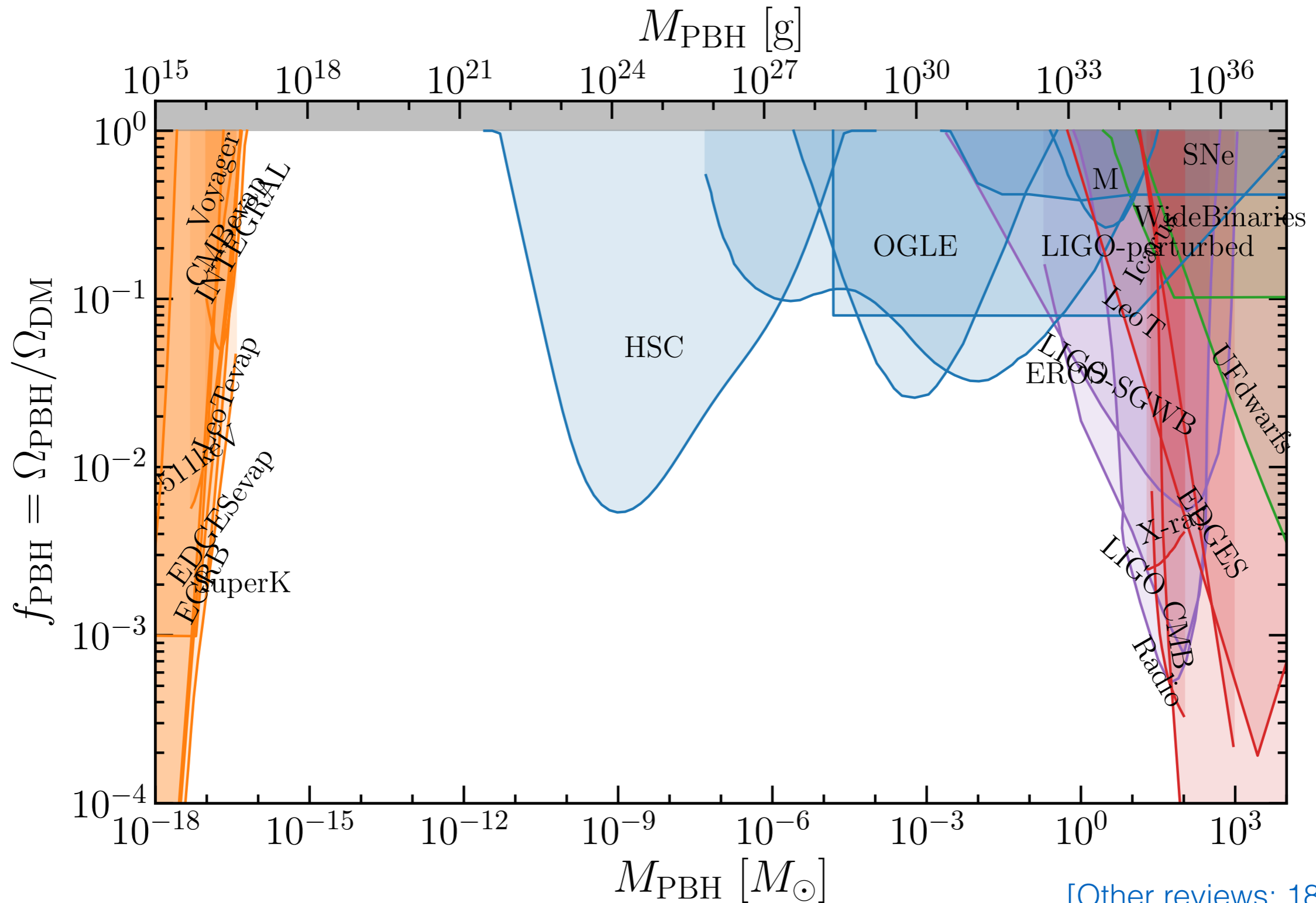
Bradley J Kavanagh  
(IFCA, UC-CSIC)

NEHOP, Naples, 20th June 2023

# PBH Constraints

[Green & **BJK**, 2007.10722]

[Code online: [github.com/bradkav/PBHbounds](https://github.com/bradkav/PBHbounds)]

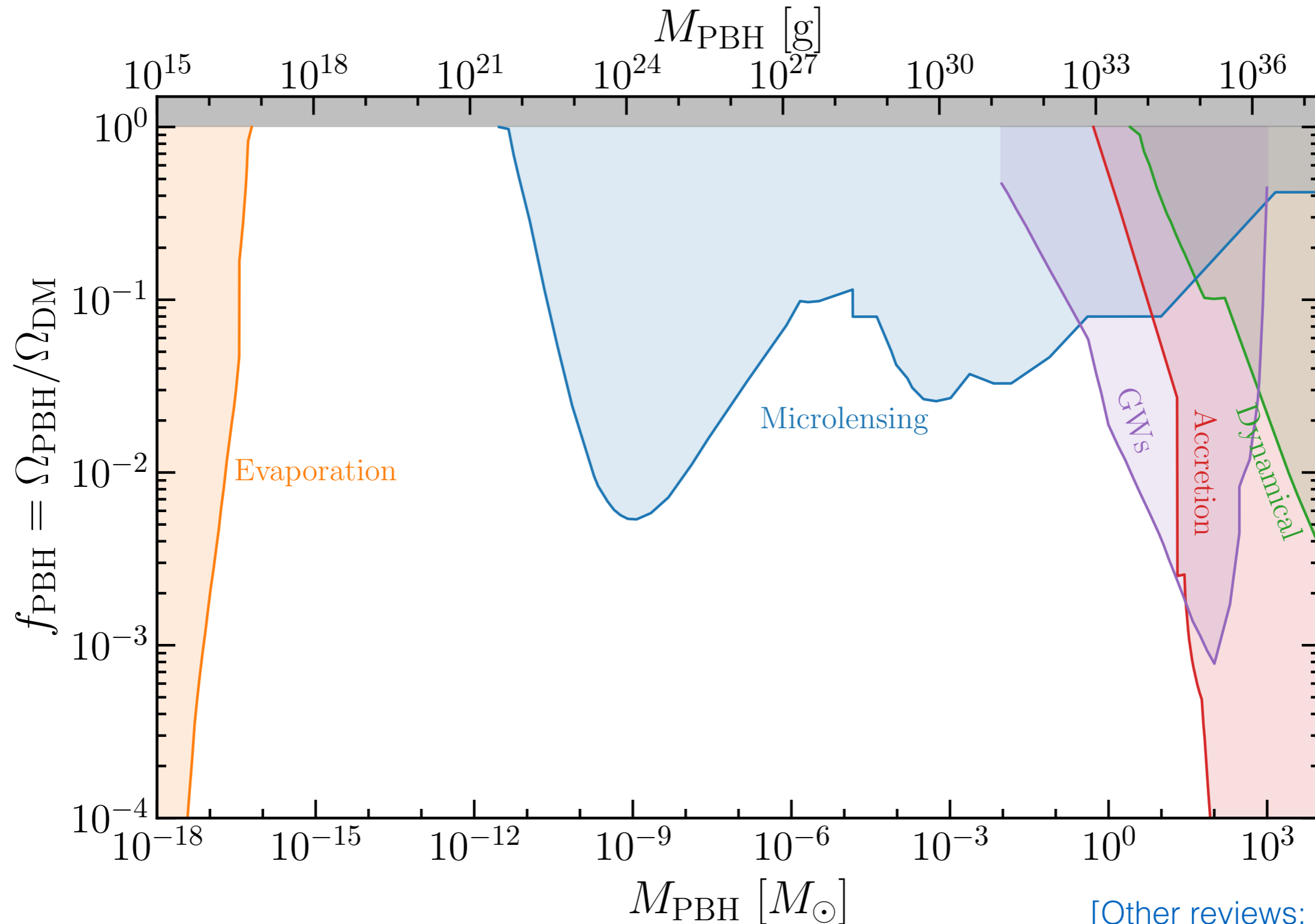


[Other reviews: [1801.05235](#),  
[2002.12778](#), [2006.02838](#)]

# PBH Constraints

[Green & **BJK**, 2007.10722]

[Code online: [github.com/bradkav/PBHbounds](https://github.com/bradkav/PBHbounds)]

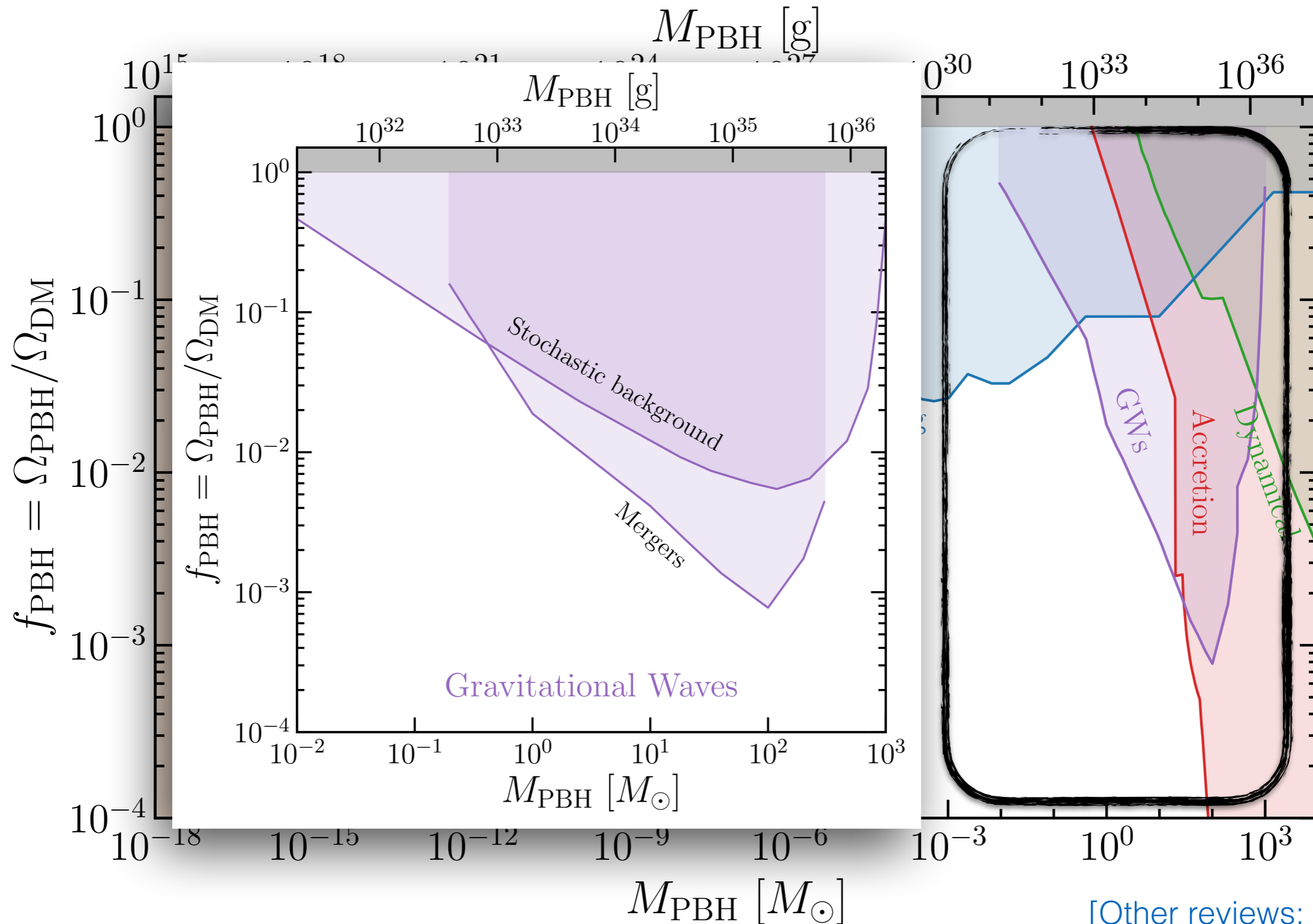


[Other reviews: [1801.05235](#),  
[2002.12778](#), [2006.02838](#)]

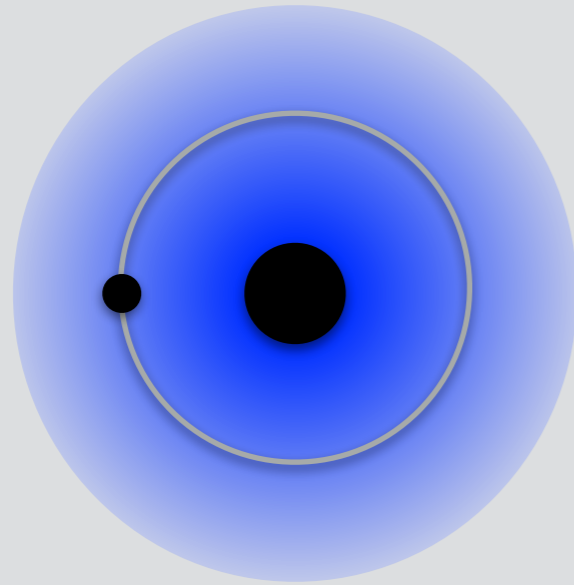
# PBH Constraints

[Green & **BJK**, 2007.10722]

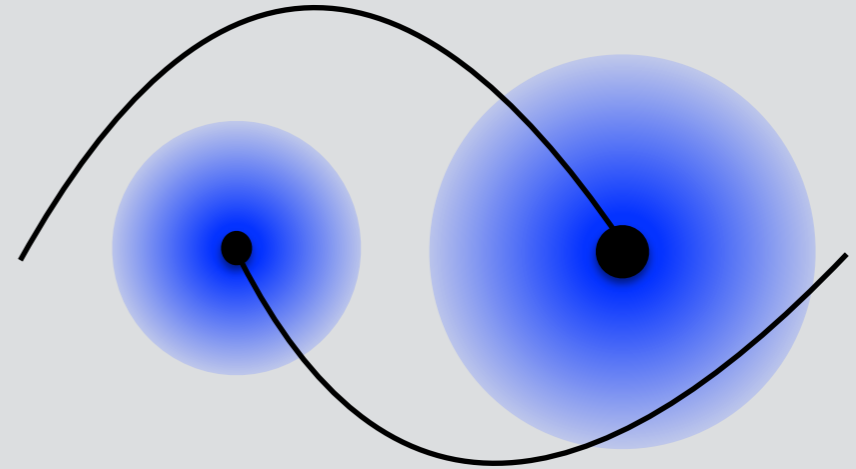
[Code online: [github.com/bradkav/PBHbounds](https://github.com/bradkav/PBHbounds)]



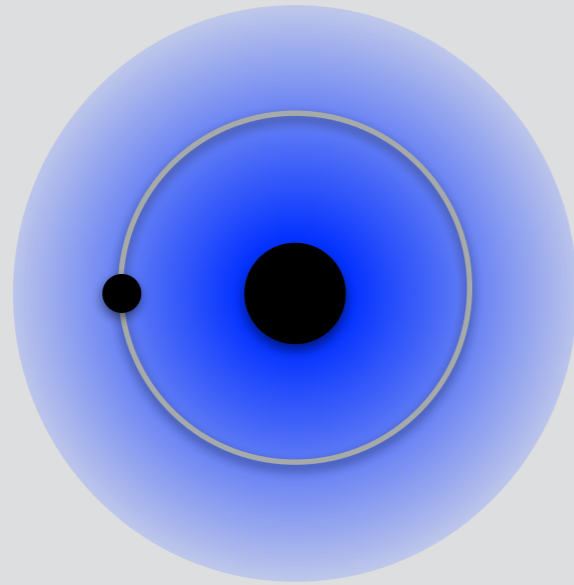
[Other reviews: [1801.05235](#), [2002.12778](#), [2006.02838](#)]



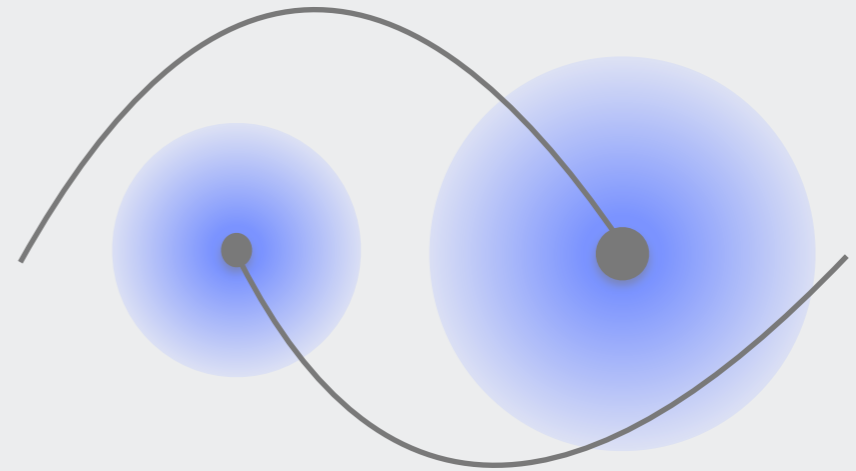
**Dark Matter  
'de-phasing' of  
GW signals**



**Merger rates  
of dressed  
PBH Binaries**



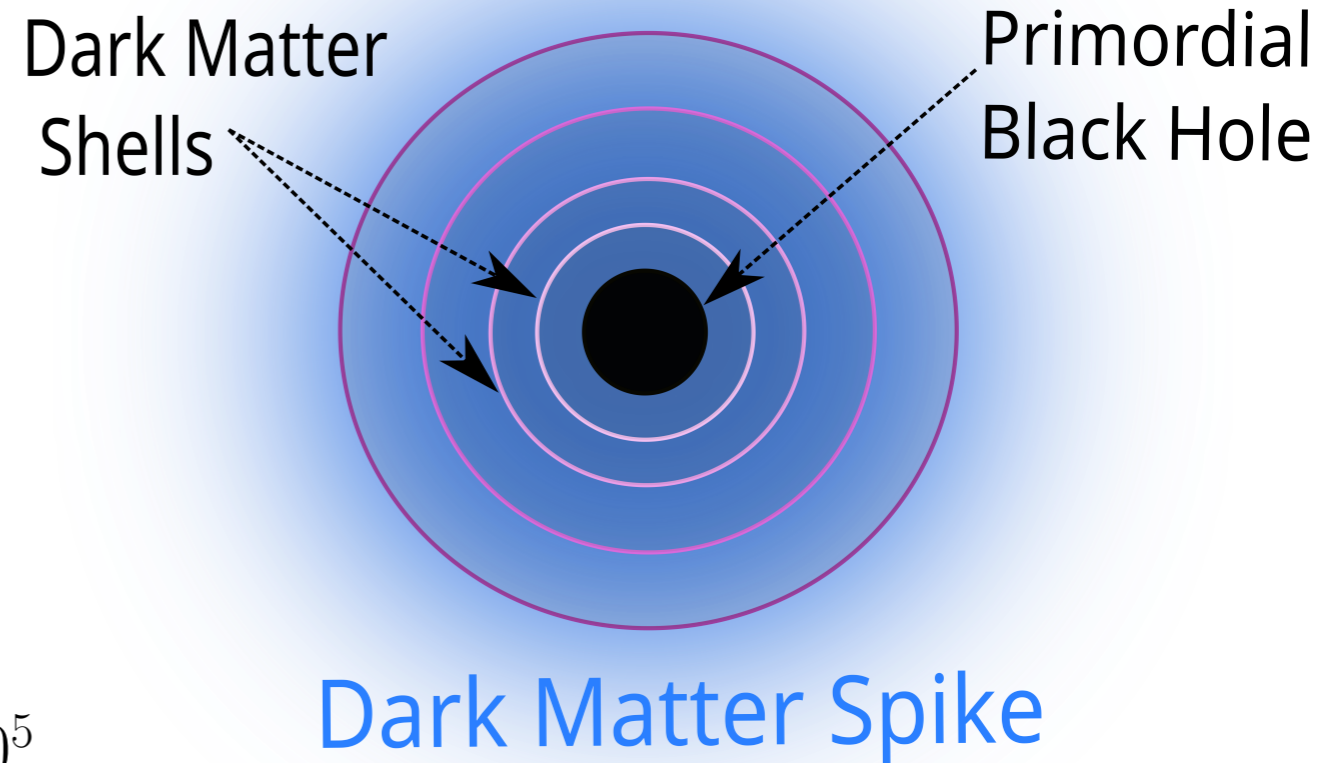
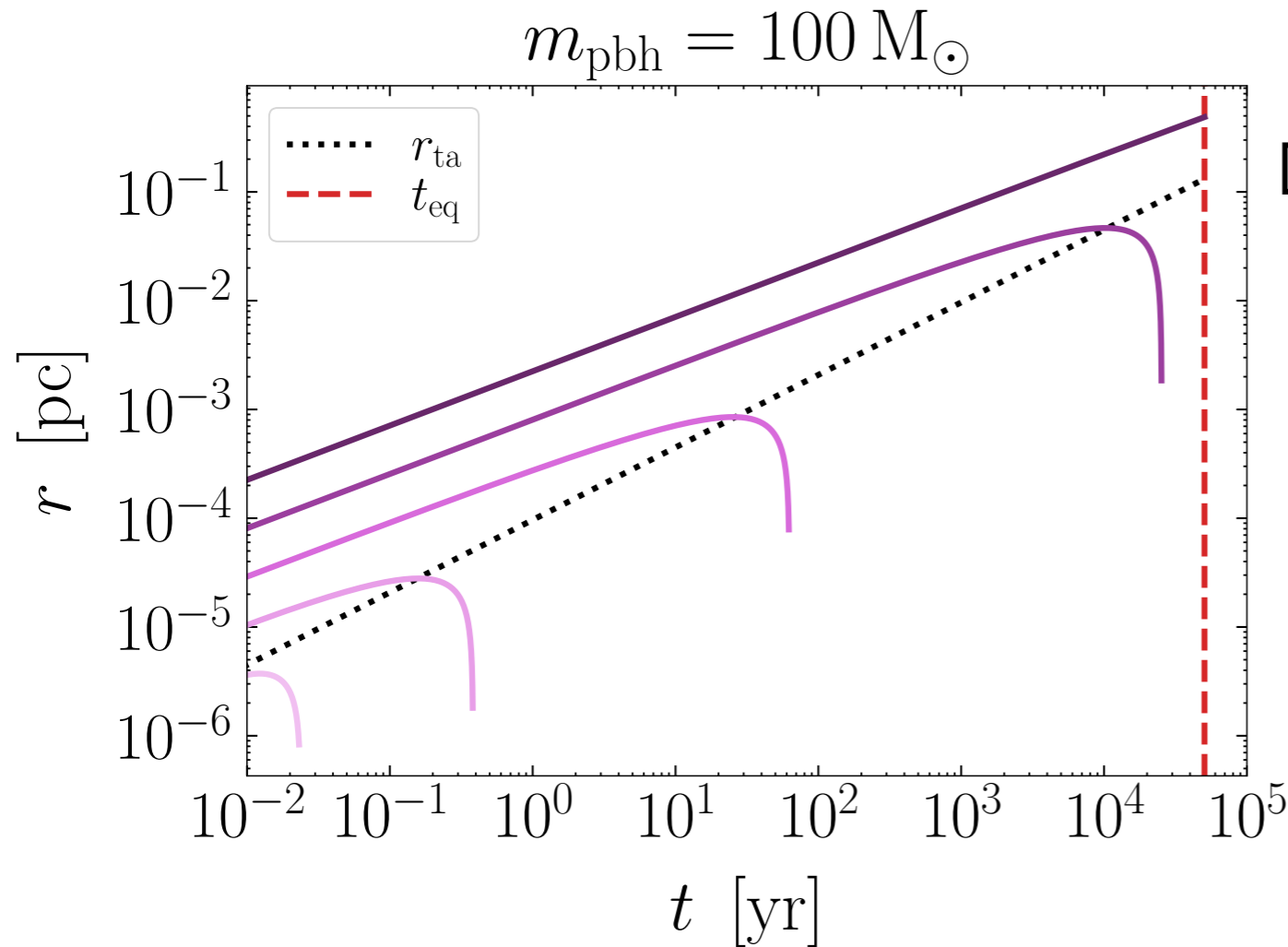
**Dark Matter  
'de-phasing' of  
GW signals**



**Merger rates  
of dressed  
PBH Binaries**

# Particle DM Spikes

[Mack et al., [astro-ph/0608642](#);  
Ricotti, [0706.0864](#),  
Sten Delos et al., [1712.05421](#),  
Boudaud et al., [2106.07480](#), ...]



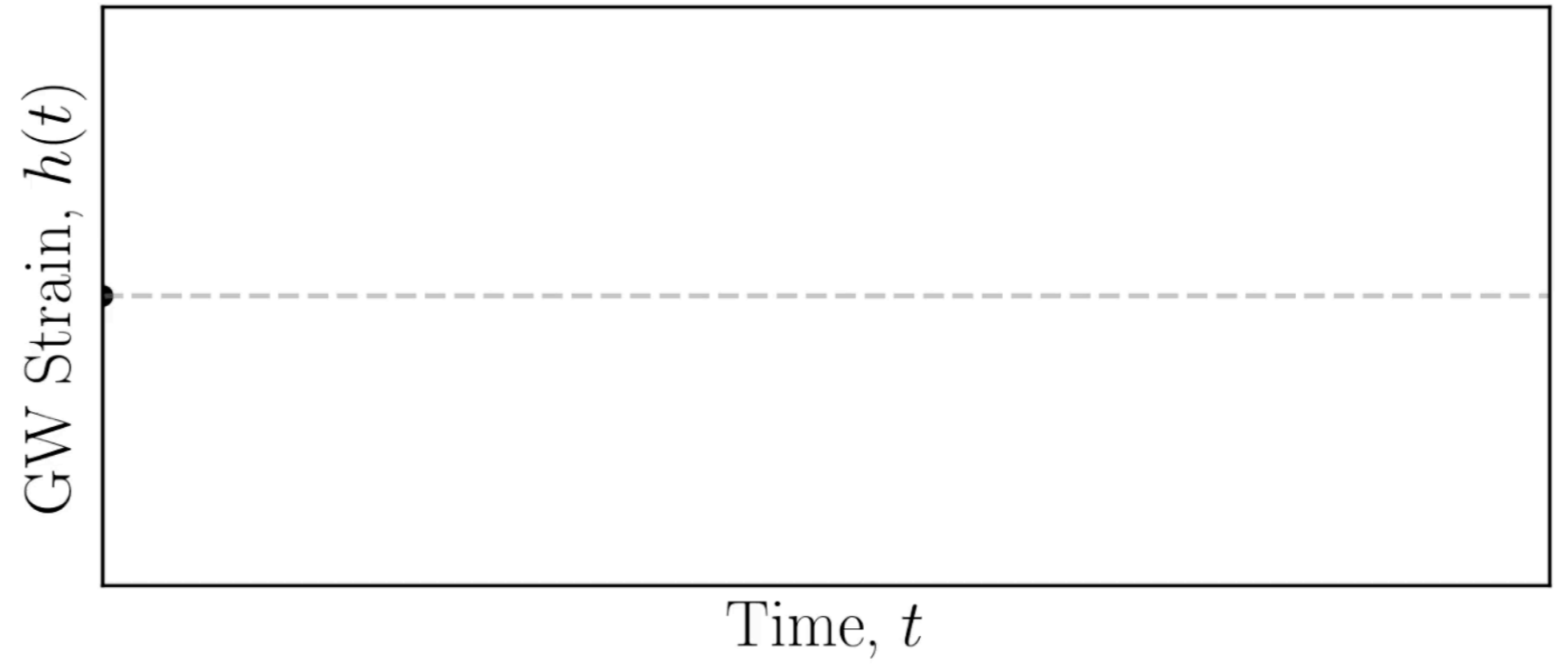
[Jangra, **BJK**, Diego, [2304.05892](#)]

Spike grows to  $M_{\text{spike}} \sim M_{\text{PBH}}$  by matter-radiation equality.

Density profile  $\rho_{\text{DM}}(r) \sim r^{-9/4}$ , can achieve densities up to  $\rho_{\text{DM}} \gtrsim 10^{20} M_{\odot}/\text{pc}^3$

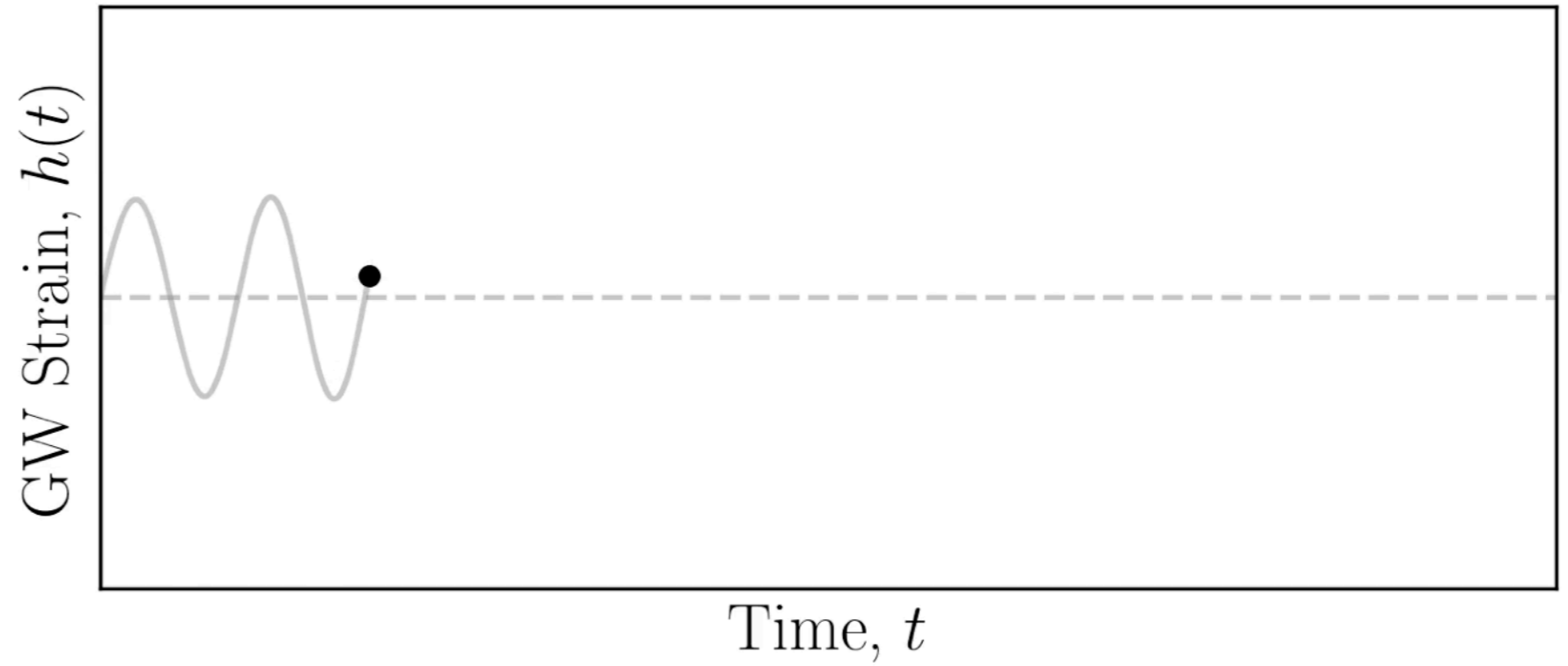
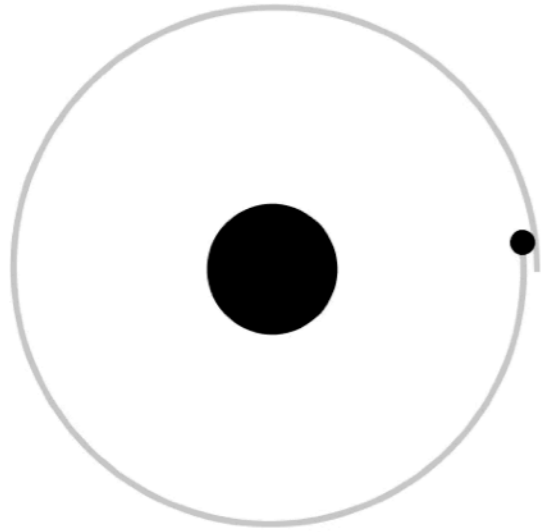
# Dark Matter “De-phasing”

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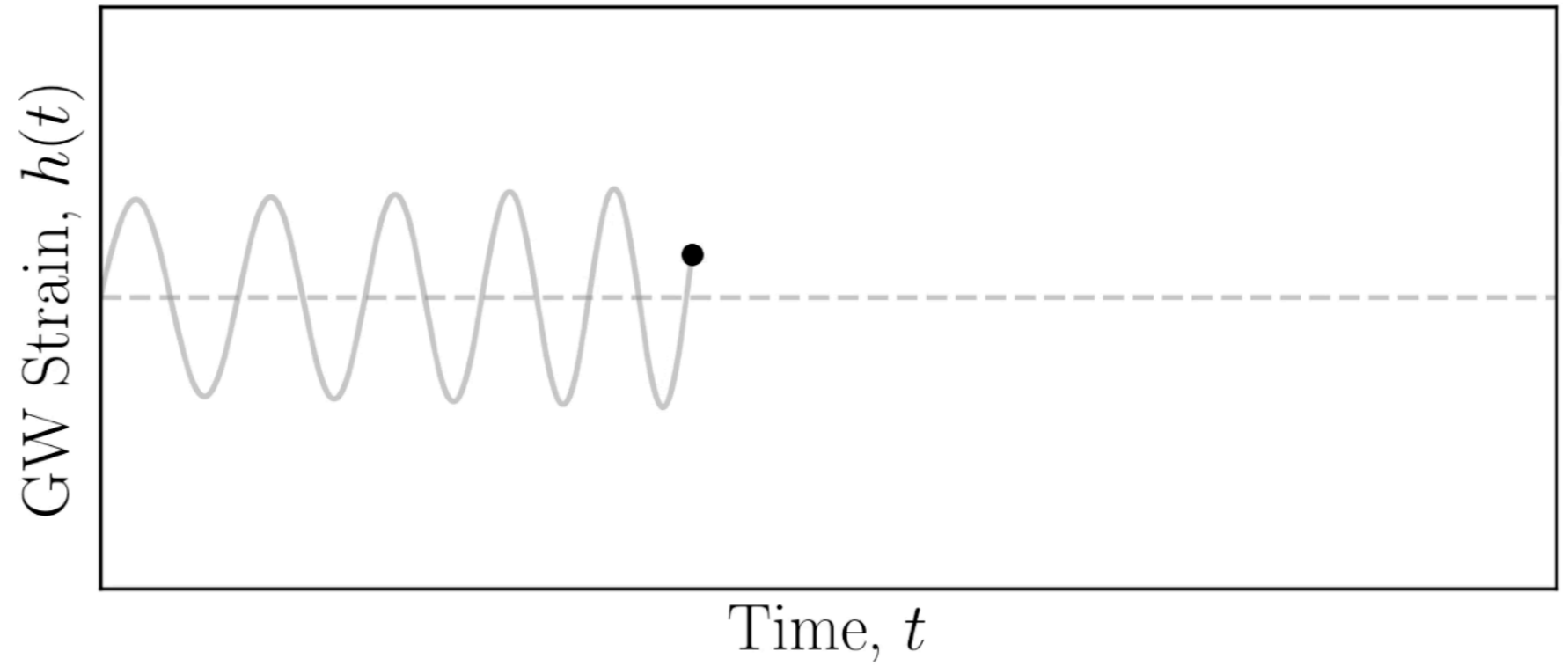
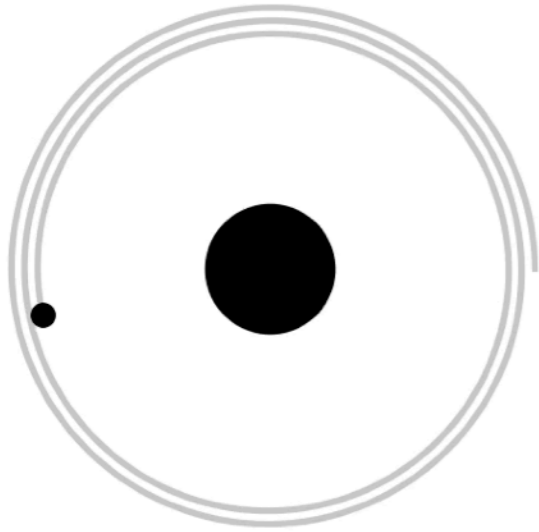




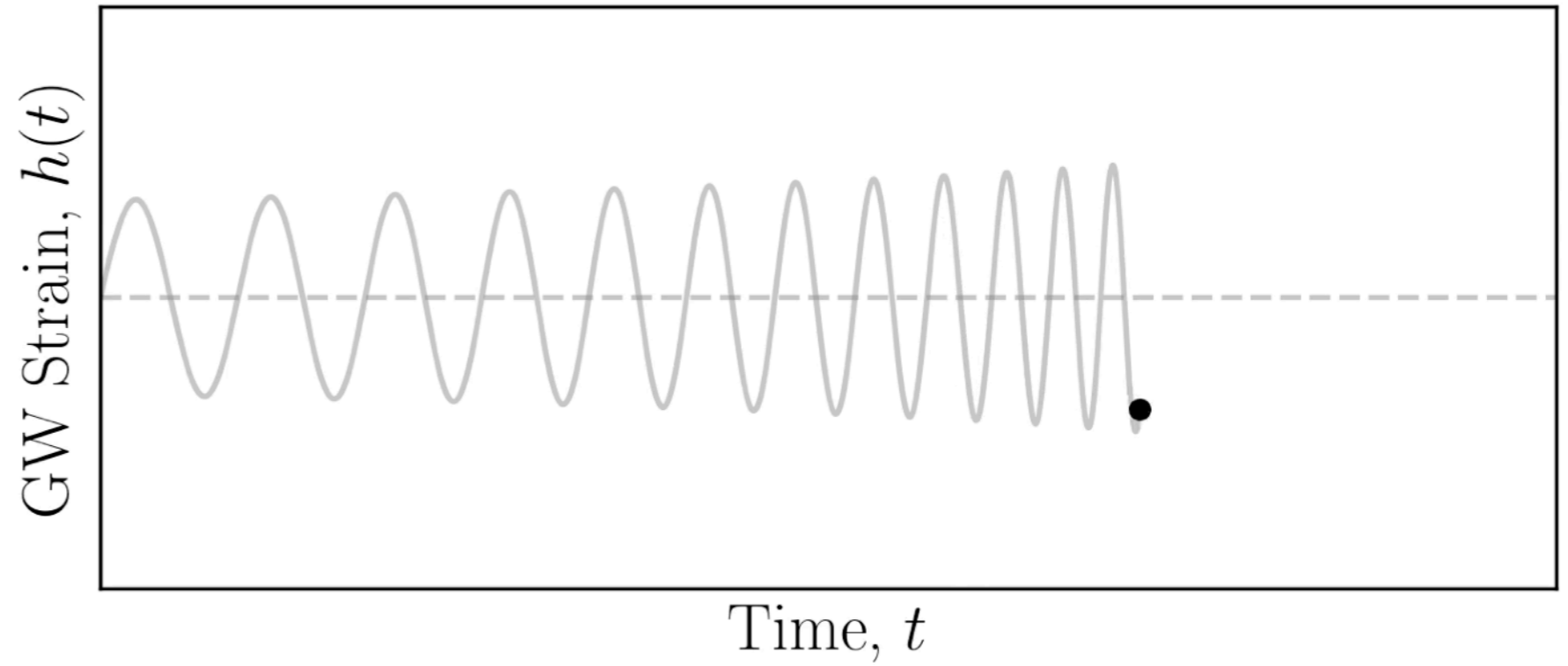
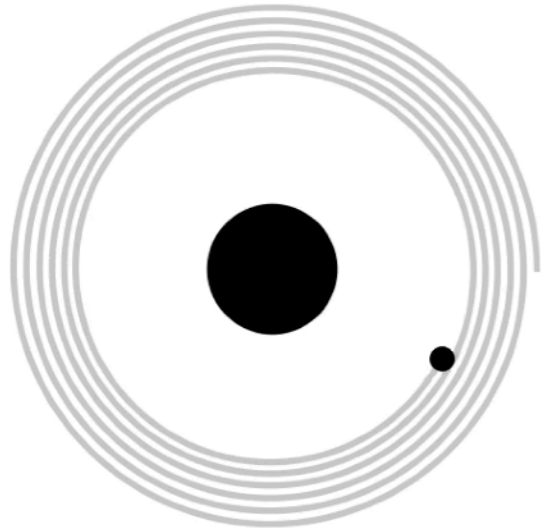
# Dark Matter “De-phasing”



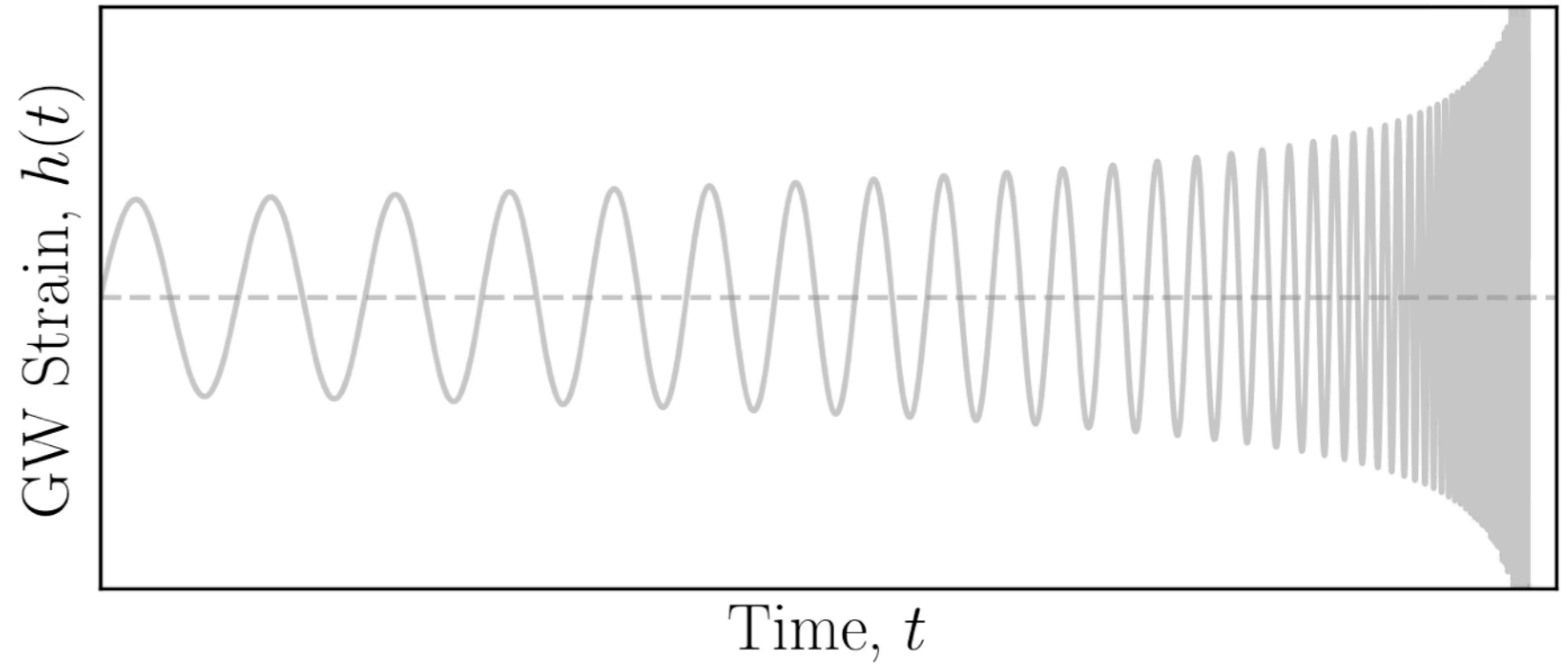
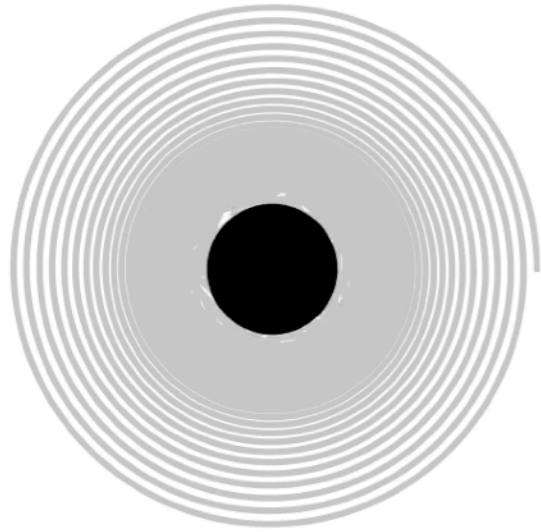
# Dark Matter “De-phasing”



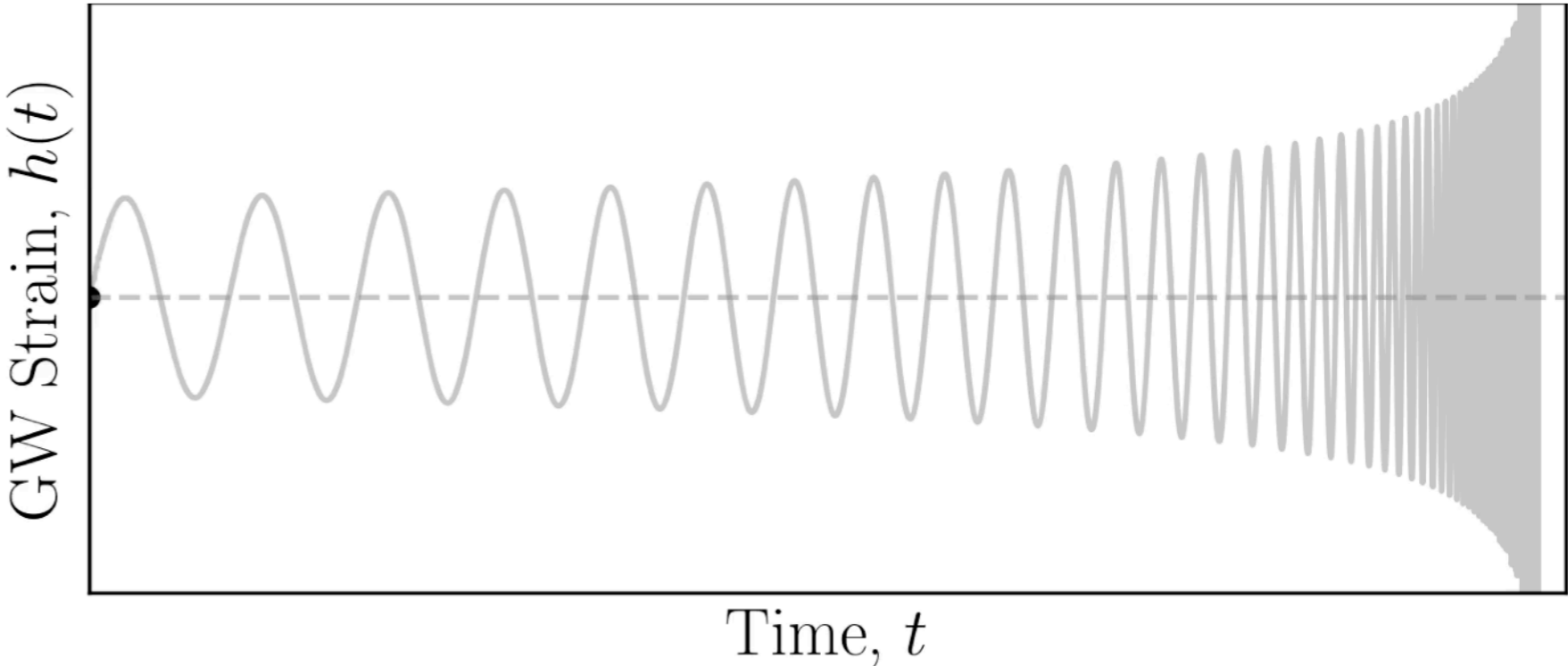
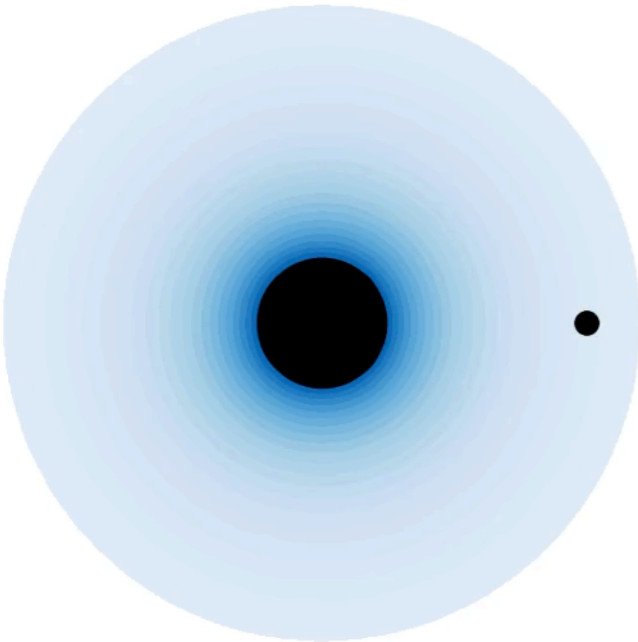
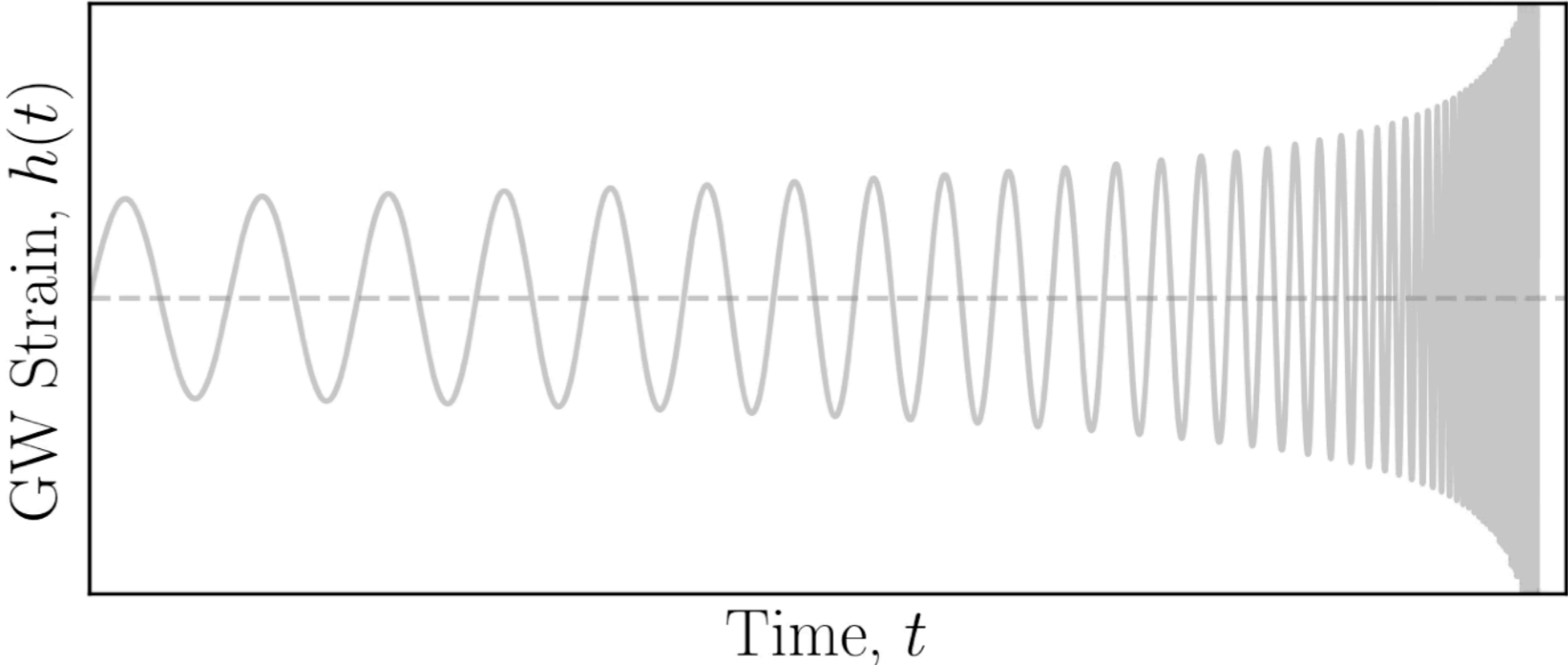
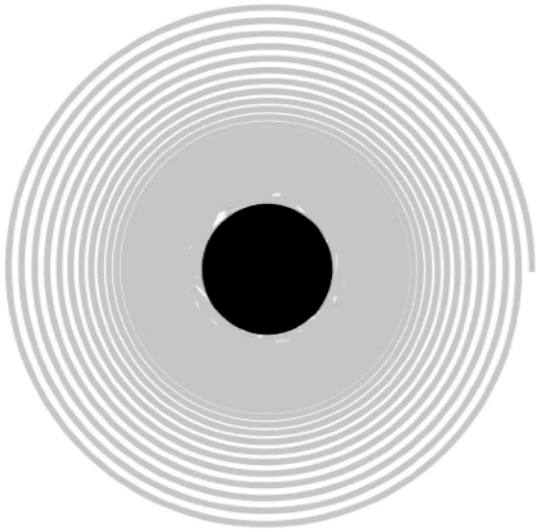
# Dark Matter “De-phasing”



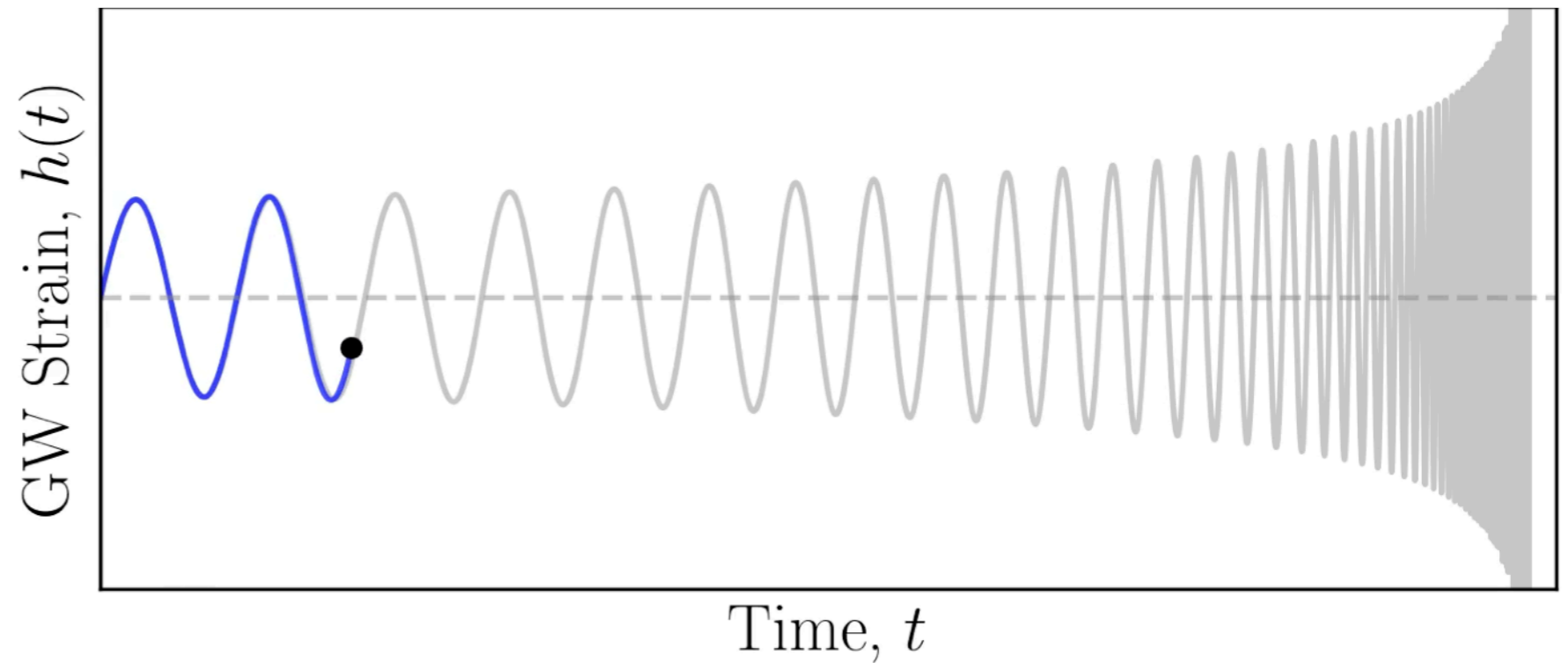
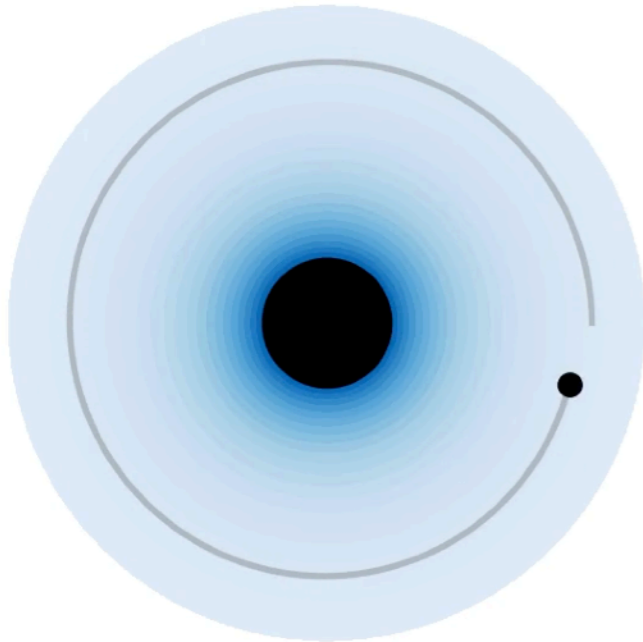
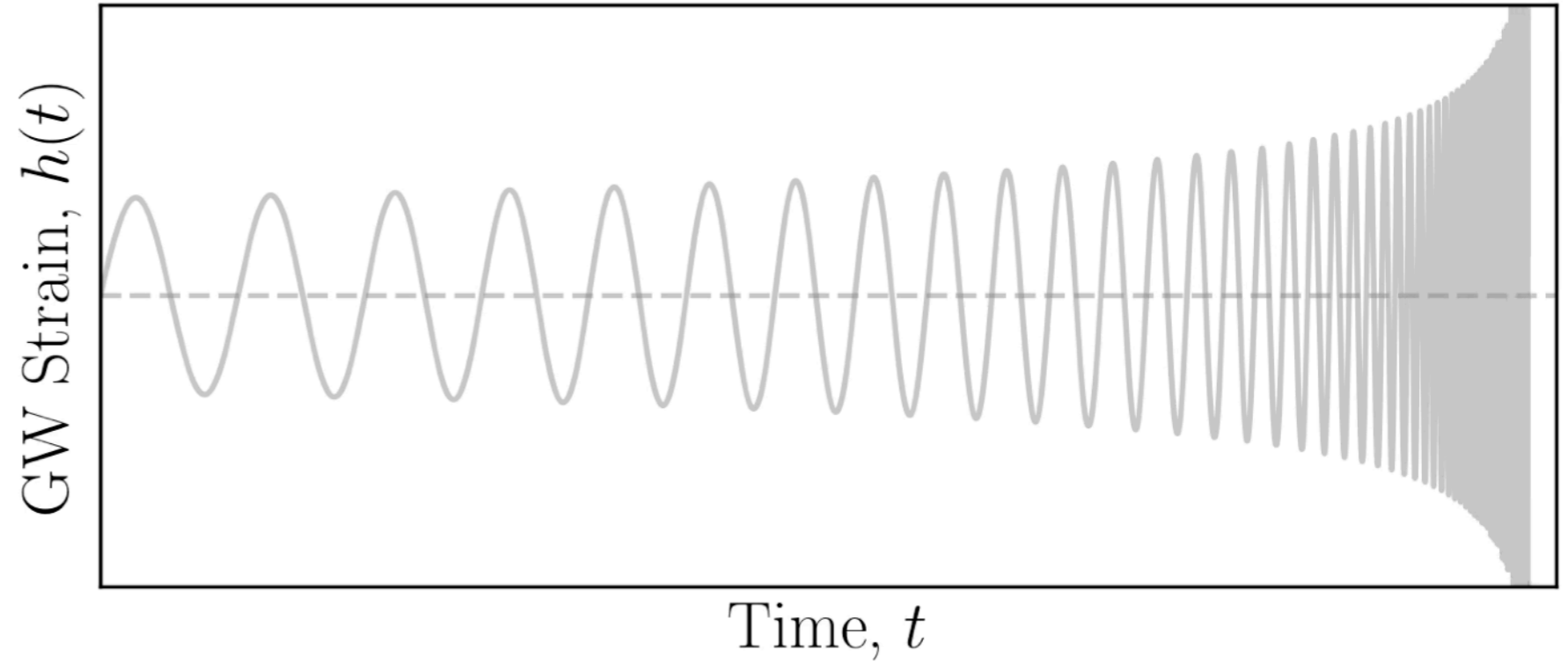
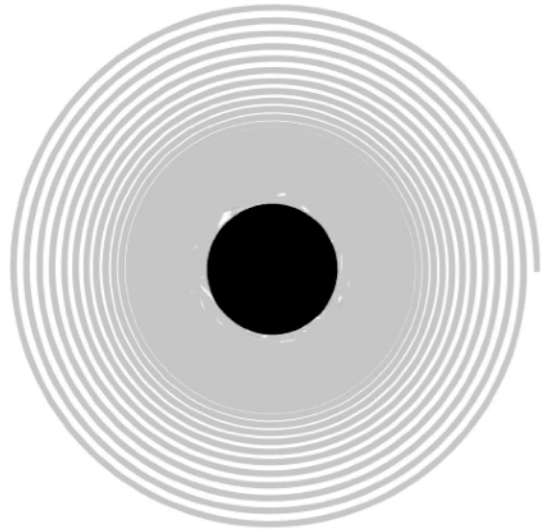
# Dark Matter “De-phasing”



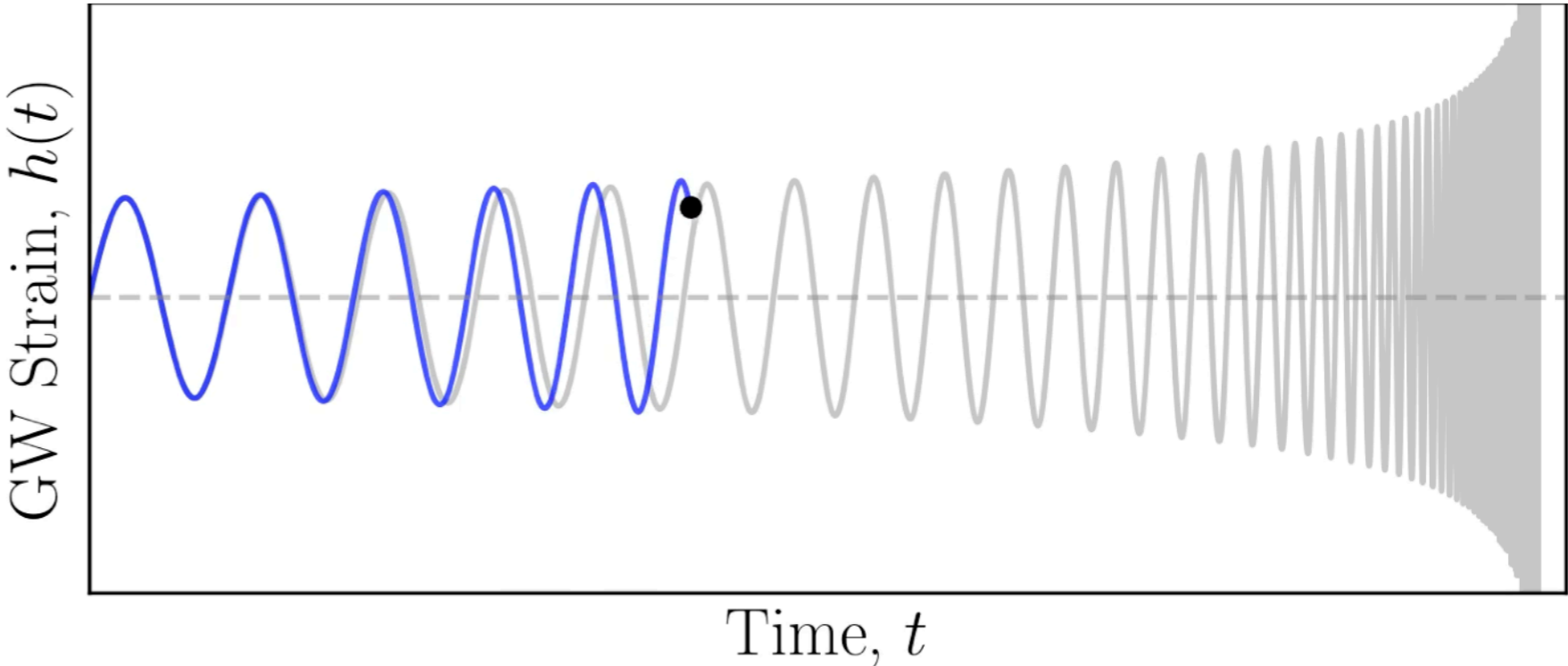
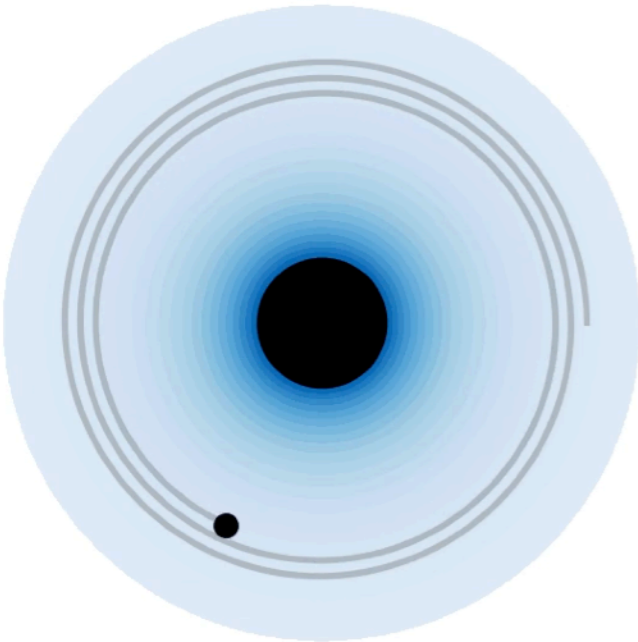
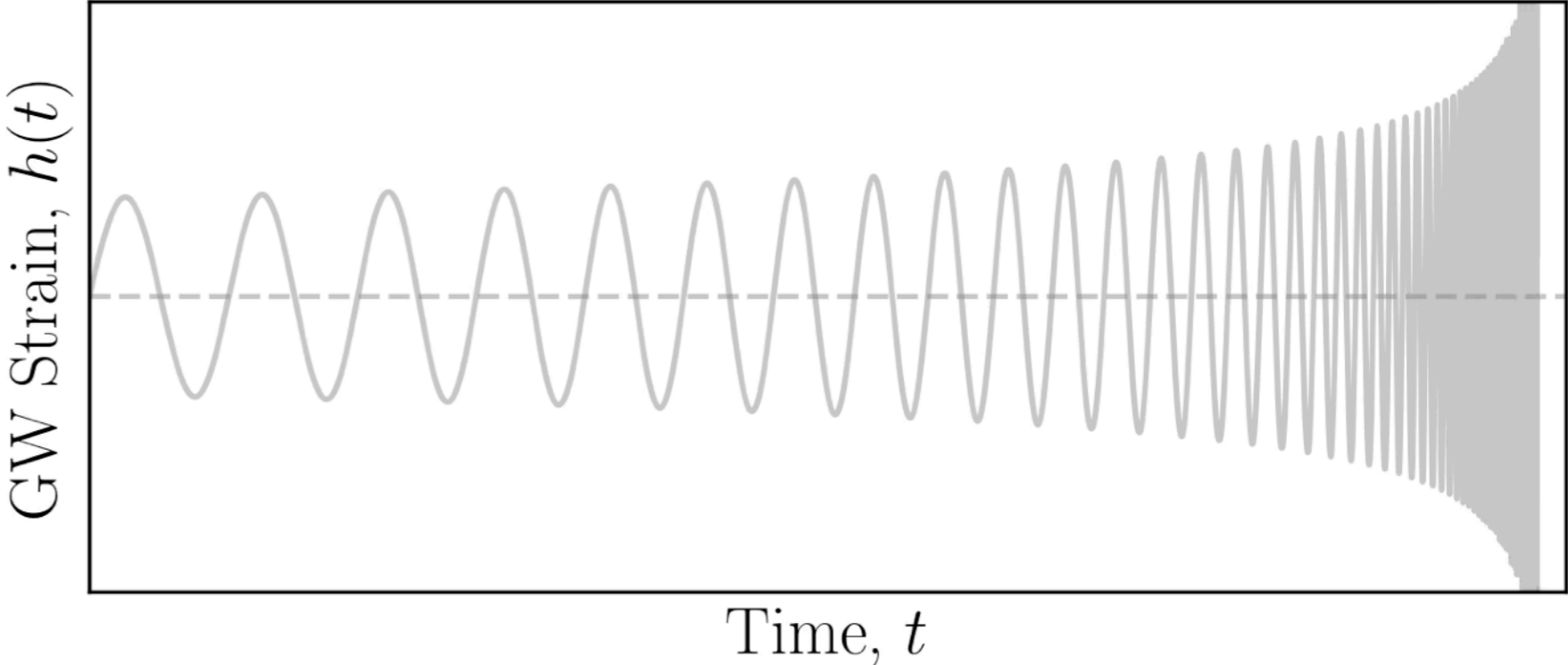
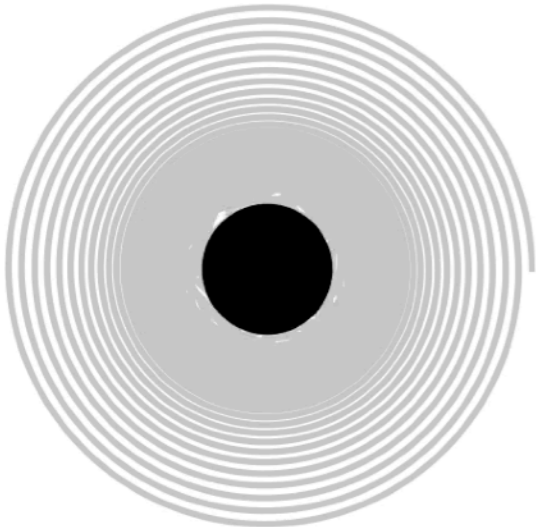
# Dark Matter “De-phasing”



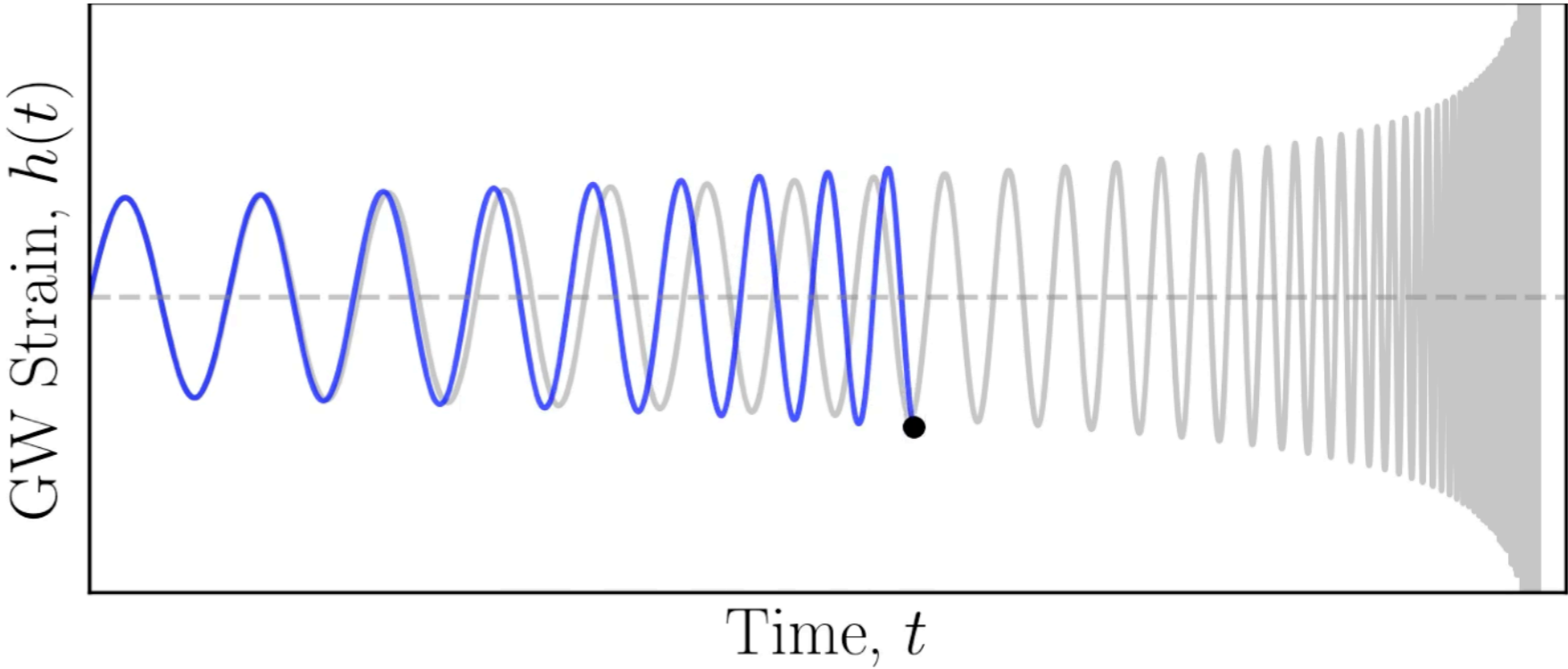
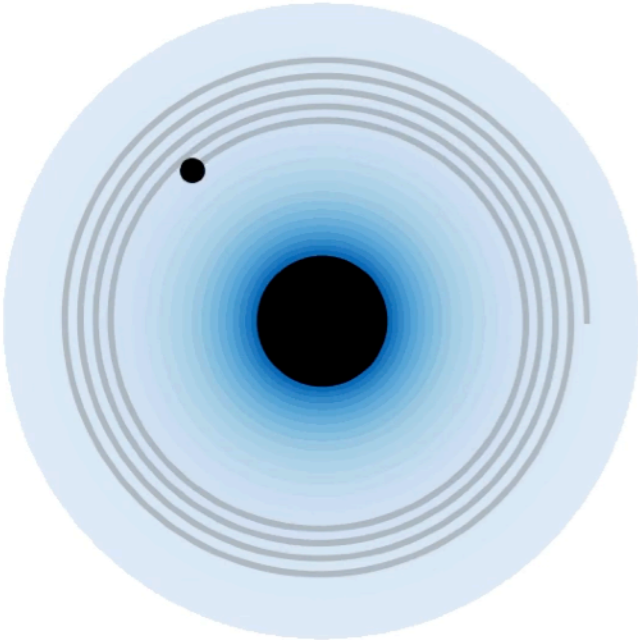
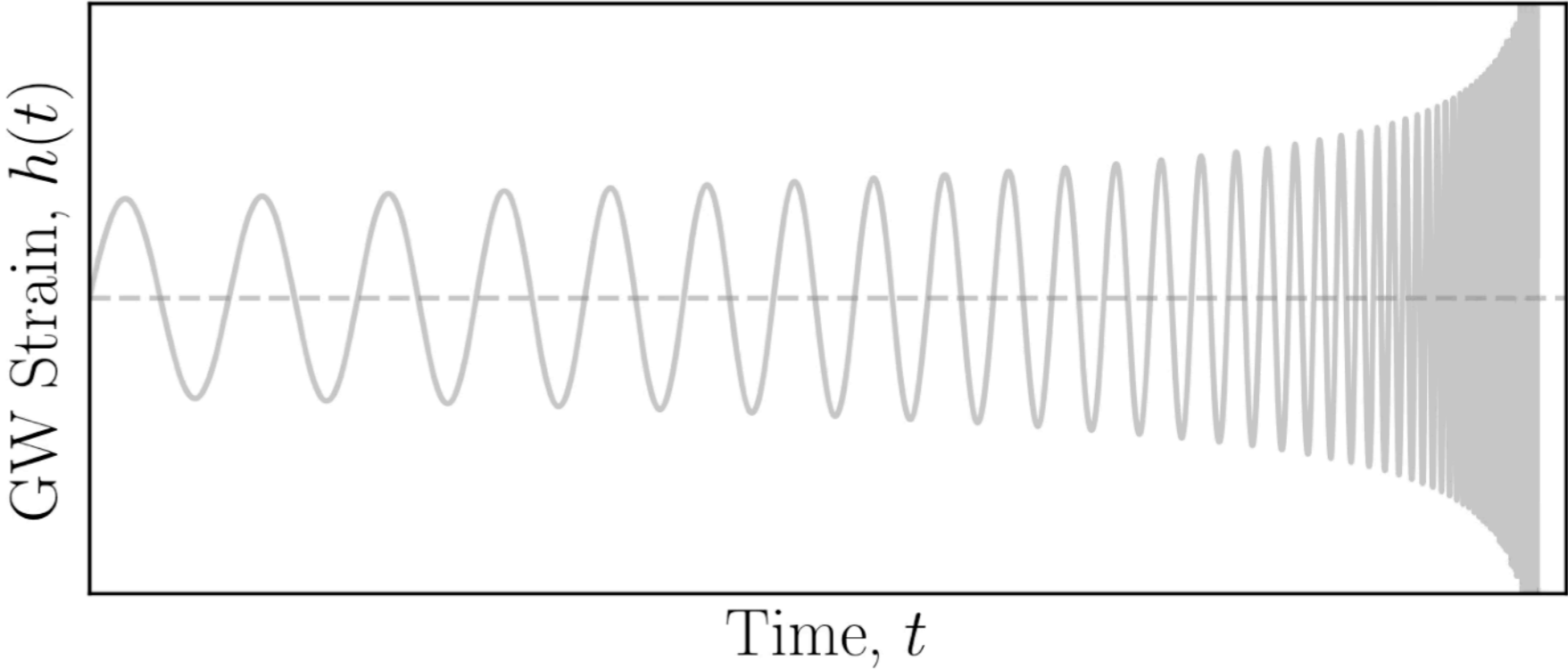
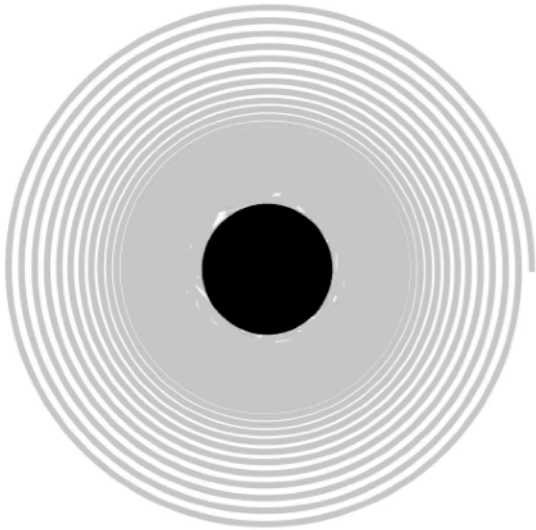
# Dark Matter “De-phasing”



# Dark Matter “De-phasing”

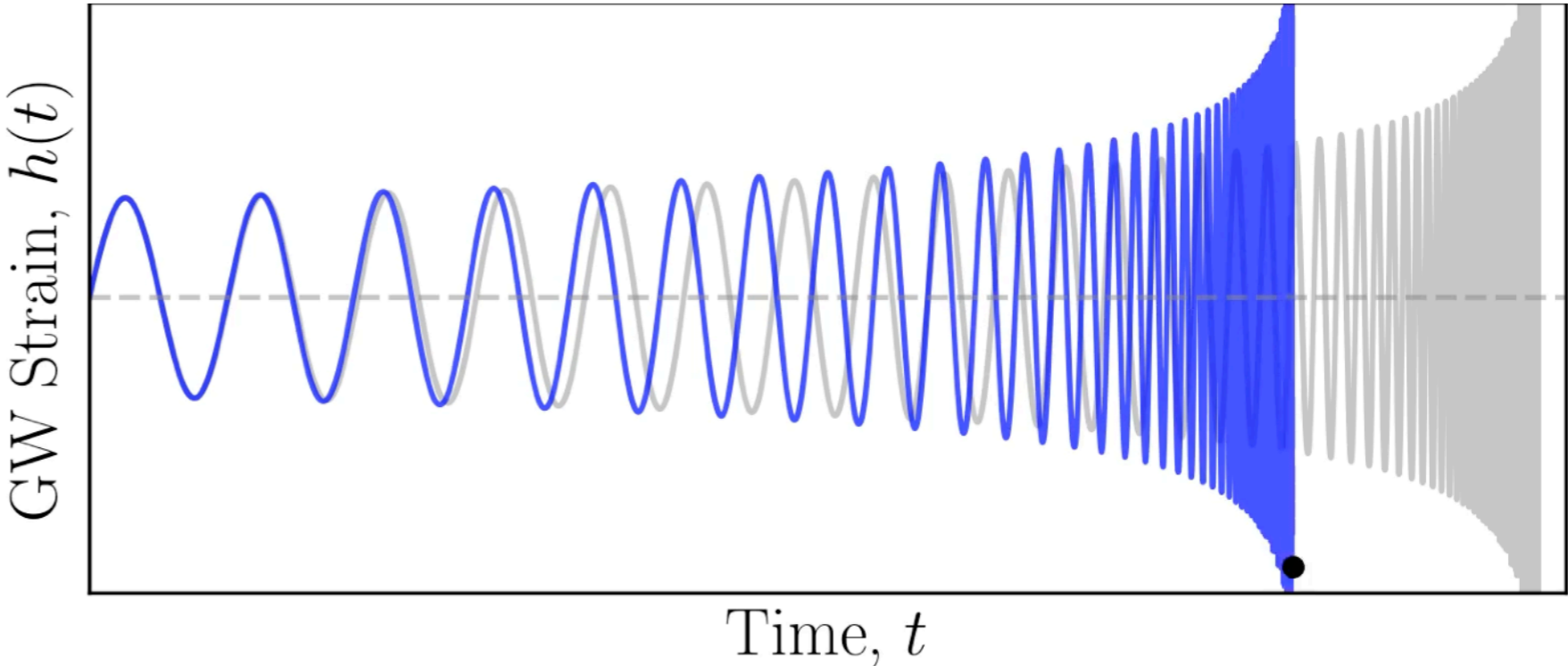
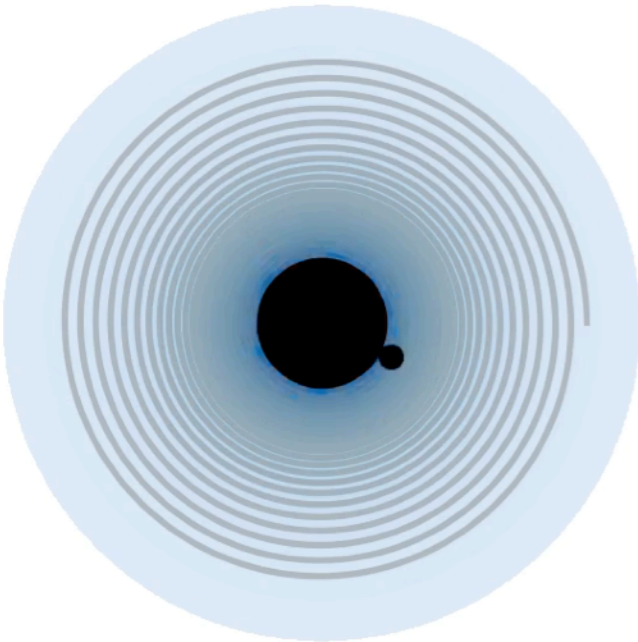
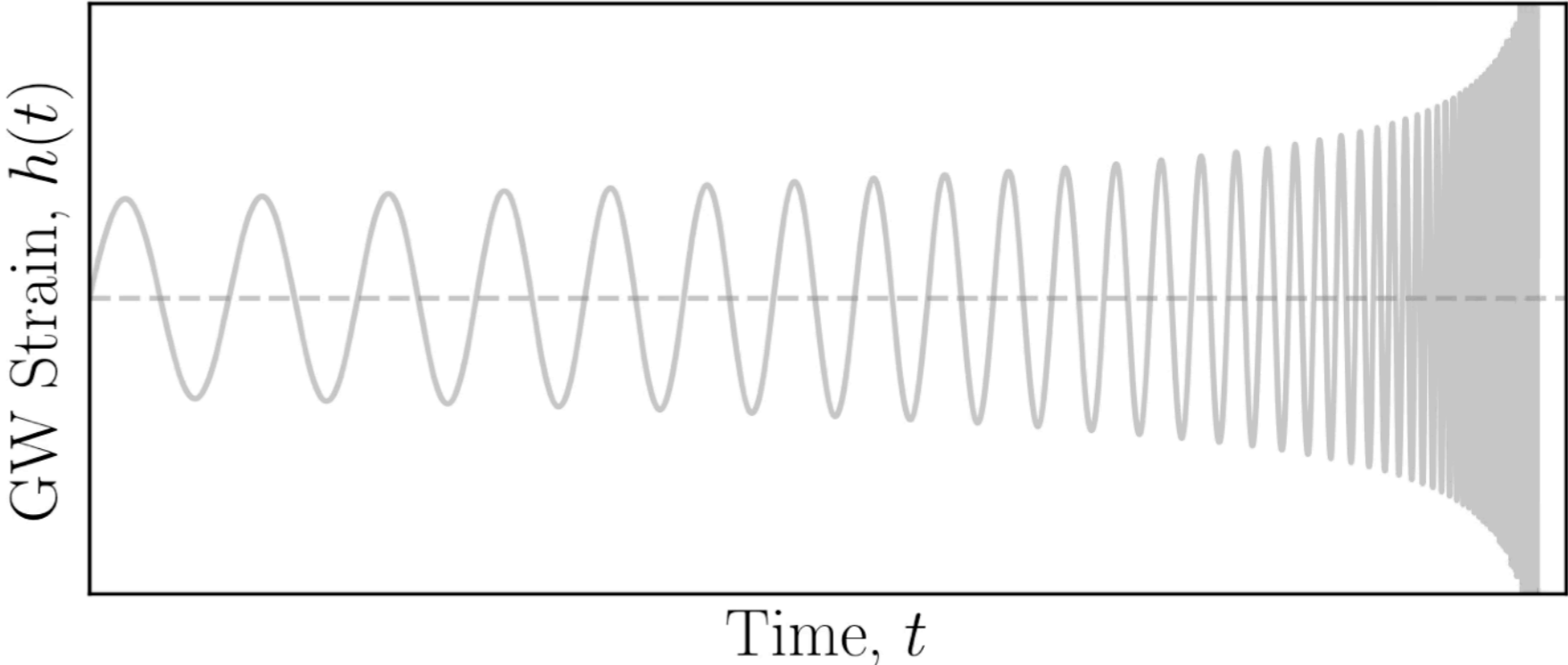
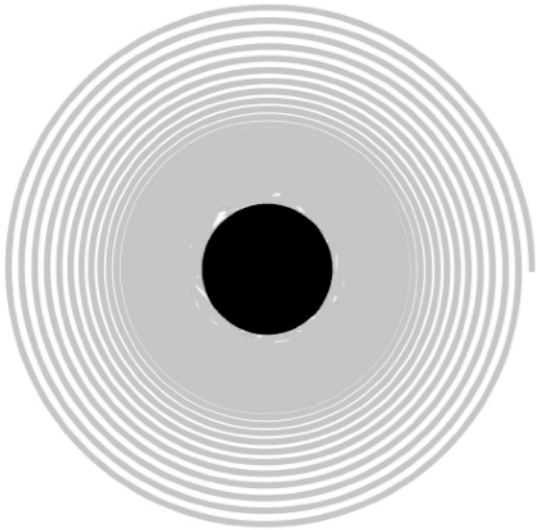


# Dark Matter “De-phasing”

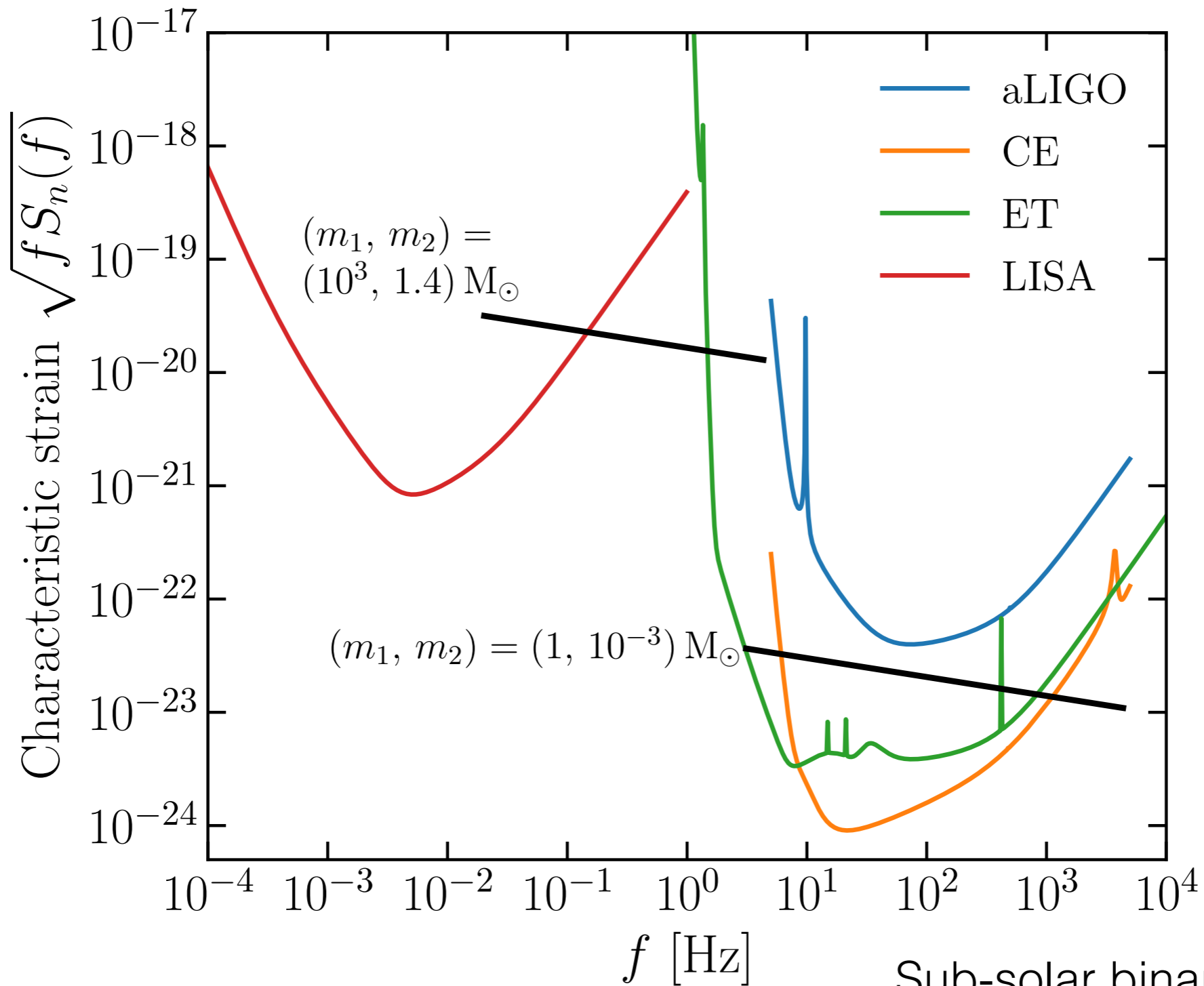




# Dark Matter “De-phasing”



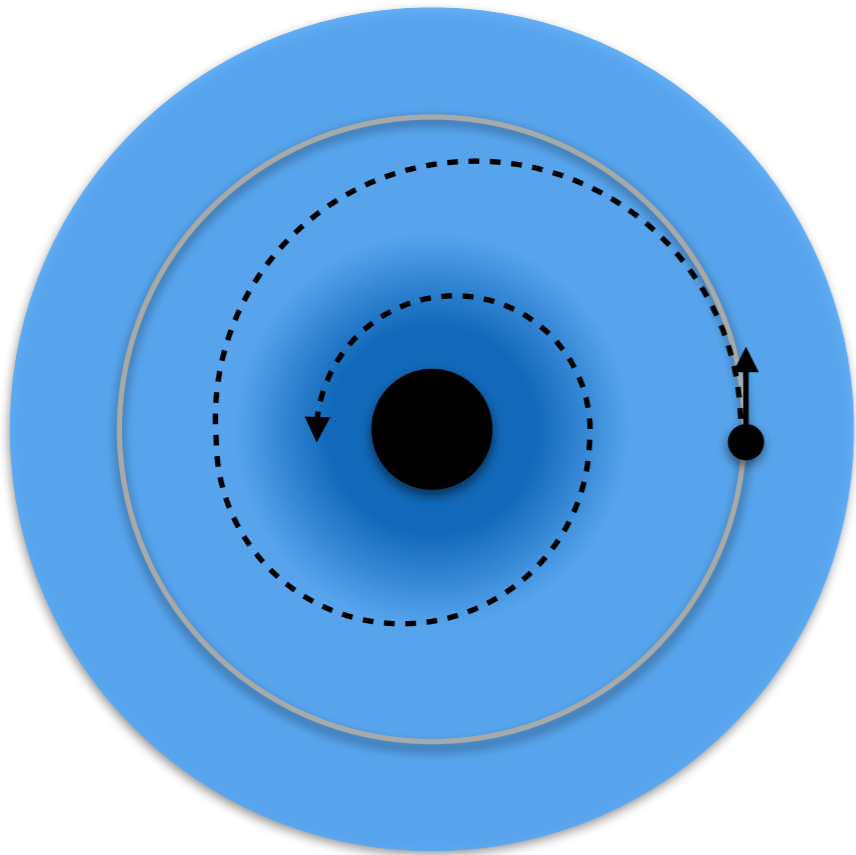
# GW Sensitivity



Sub-solar binaries detectable  
out to  $\sim 100$  Mpc with future  
GW observatories

# Modelling the dephasing

Assume quasi-circular, Newtonian orbits in the limit  $q = m_2/m_1 \ll 1$ :

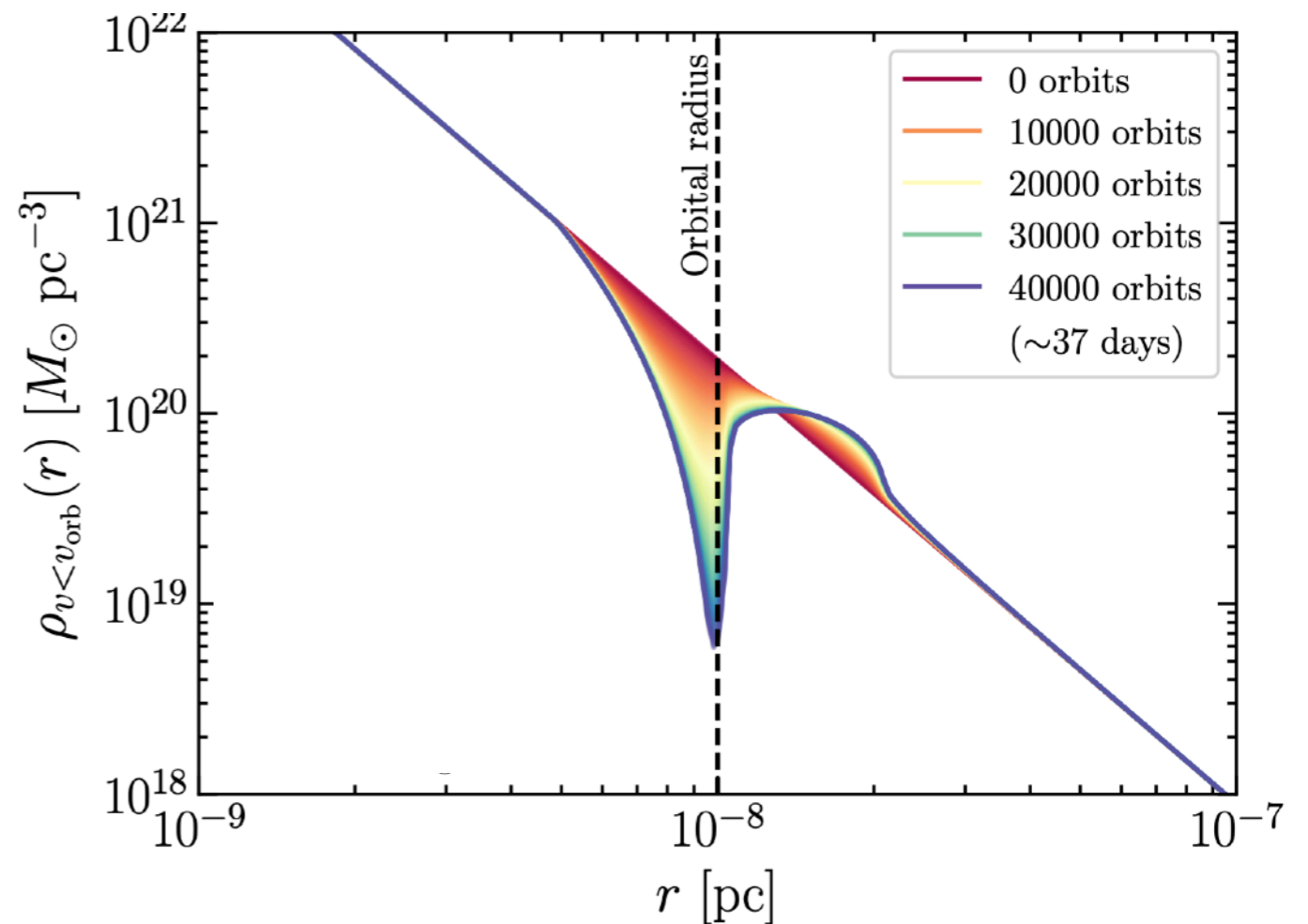


$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$

$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$

Model also the feedback on the DM spike:

$$\frac{df(\mathcal{E})}{dt} \rightarrow \rho_{\text{DM}}(r, t)$$

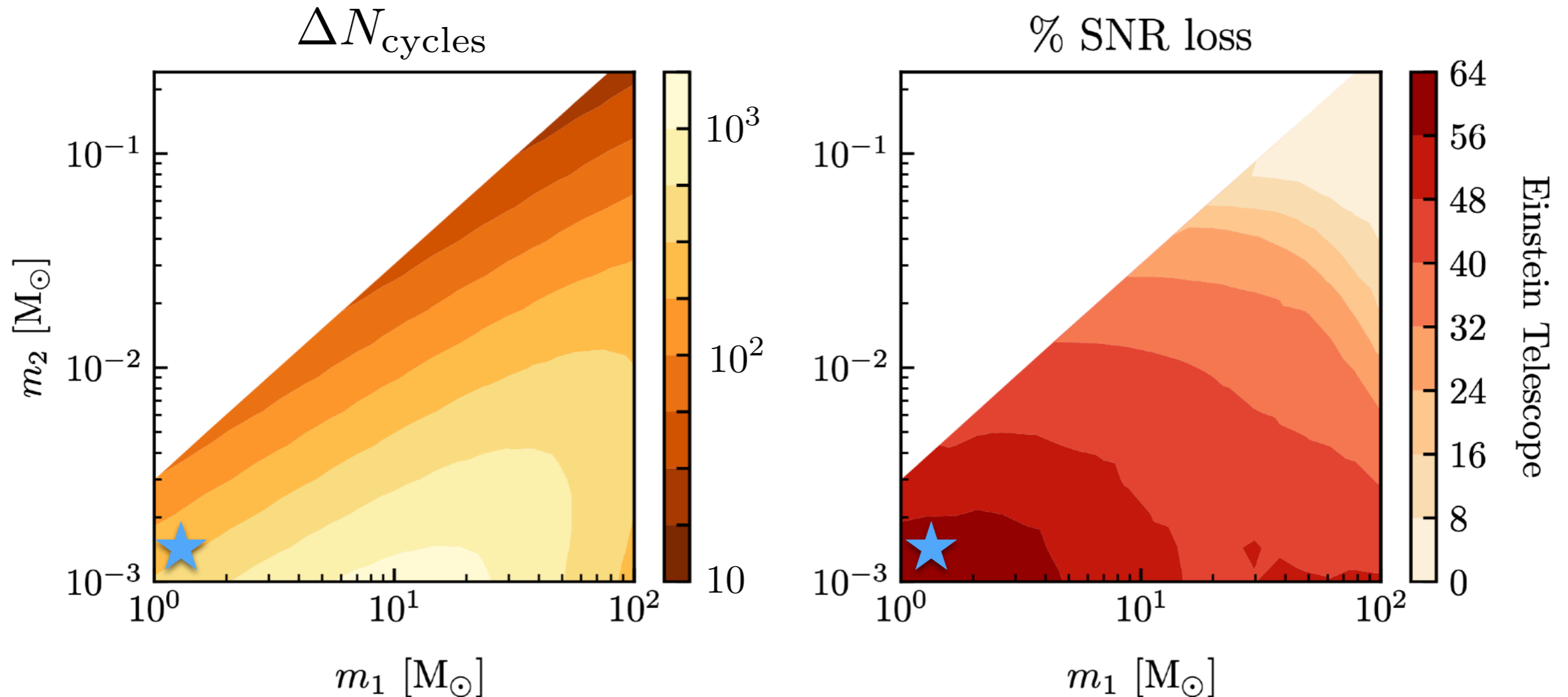


[[HaloFeedback Code](#), **BJK** et al., [2002.12811](#);  
Coogan et al. (including **BJK**), [2108.04154](#);  
See Backup Slides]

# Quantifying the dephasing

$$\star (m_1, m_2) = (1, 10^{-3}) M_\odot$$

In vacuum, expect  $\sim 10^7$  GW cycles during a one year observation.  
But with a DM spike:



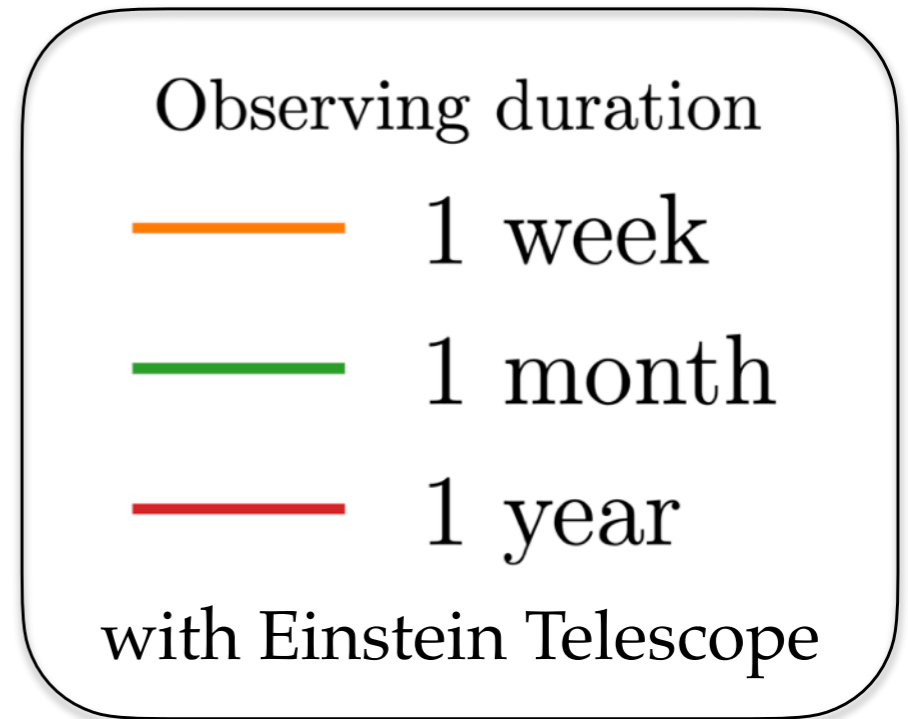
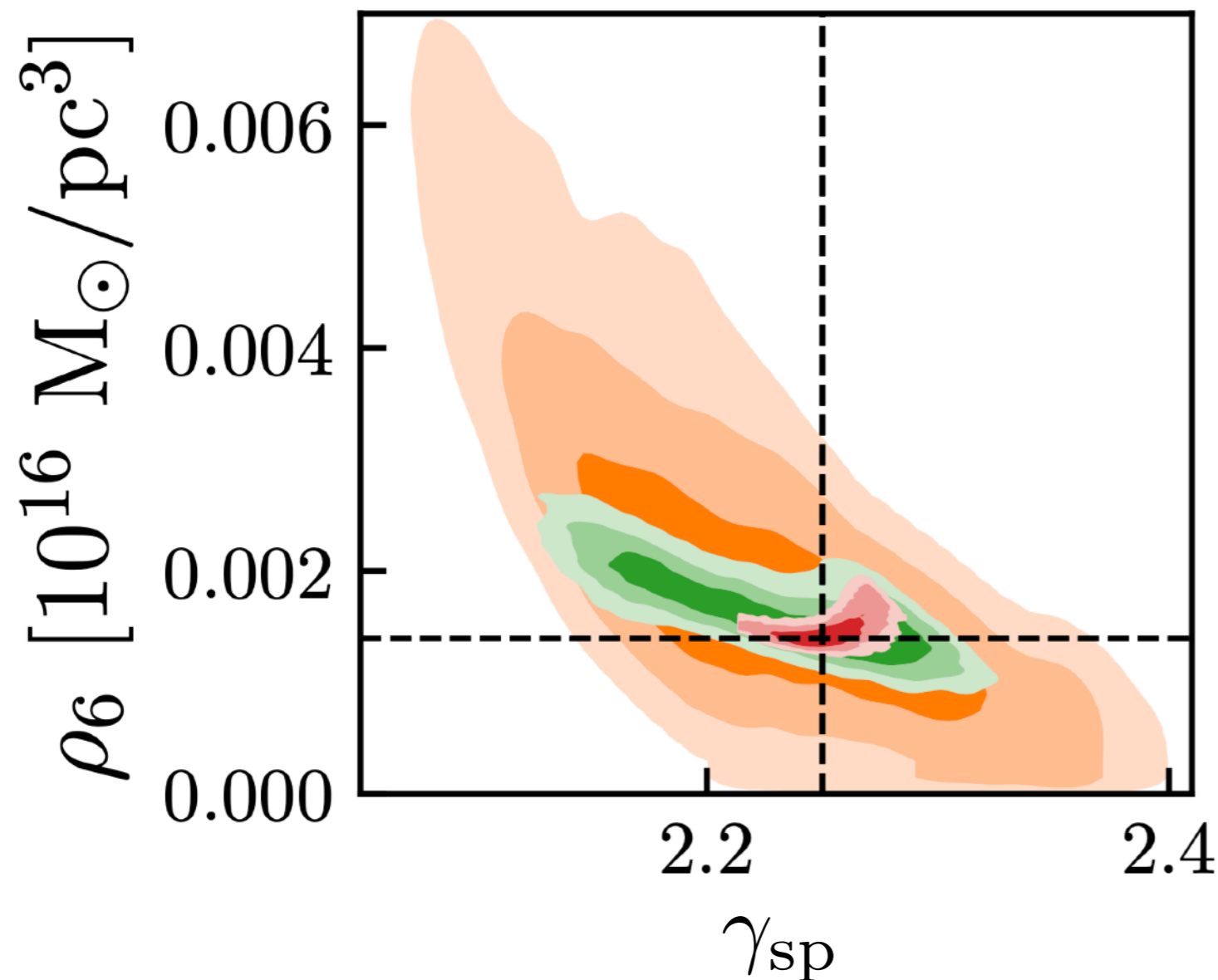
With sufficiently large SNR loss, signal could be missed altogether!

[Cole et al. (including **BJK**), [2207.07576](#)]

# Measuring the DM Spike

$$\rho_{\text{DM}}(r) = \rho_6 \left( \frac{r}{10^{-6} \text{ pc}} \right)^{-\gamma_{\text{sp}}}$$

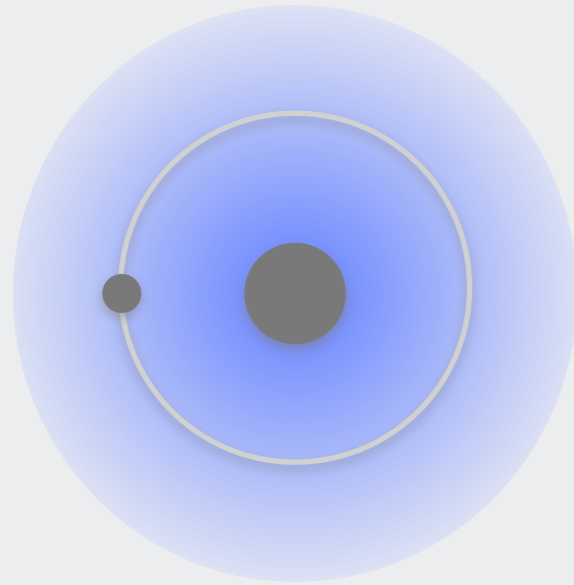
$$(m_1, m_2) = (1, 10^{-3}) M_{\odot}$$



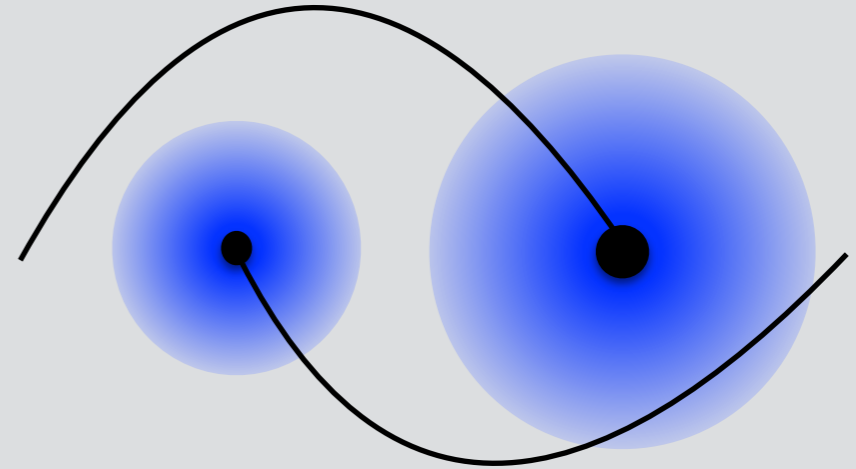
[Cole et al. (including **BJK**), [2207.07576](#)]

Distinguishable from other environmental effects (baryons, light bosonic fields, ...)!

[Cole et al. (including **BJK**), [2211.01362](#)]

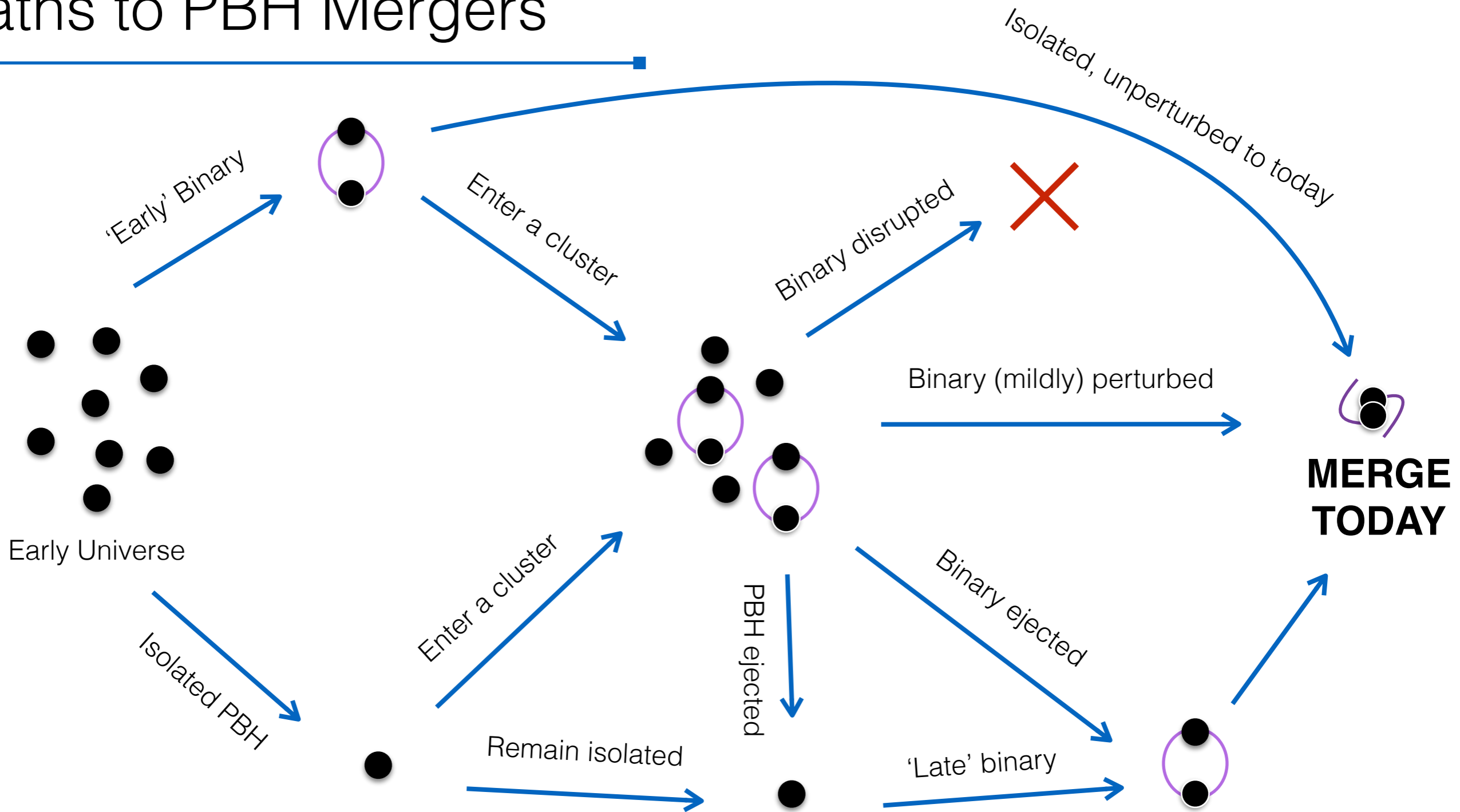


**Dark Matter  
'de-phasing' of  
GW signals**



**Merger rates  
of dressed  
PBH Binaries**

# Paths to PBH Mergers



+ particle DM halos?

[**BJK**, Gaggero & Bertone, [1805.09034](#);  
Jangra, **BJK**, Diego, [2304.05892](#)]

+ baryonic accretion?

[De Luca et al., [2003.12589](#)]

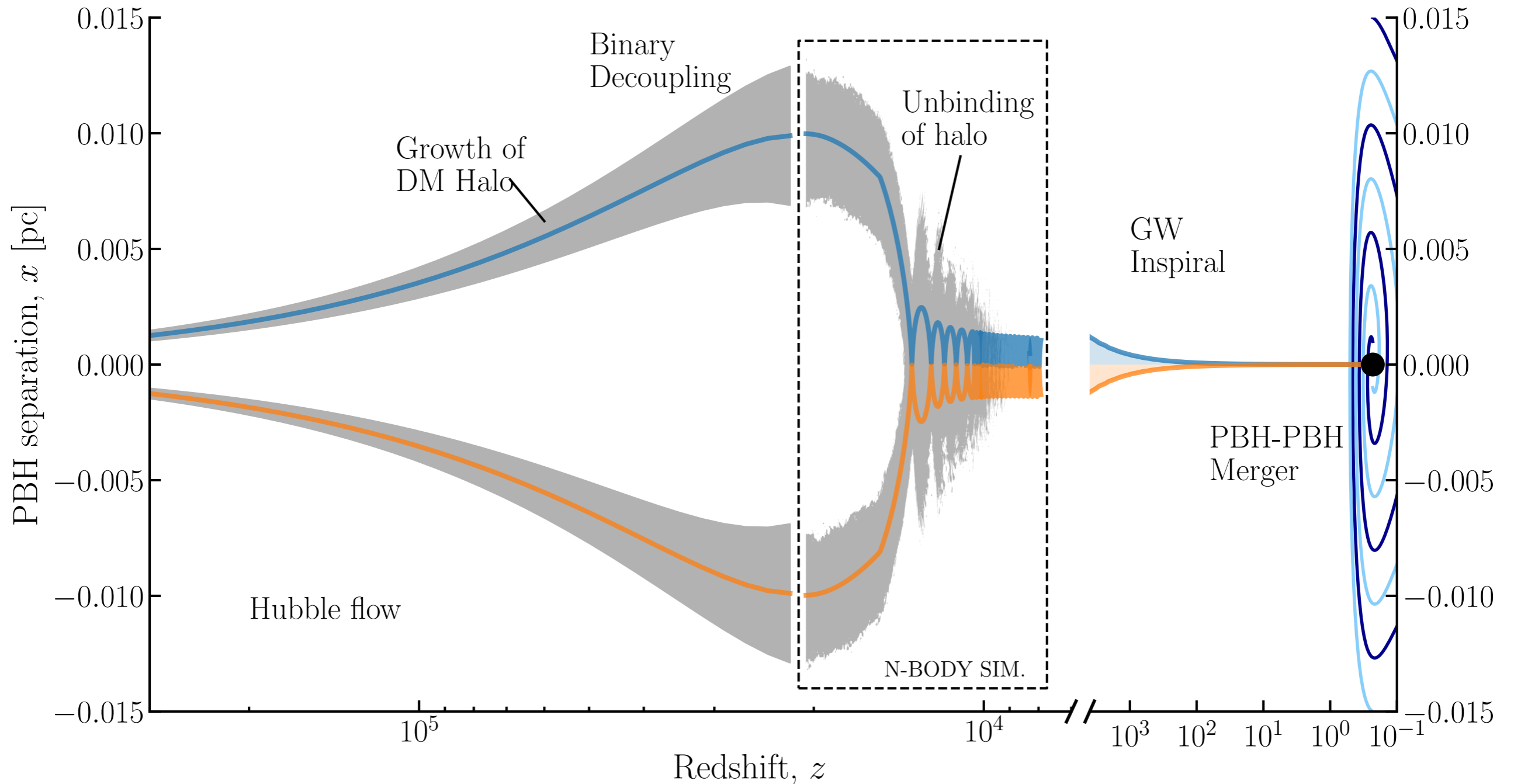
[Raidal+, [1812.01930](#); Vaskonen & Veermäe, [1908.09752](#);  
Atal+, [2007.07212](#); De Luca+, [2009.04731](#) and others...]

# Life of a PBH binary

$$a_i = 0.01 \text{ pc}$$

$$e_i = 0.995$$

Consider a  $\sim$ -equal-mass PBH binary:



[[Movies here](#)]

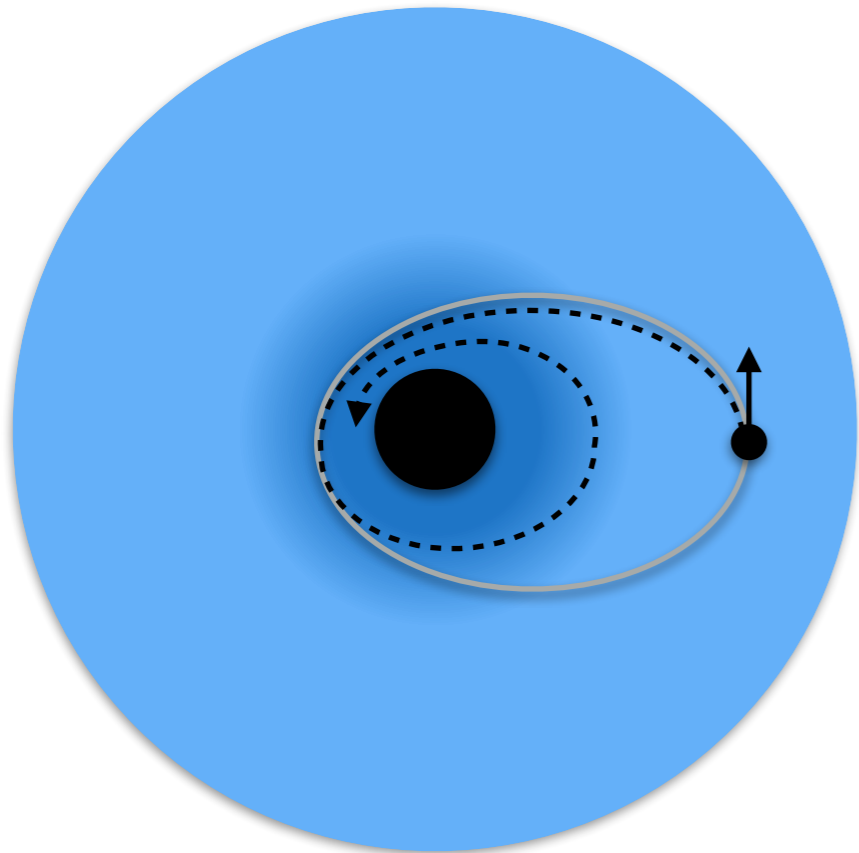
[[BJK, Gaggero & Bertone, 1805.09034](#)]



# Large mass-ratio binaries

[Jangra, **BJK**, Diego, [2304.05892](#)]

Starting from binary formation, model **highly-eccentric** Newtonian orbits:

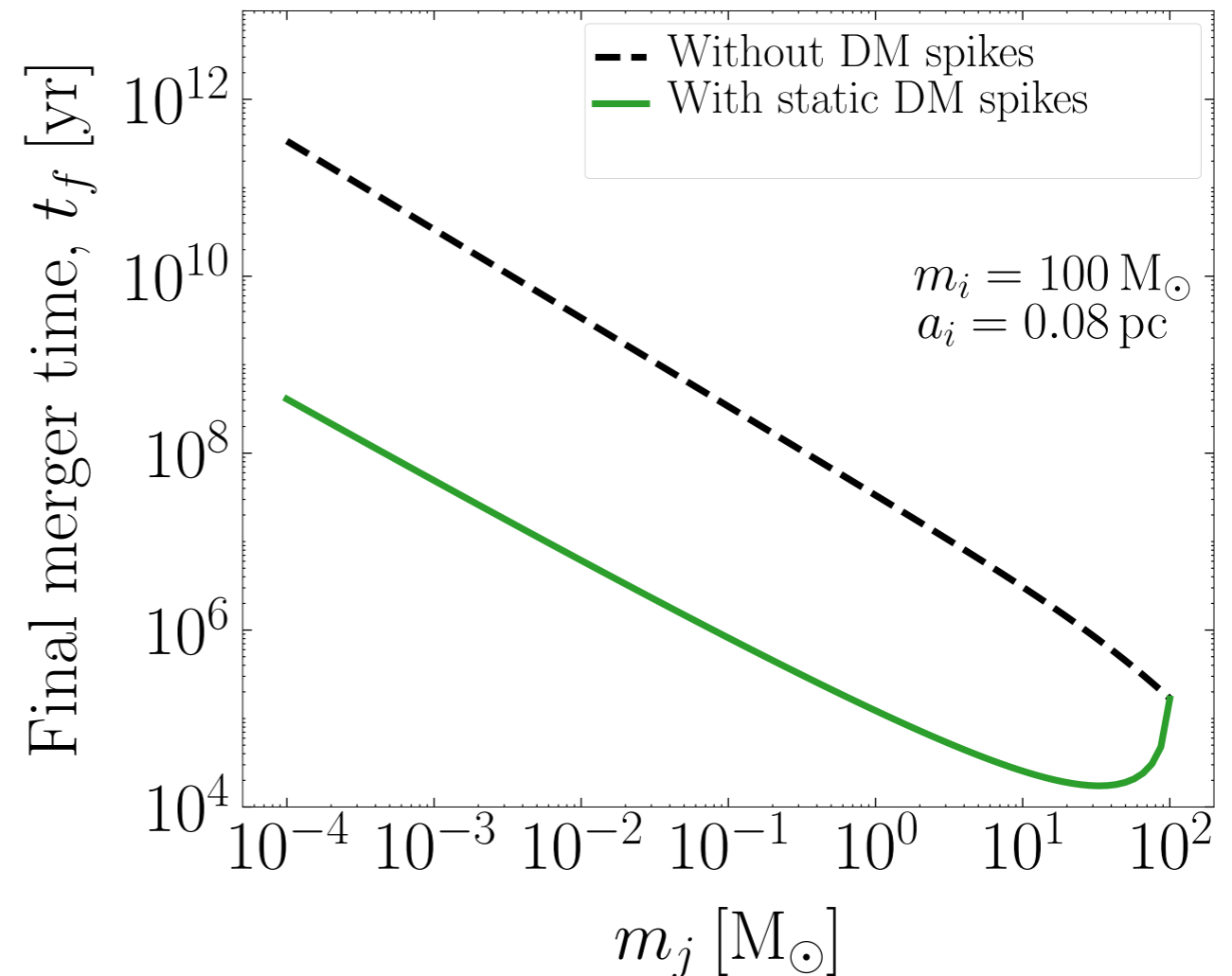


Assume “**static spikes**” (i.e. neglecting feedback). Formalism for eccentric orbits is being developed...

[[IMRlpy Code](#); Becker et al., [2112.09586](#)]

$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$

$$-\dot{L}_{\text{orb}} = \dot{L}_{\text{GW}} + \dot{L}_{\text{DF}} \quad \longrightarrow \quad t_m$$

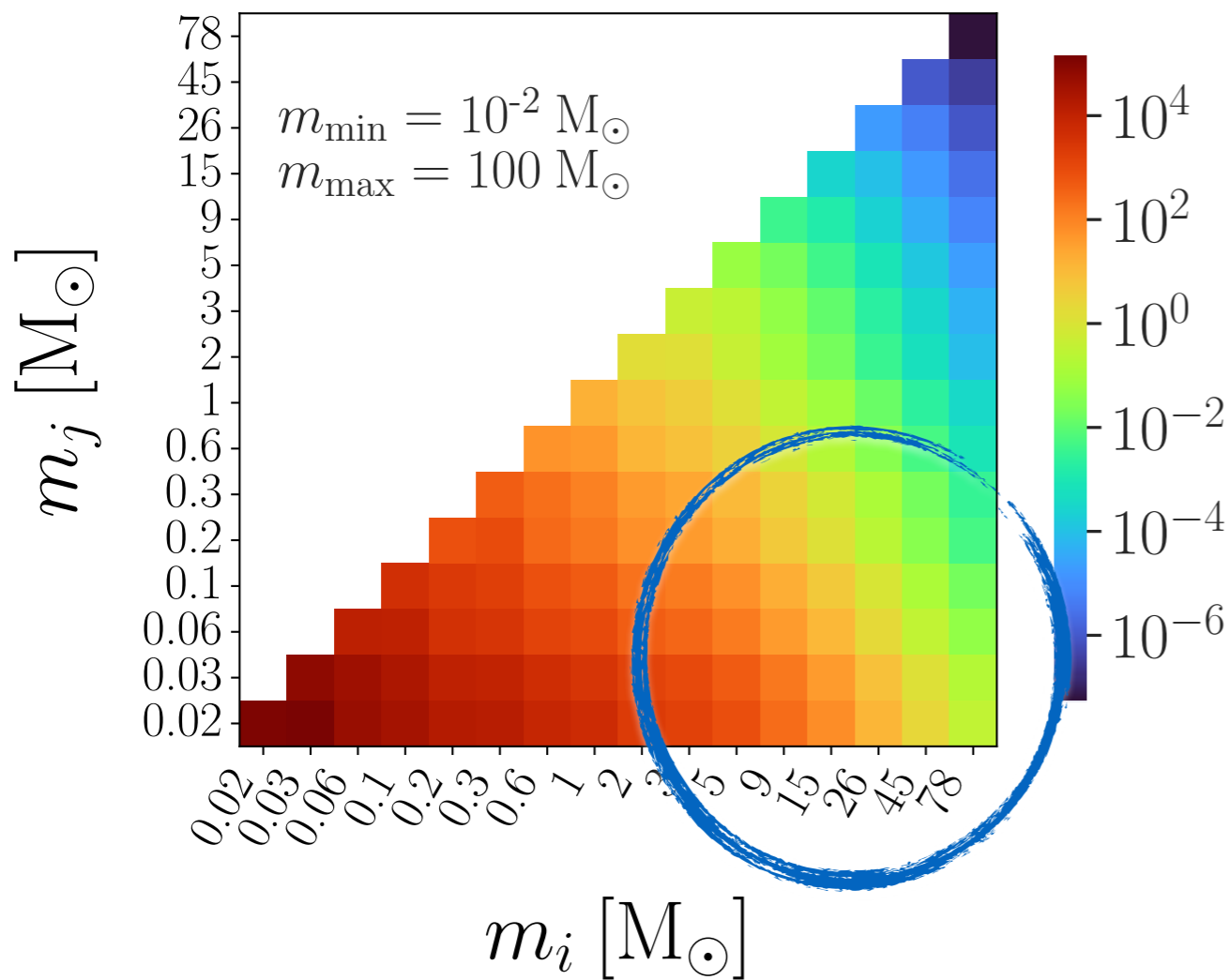


$$\frac{d\mathcal{R}_{ij}}{dm_i dm_j} \sim \rho_m^0 \min\left(\frac{f_i}{m_i}, \frac{f_j}{m_j}\right) P(t_m)$$

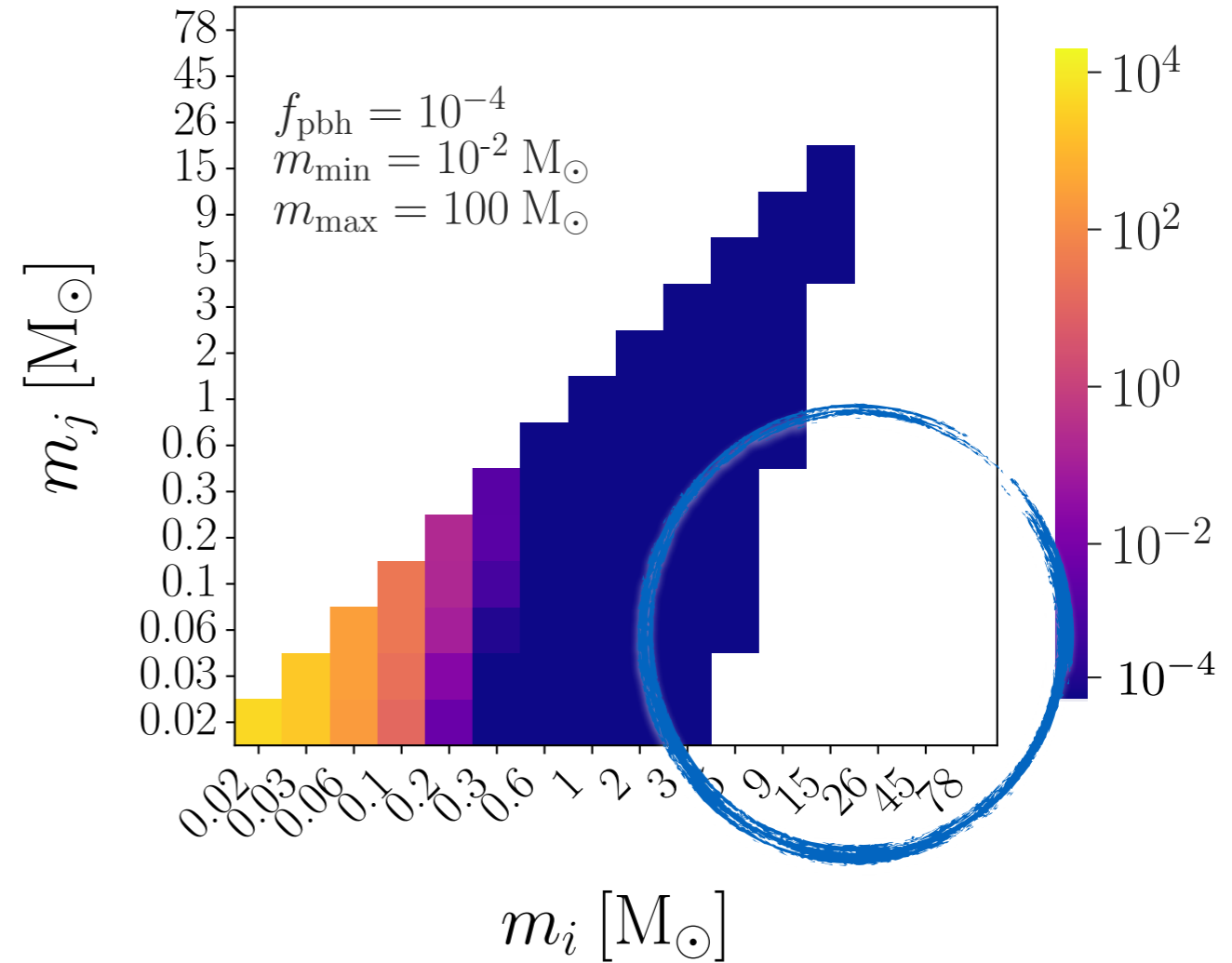
# Merger rate ratios

Assuming a power-law mass function,  
just below current LVK constraints:

$$\mathcal{R}_{ij,0} [\text{Gpc}^{-3}\text{yr}^{-1}]$$



$$\mathcal{R}_{ij, \text{static}} / \mathcal{R}_{ij,0}$$



Merger can be accelerated by a huge amount.  
Binaries which would have merged today have already merged!

[Jangra, **BJK**, Diego, [2304.05892](https://arxiv.org/abs/2304.05892)]

# Many collaborators...

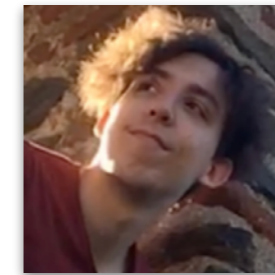
...and others...

Gianfranco Bertone  
(GRAPPA, Amsterdam)



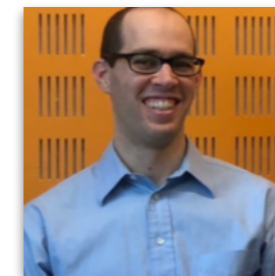
Pratibha Jangra  
(IFCA, Santander)

Pippa Cole  
(GRAPPA, Amsterdam)



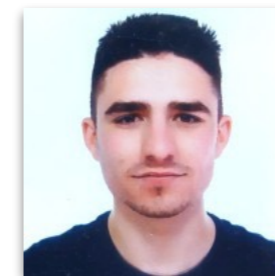
Theophanes Karydas  
(GRAPPA, Amsterdam)

Adam Coogan  
(Mila, Montreal)



David Nichols  
(U. Virginia)

Jose Maria Diego  
(IFCA, Santander)



Abram Perez Herrero  
(IFCA, Santander)

Daniele Gaggero  
(INFN, Pisa)

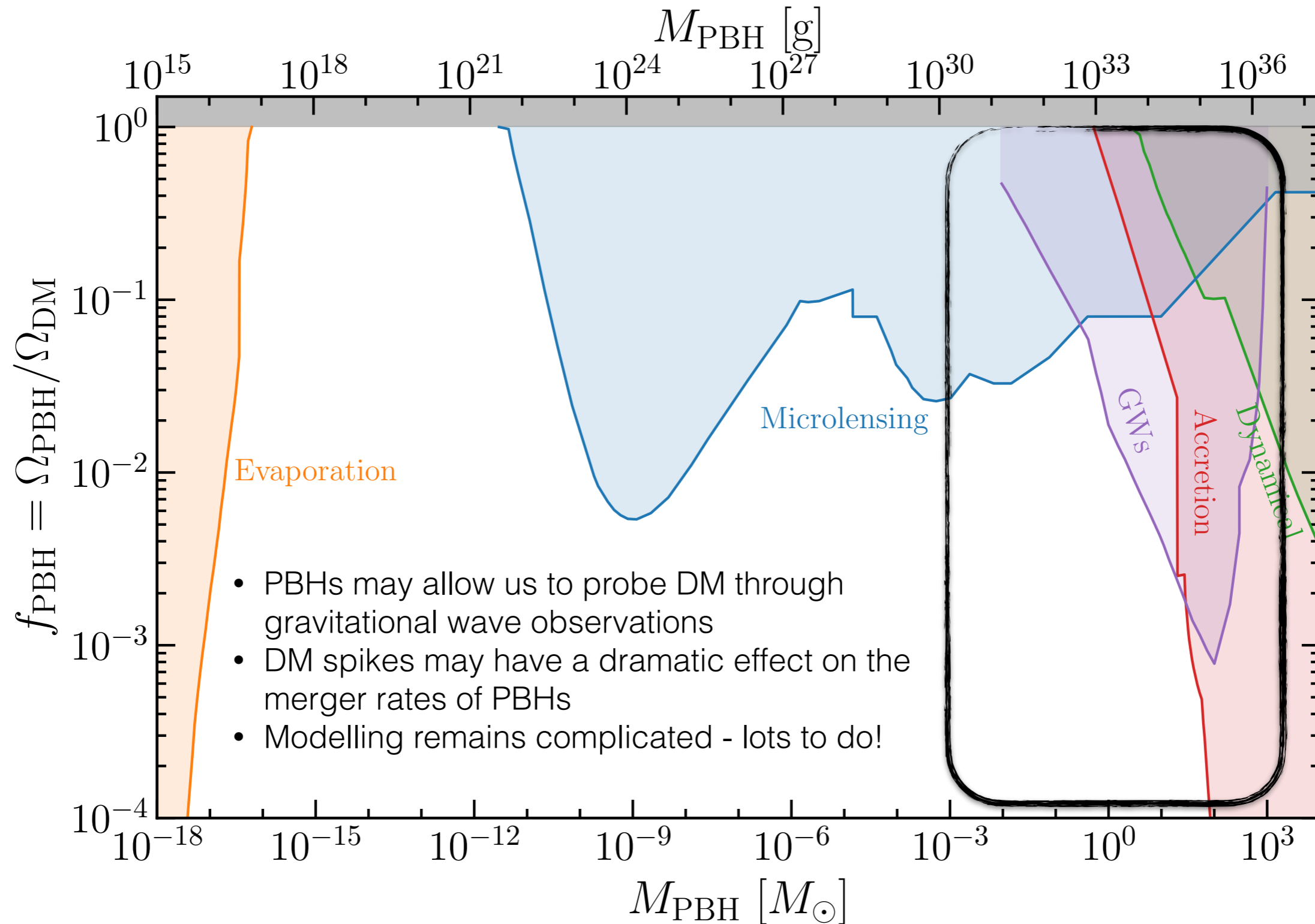


Francesca Scarcella  
(IFT, Madrid)

# Conclusions

[Green & **BJK**, 2007.10722]

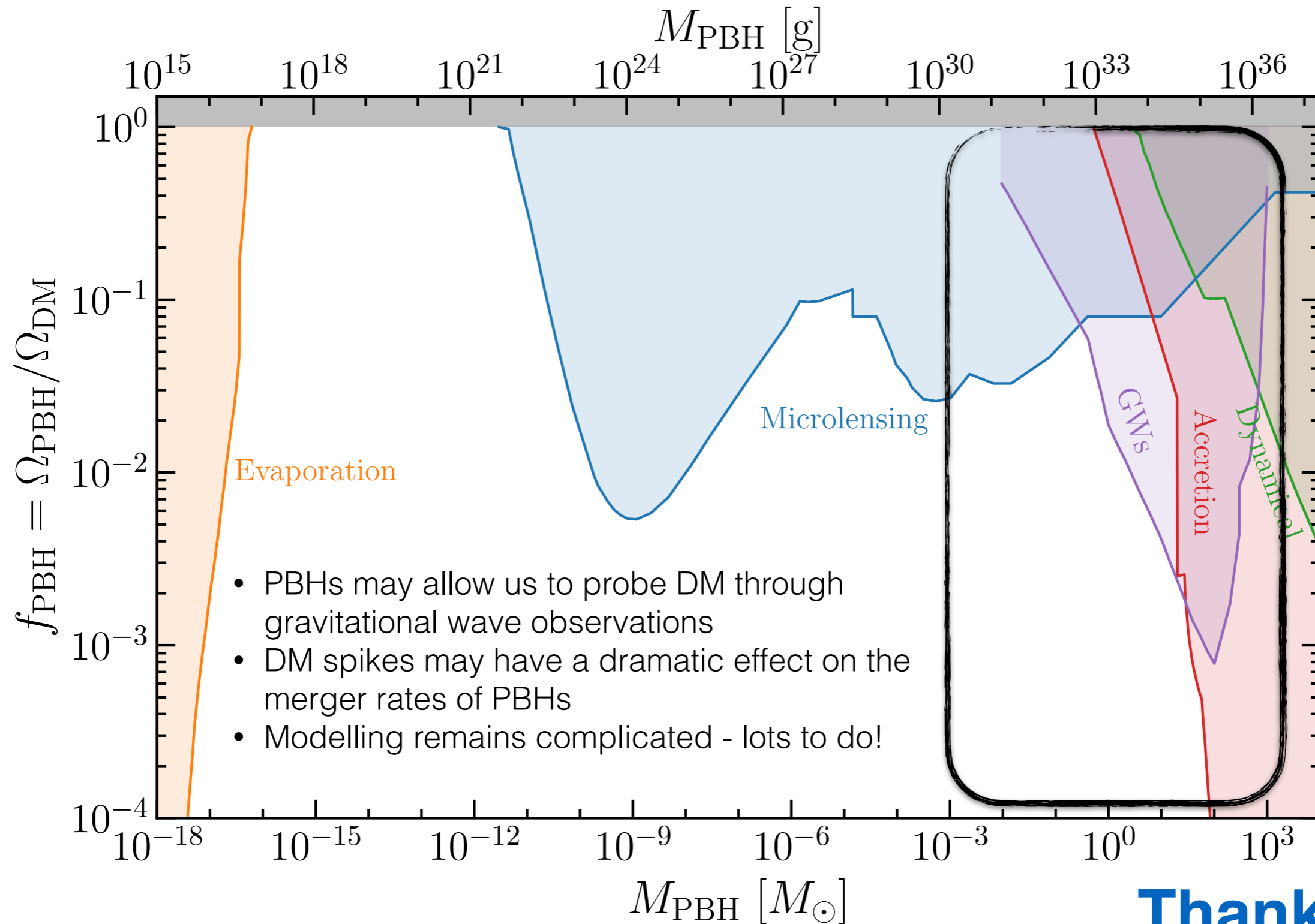
[Code online: [github.com/bradkav/PBHbounds](https://github.com/bradkav/PBHbounds)]



# Conclusions

[Green & **BJK**, 2007.10722]

[Code online: [github.com/bradkav/PBHbounds](https://github.com/bradkav/PBHbounds)]



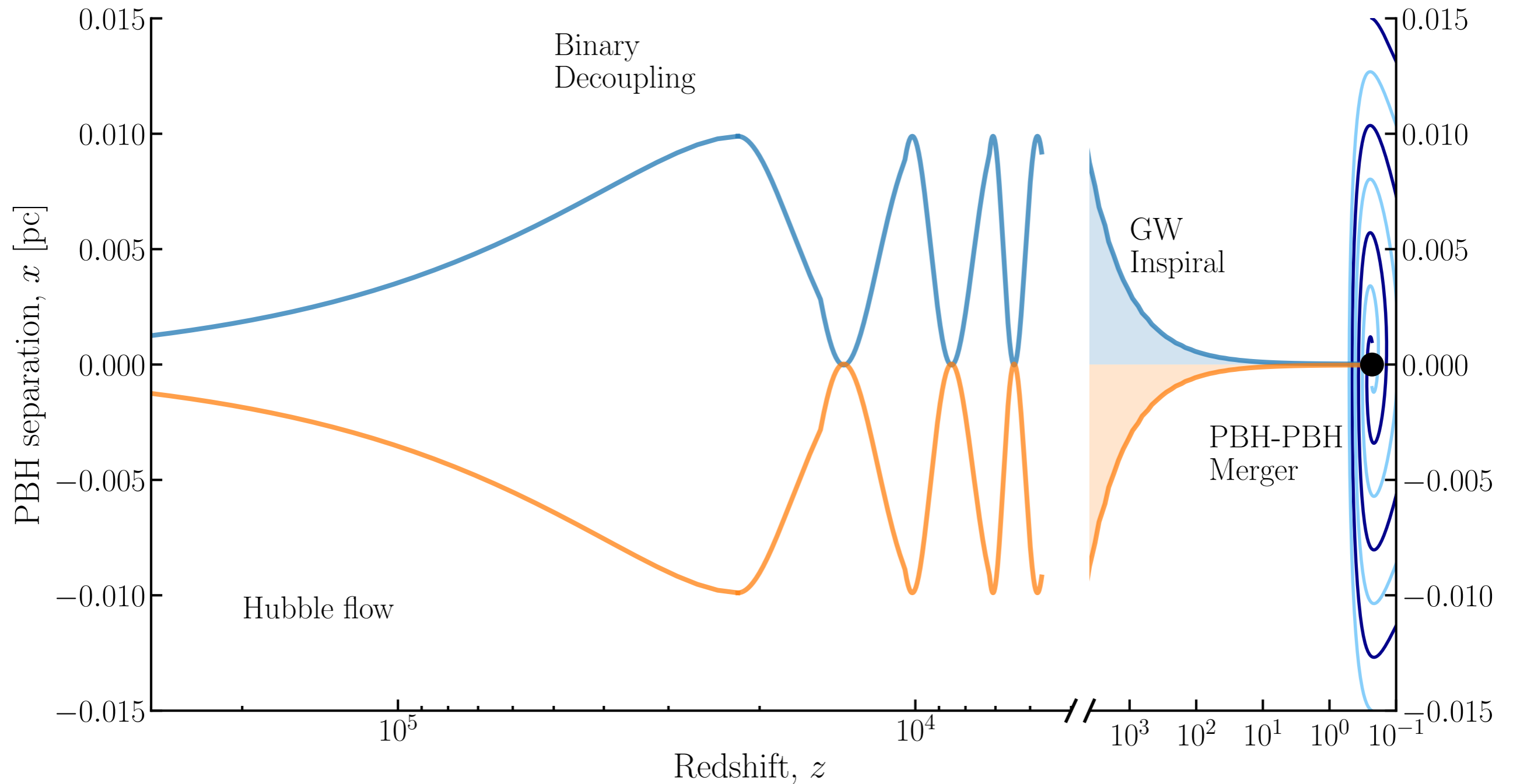
# Thank you!

# Backup Slides

# Life of a PBH binary

$$a_i = 0.01 \text{ pc}$$

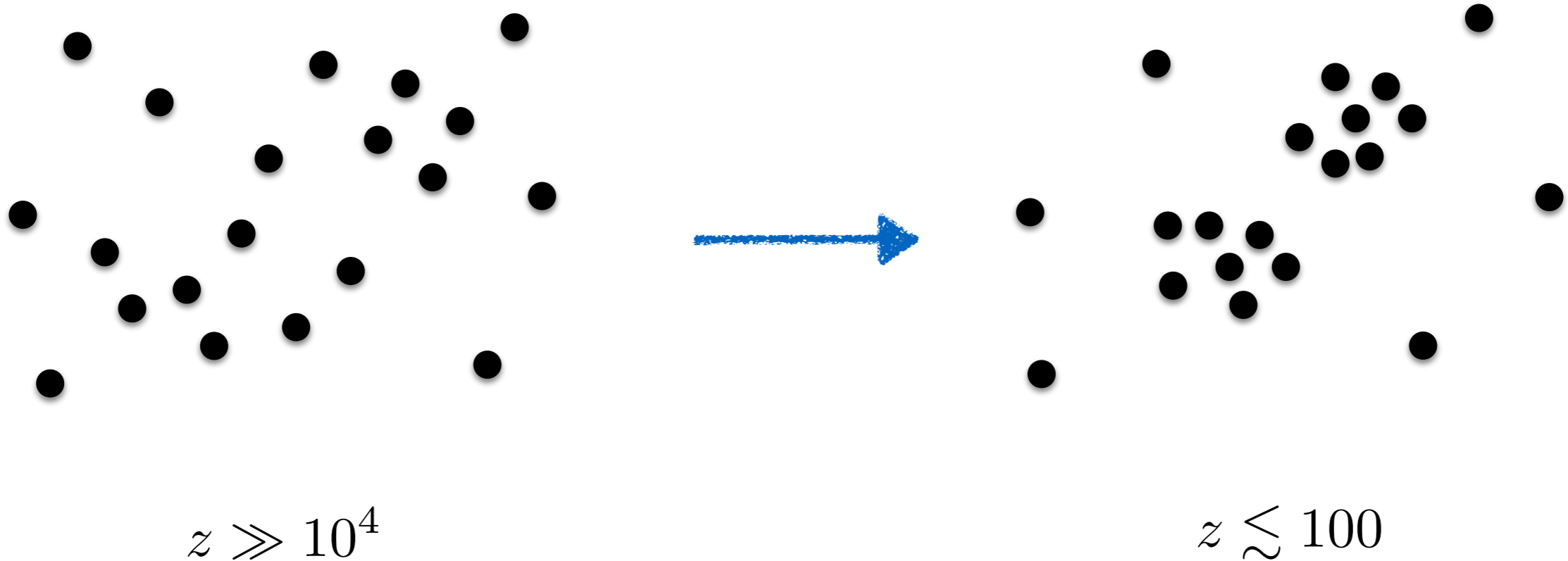
$$e_i = 0.995$$



[Ali-Haïmoud et al., [1709.06576](#),  
**BJK**, Gaggero & Bertone, [1805.09034](#)]

# PBH Clustering

Even a distribution of PBHs which is initially Poisson distributed will cluster at late times

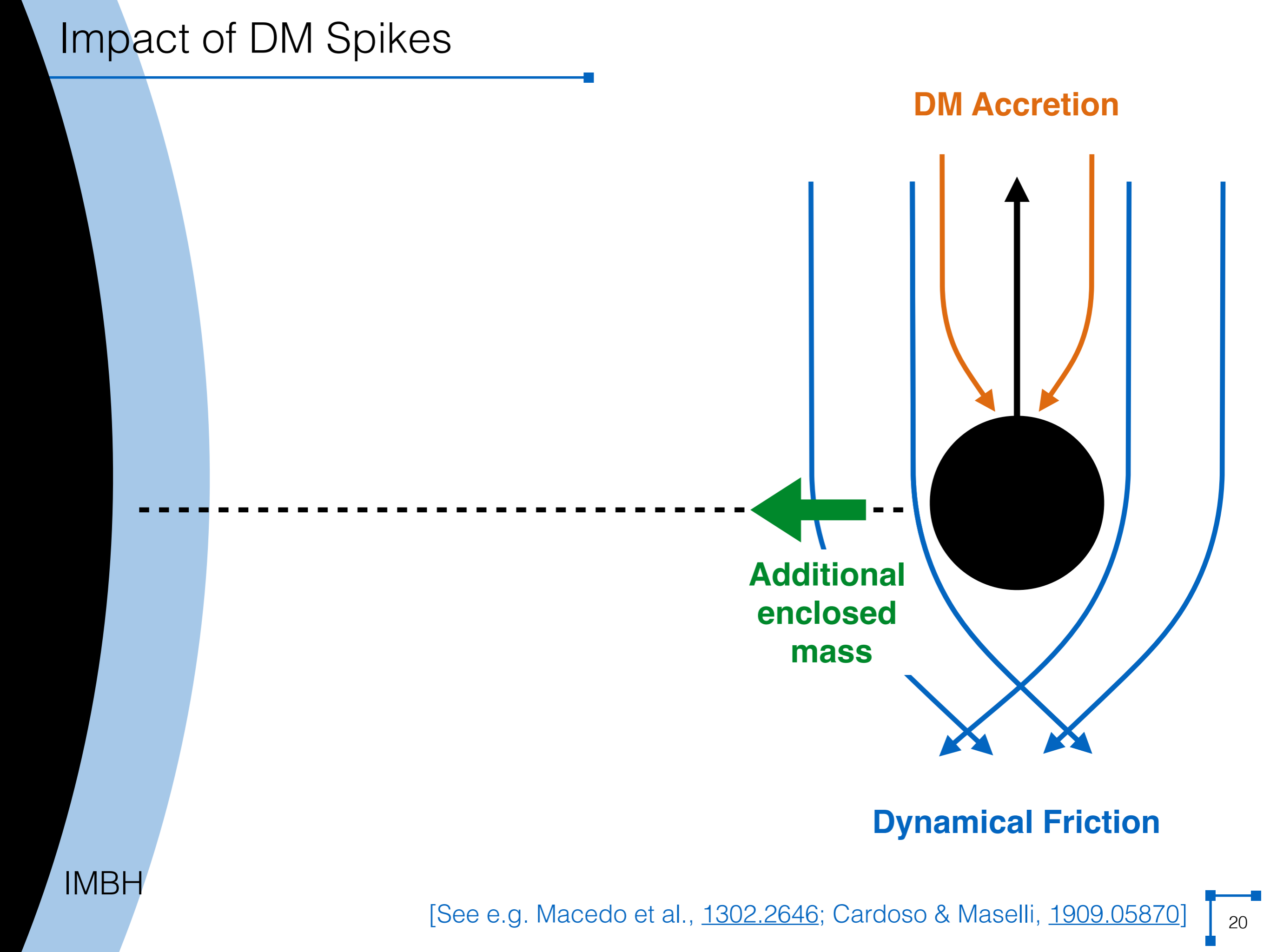


A cluster of  $N$  PBHs is expected to form around  $z_c \sim z_{\text{eq}} f_{\text{PBH}} / \sqrt{N}$

[Chisholm - [astro-ph/0509141](https://arxiv.org/abs/astro-ph/0509141), [1110.4402](https://arxiv.org/abs/1110.4402); Inman & Ali-Haïmoud, [1907.08129](https://arxiv.org/abs/1907.08129)]



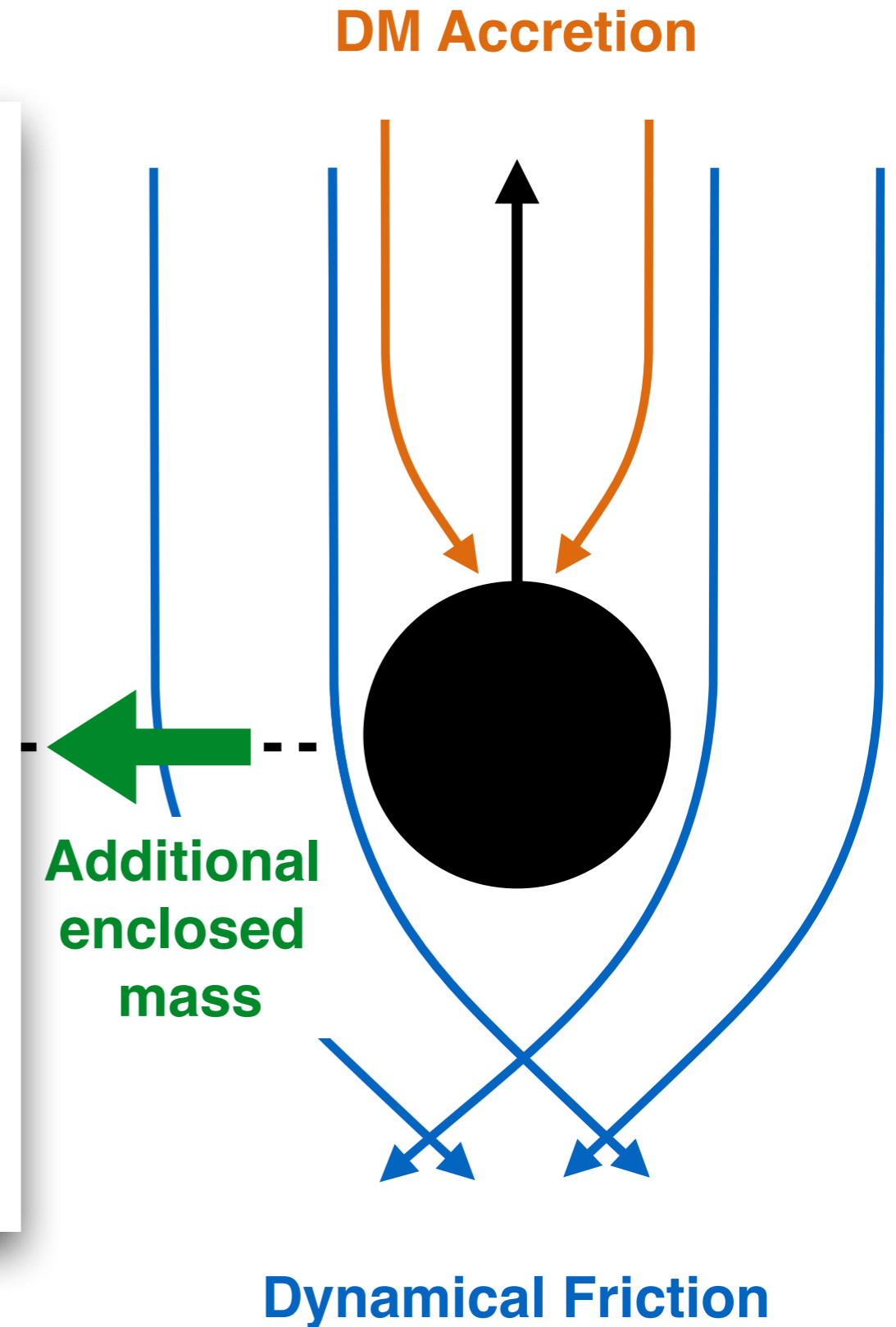
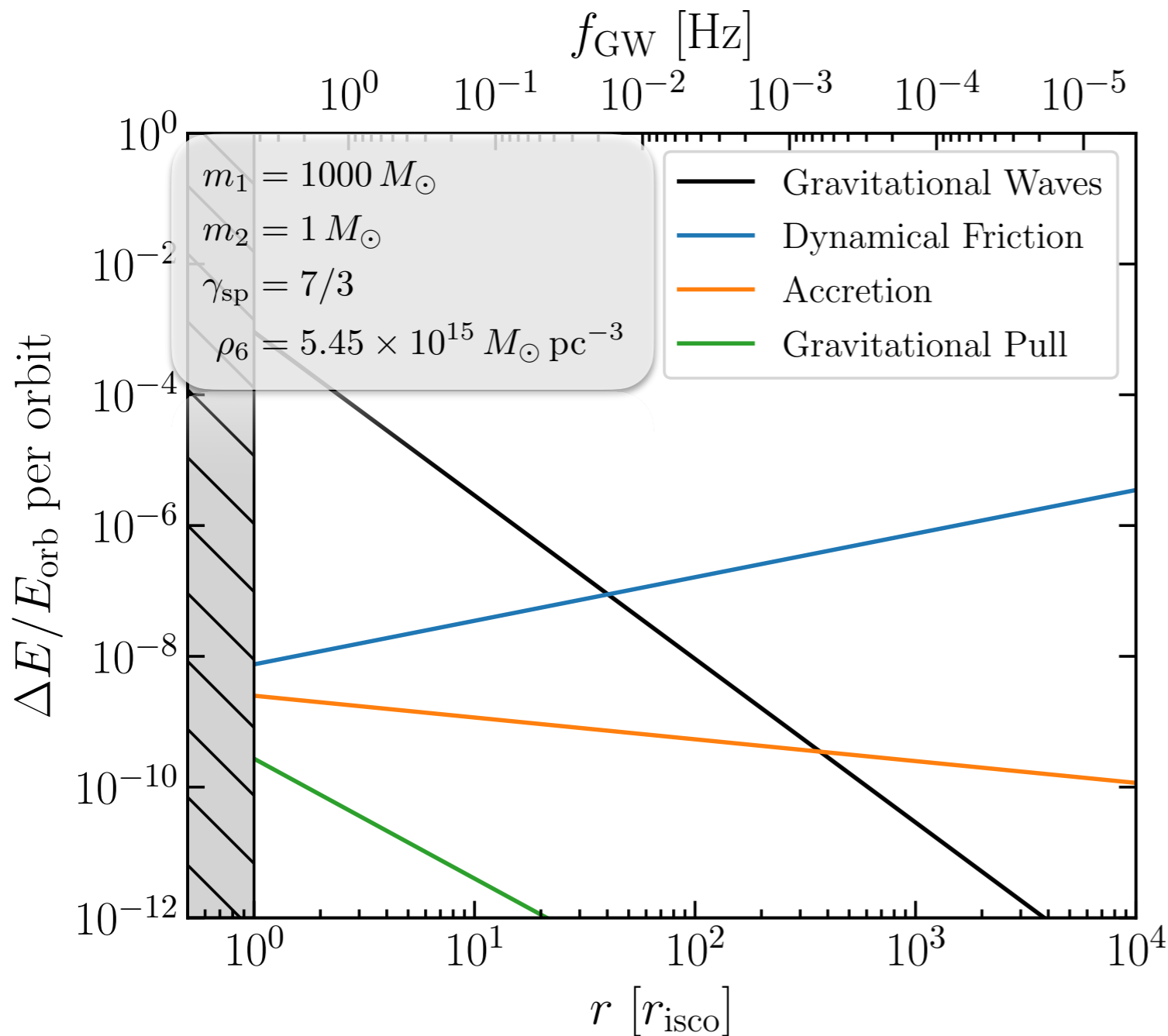
# Impact of DM Spikes



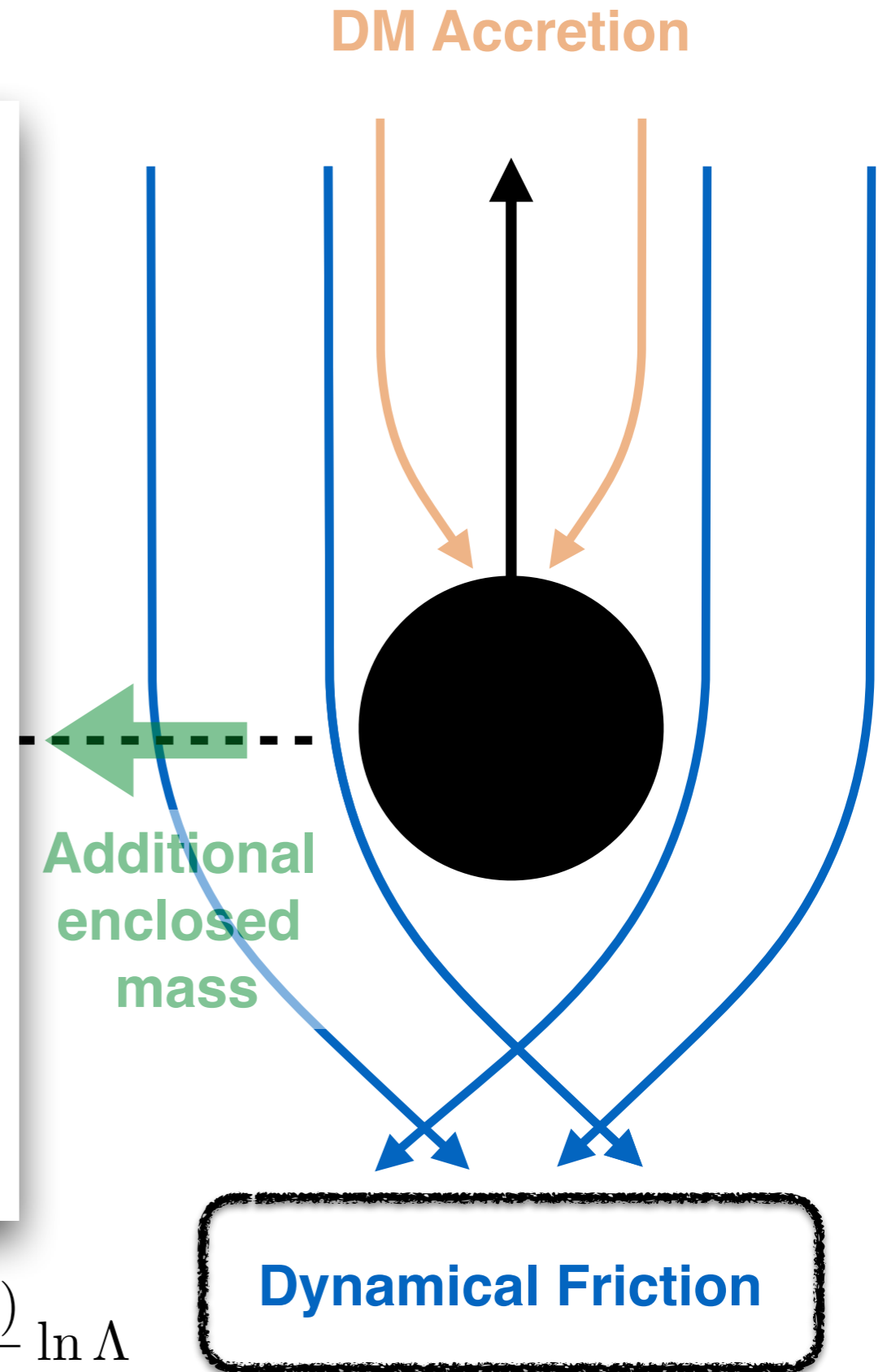
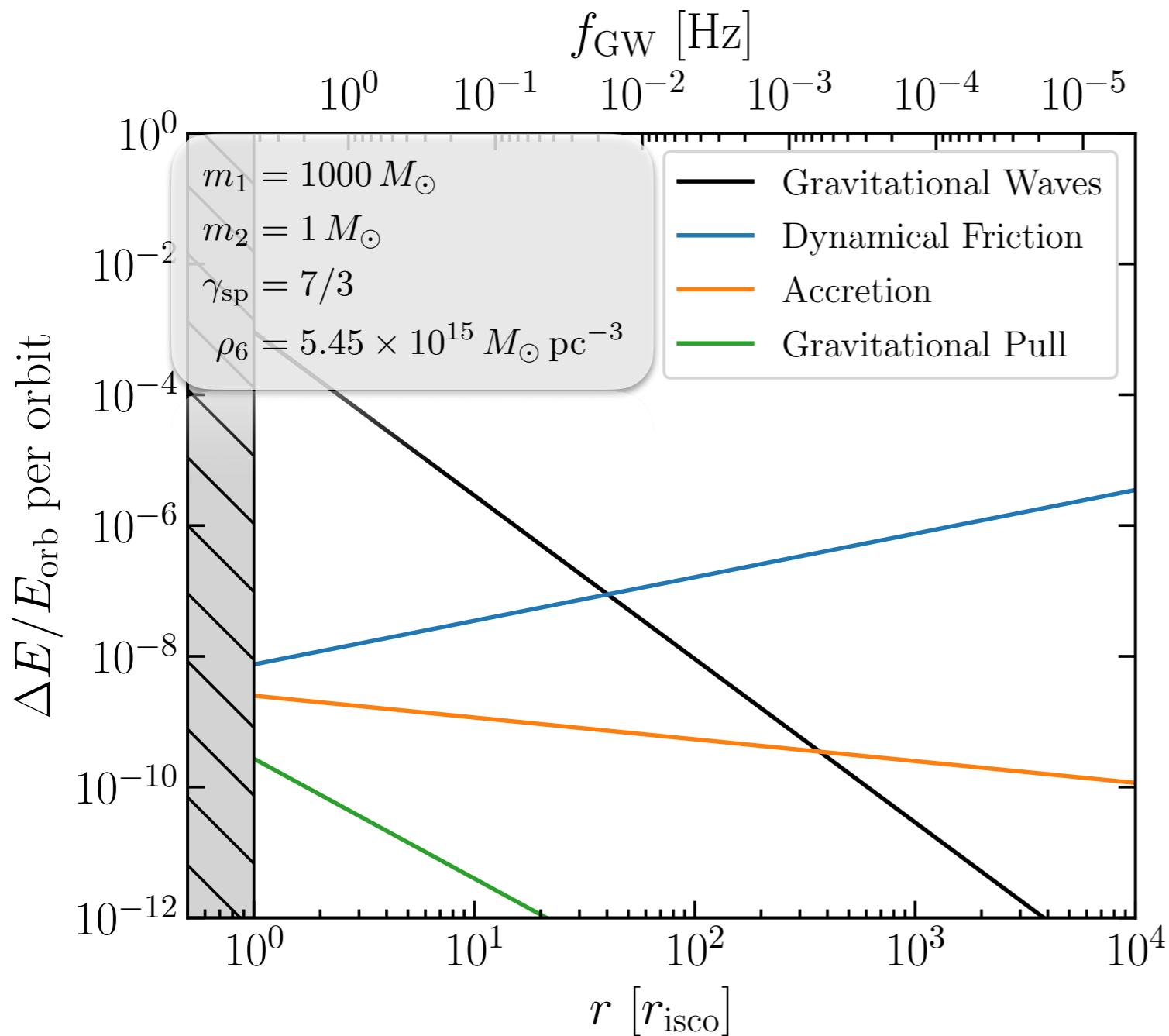
IMBH

[See e.g. Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]

# Impact of DM Spikes



# Impact of DM Spikes



$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$

[See e.g. Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]

# DM Spike Feedback (1)

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

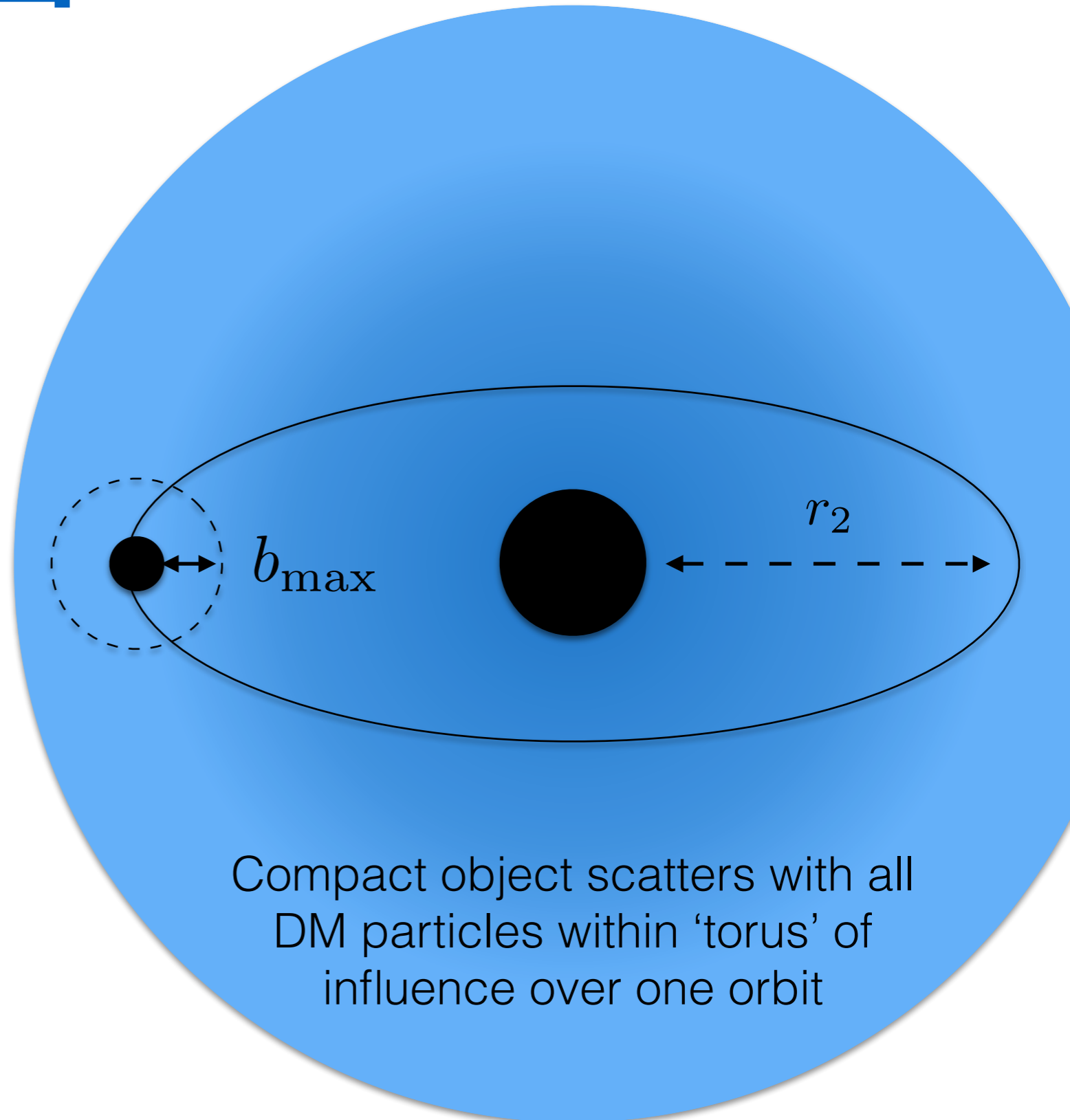
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a 'kick' through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$



# DM Spike Feedback (2)

Assuming everything evolves slowly compared to the orbital period:

$$\Delta f(\mathcal{E}) = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left( \frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E} - \Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$  - probability for a particle with energy  $\mathcal{E}$  to scatter and receive a 'kick'  $\Delta\mathcal{E}$

$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$  - total probability for a particle with energy  $\mathcal{E}$  to scatter

Formalism valid for small  $q = m_2/m_1$

Code available online:

[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)

# DM Spike Feedback (2)

Assuming everything evolves slowly compared to the orbital period:

$$\Delta f(\mathcal{E}) = -p_{\mathcal{E}} f(\mathcal{E}) +$$

Particles scattering from  
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

$$\int \left( \frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E} - \Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

Particles scattering from  
 $\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

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# DM Spike Feedback (2)

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{df(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) +$$

Particles scattering from  
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

$$\int \left( \frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E} - \Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

Particles scattering from  
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$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E} \quad \text{- total probability for a particle with energy } \mathcal{E} \text{ to scatter}$$

Formalism valid for small  $q = m_2/m_1$

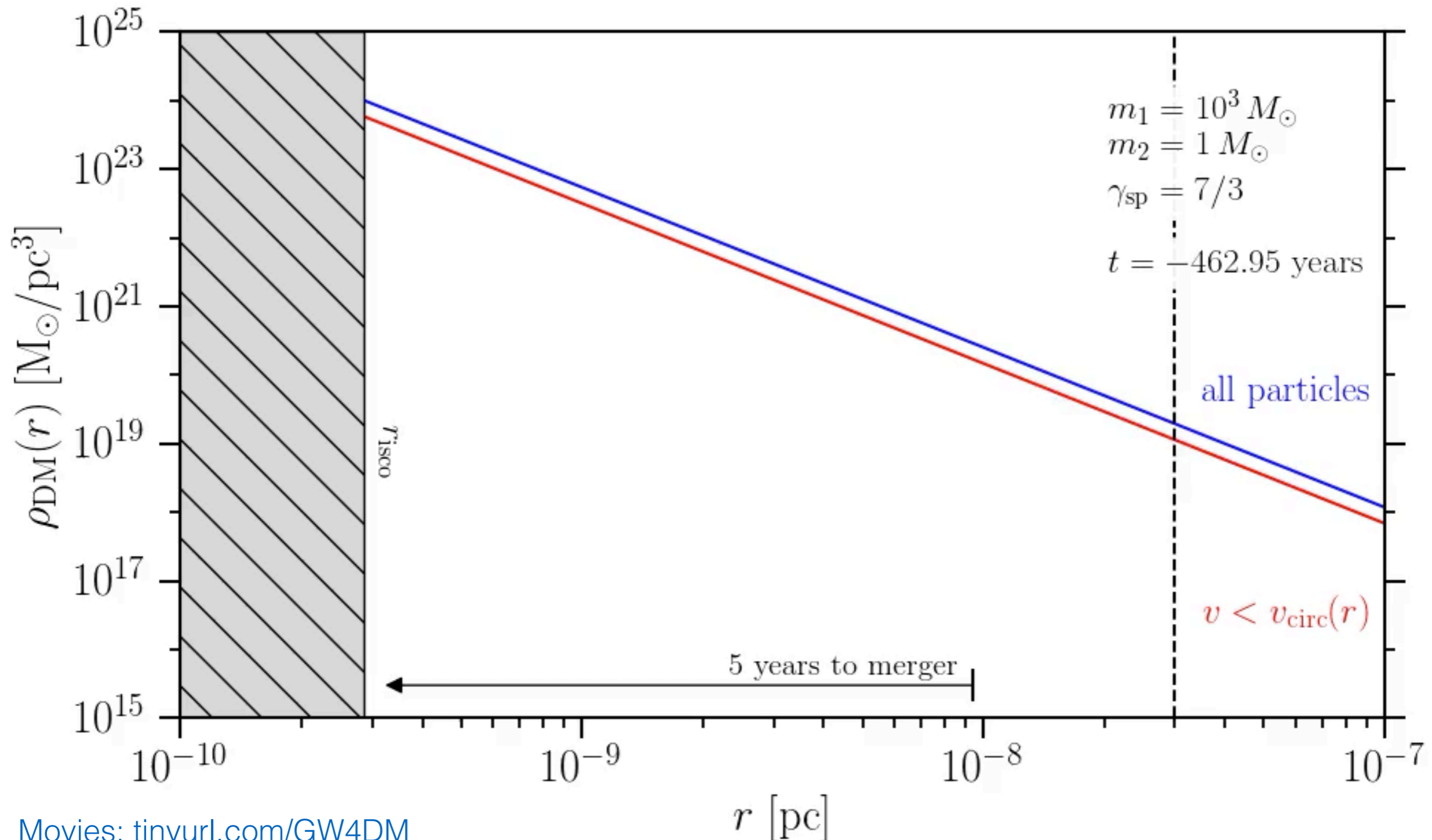
Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)

# Feedback in Action

[BJK, Nichols, Gaggero & Bertone, 2002.12811]

[Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

Need to include **feedback** on the DM spike:



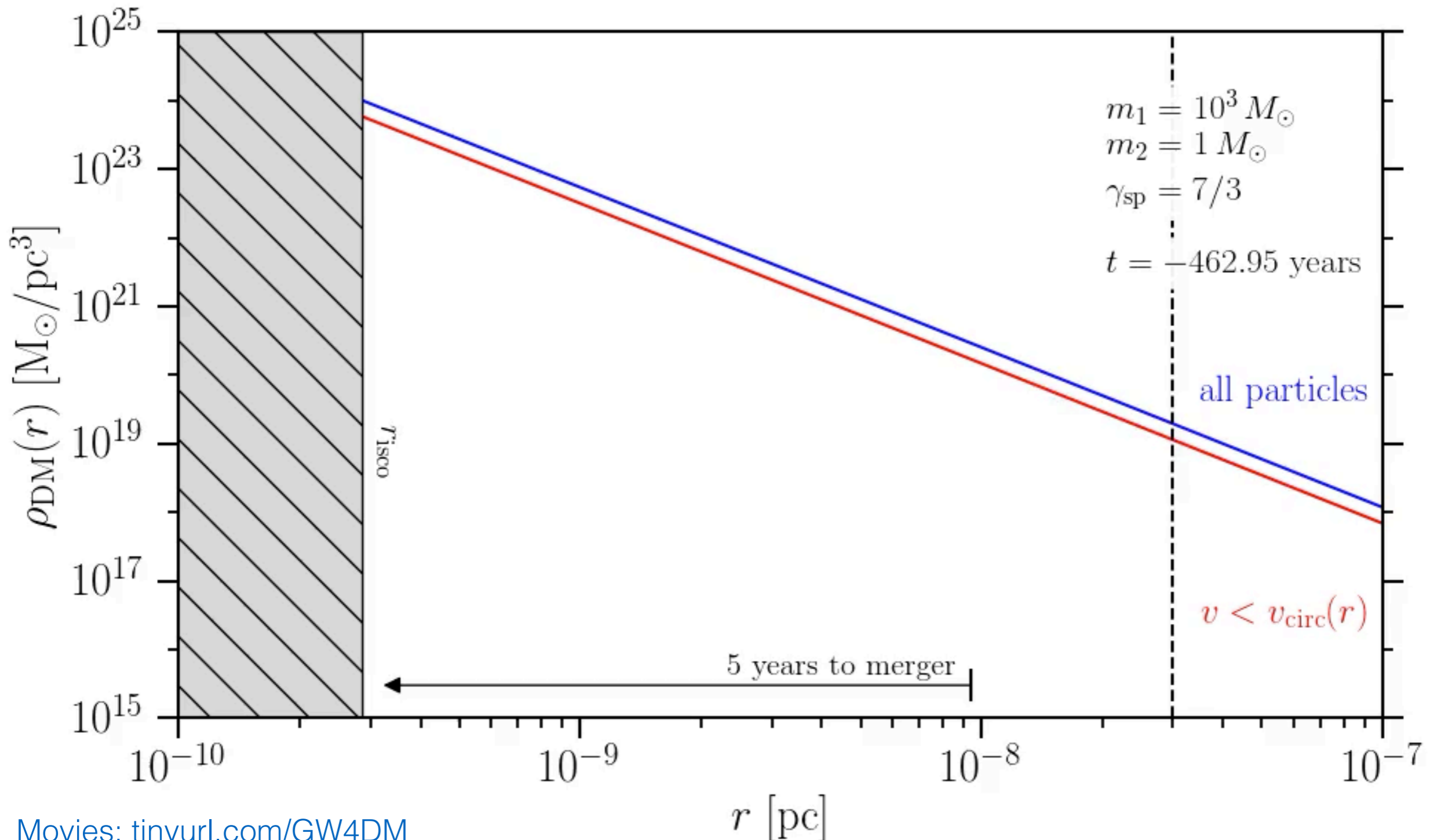


# Feedback in Action

[BJK, Nichols, Gaggero & Bertone, 2002.12811]

[Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

Need to include **feedback** on the DM spike:

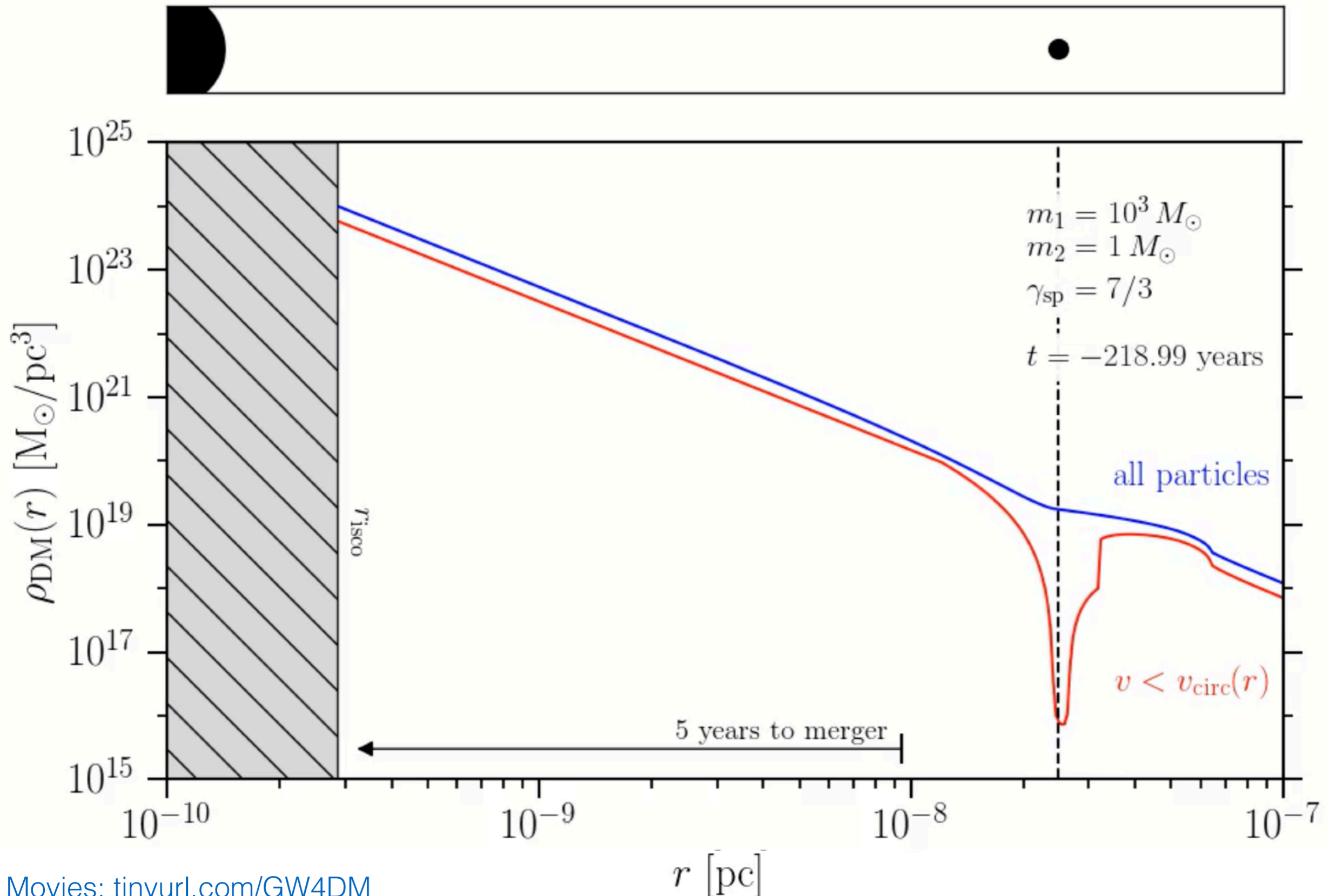


# Feedback in Action

[BJK, Nichols, Gaggero & Bertone, 2002.12811]

[Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

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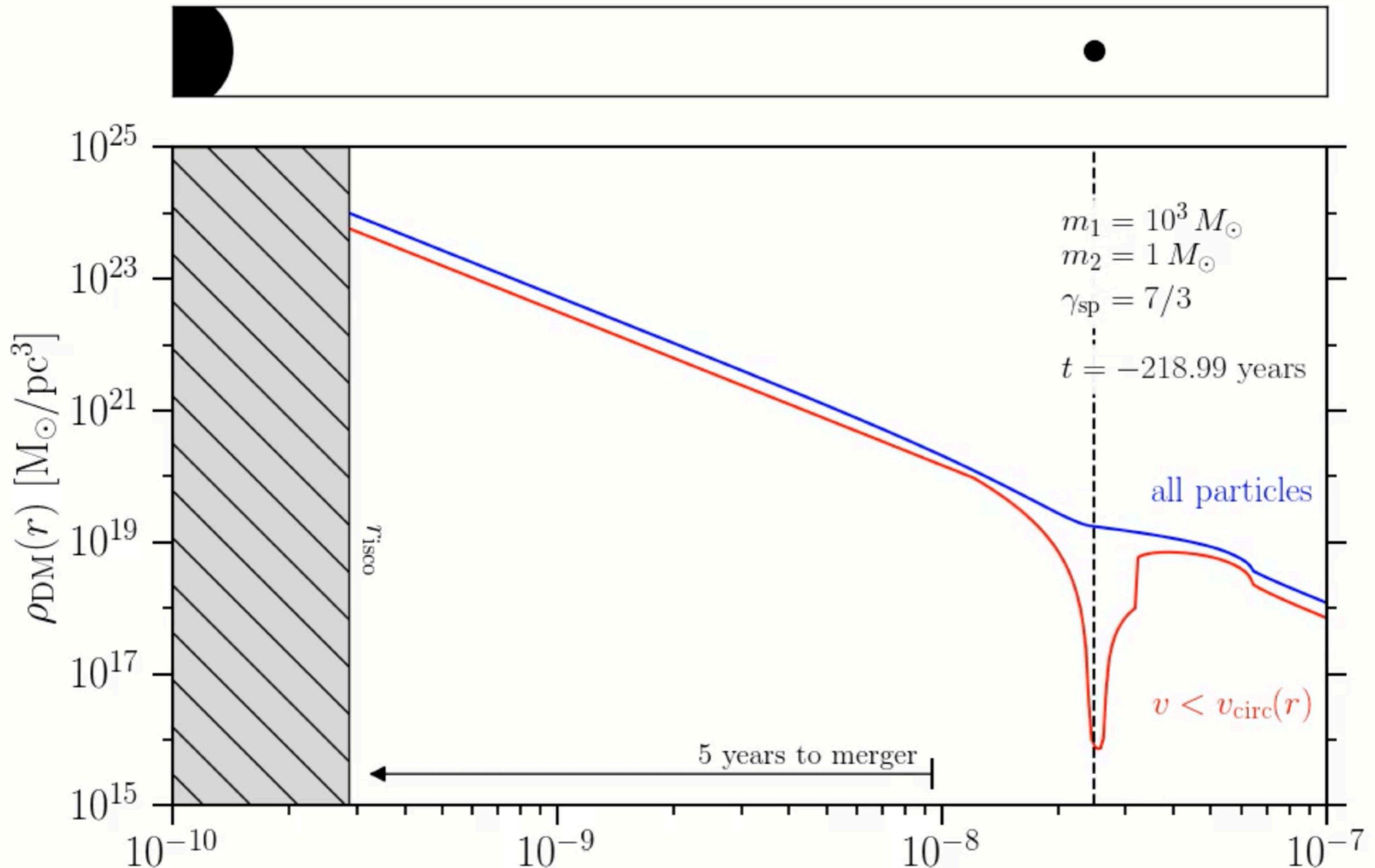


# Feedback in Action

[BJK, Nichols, Gaggero & Bertone, 2002.12811]

[Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

Need to include **feedback** on the DM spike:

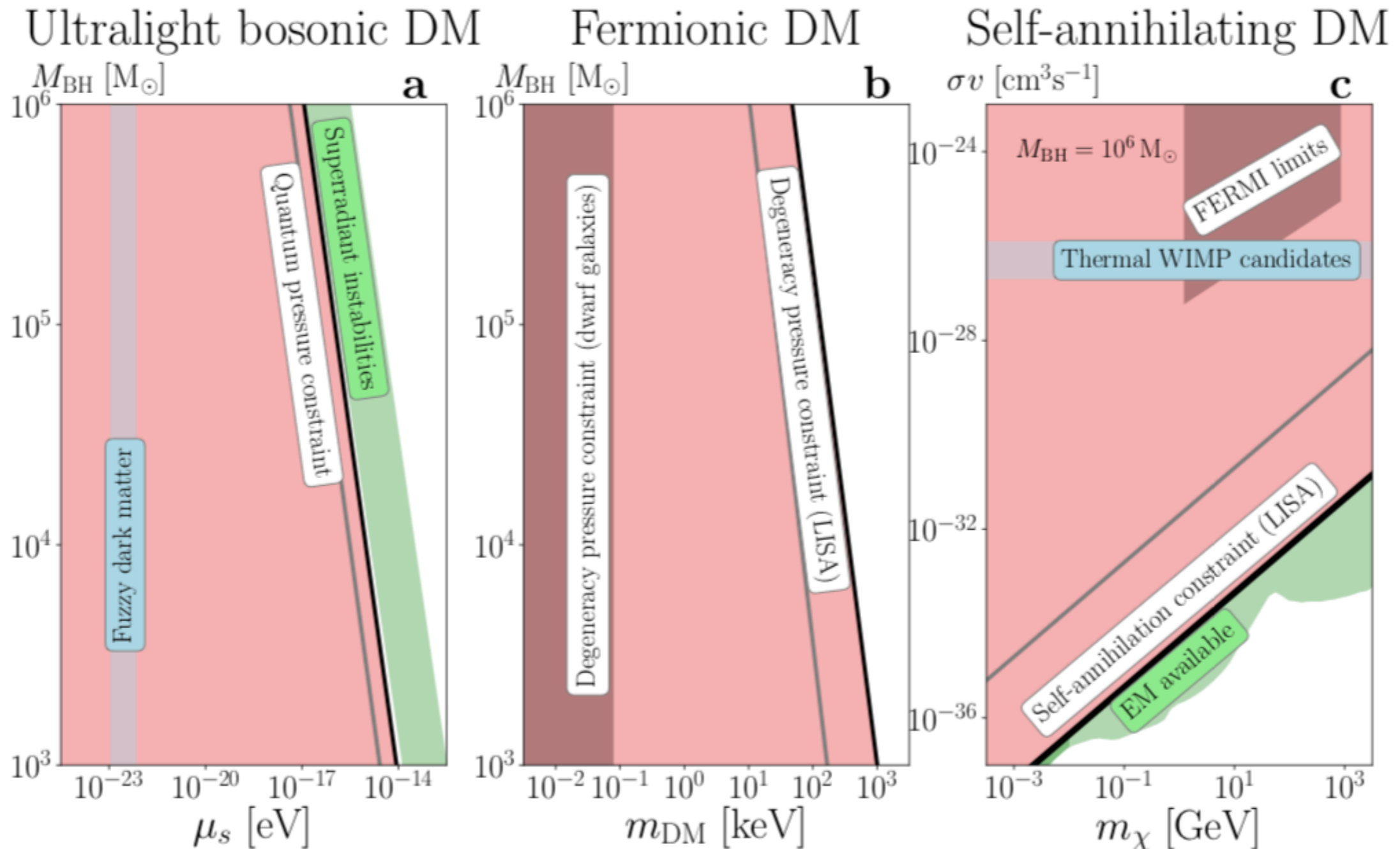


Movies: [tinyurl.com/GW4DM](https://tinyurl.com/GW4DM)

# Probe the nature of DM!

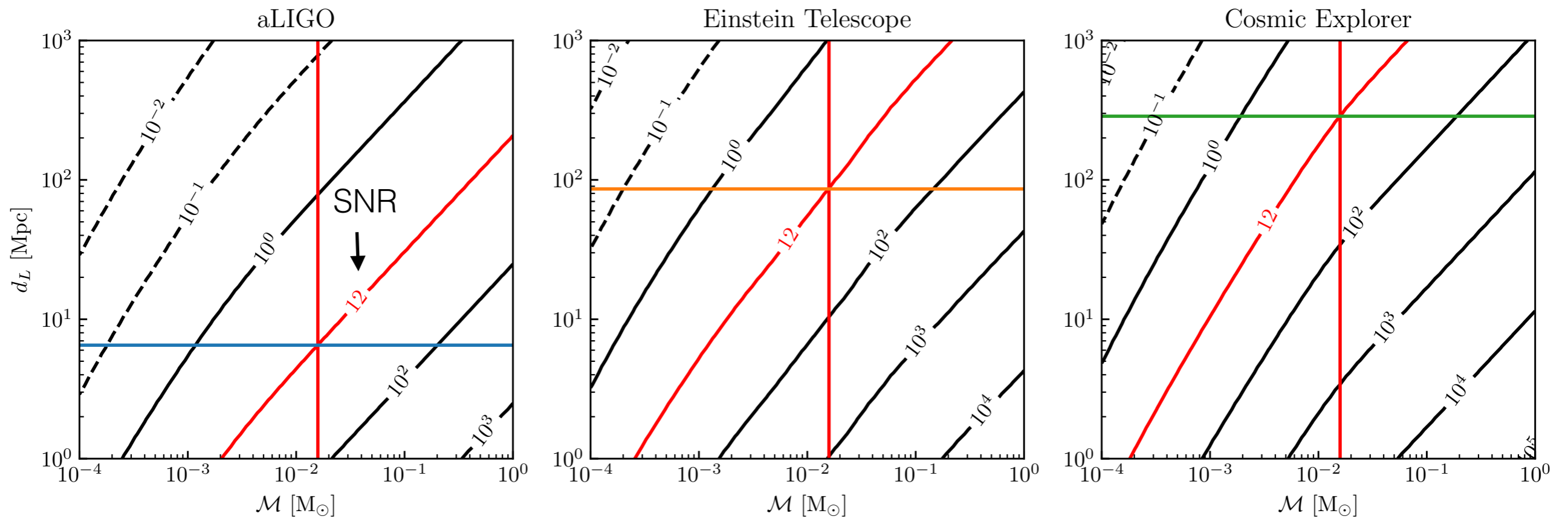
Red regions would be ruled out by observation of a DM spike!

[[1906.11845](#)]



[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, [1905.01238](#)]

SNR as a function of chirp mass and luminosity distance:

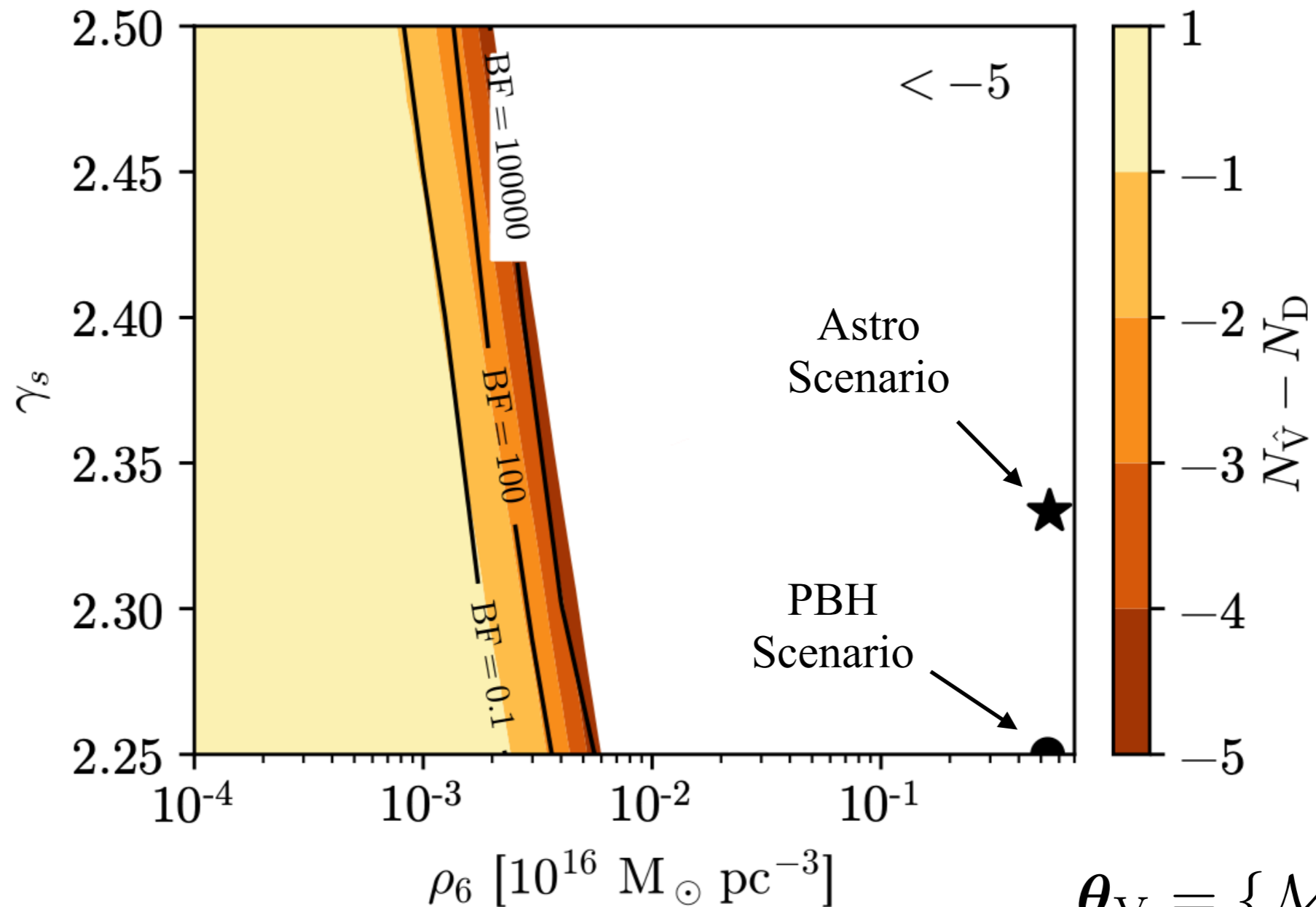


# Discoverability

[Code available online:  
<https://github.com/adam-coogan/pydd>]

$$q = m_2/m_1$$

We'll call a DM spike **discoverable** if it can be distinguished from a GR-in-vacuum system.

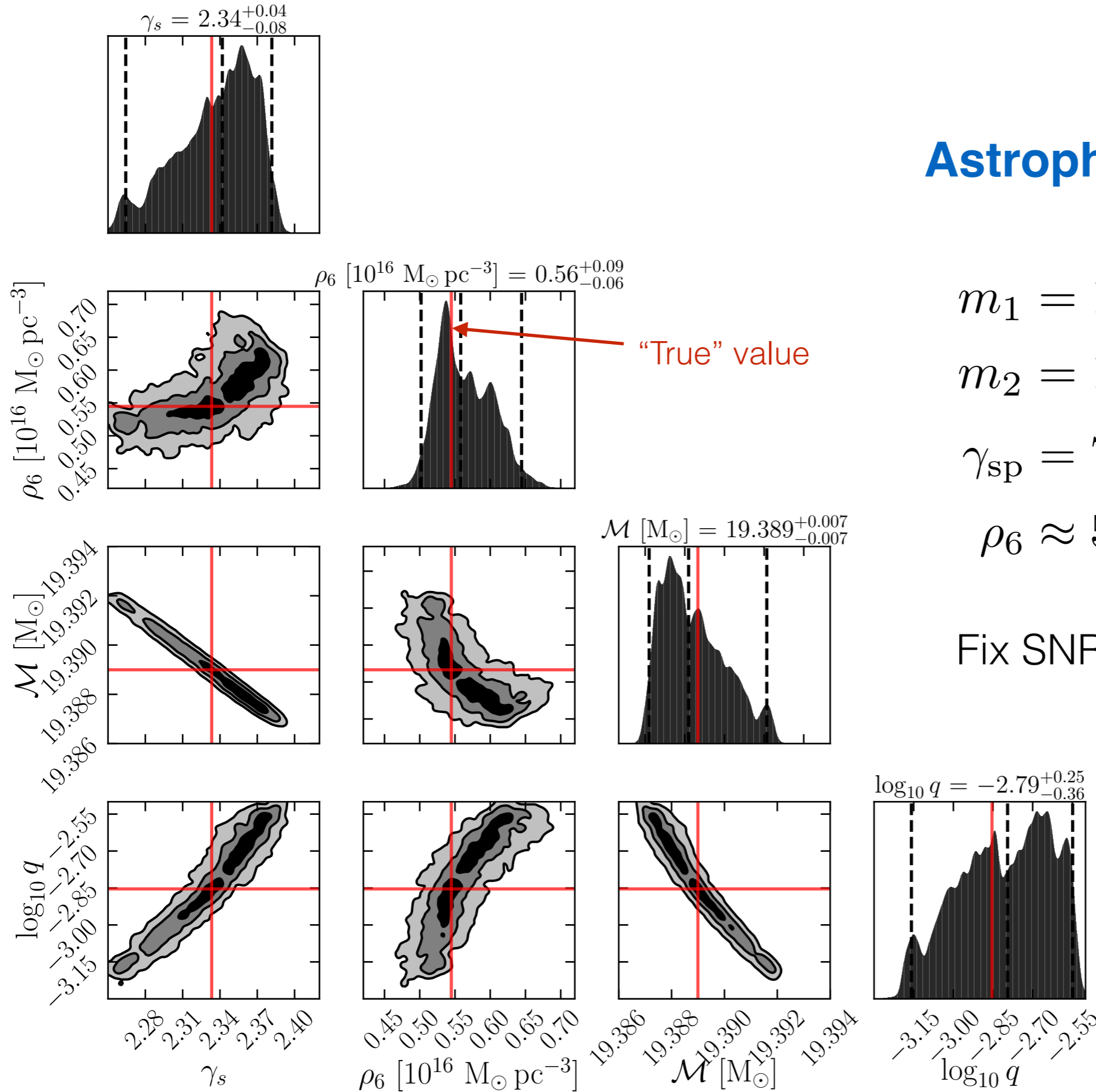


Compute Bayes Factor (BF) comparing Bayesian evidence for **V**acuum and **D**ressed systems, with parameters:

$$\theta_V = \{\mathcal{M}\}$$

vs.

$$\theta_D = \{\gamma_{sp}, \rho_6, \mathcal{M}, \log_{10} q\}$$



## Astrophysical scenario

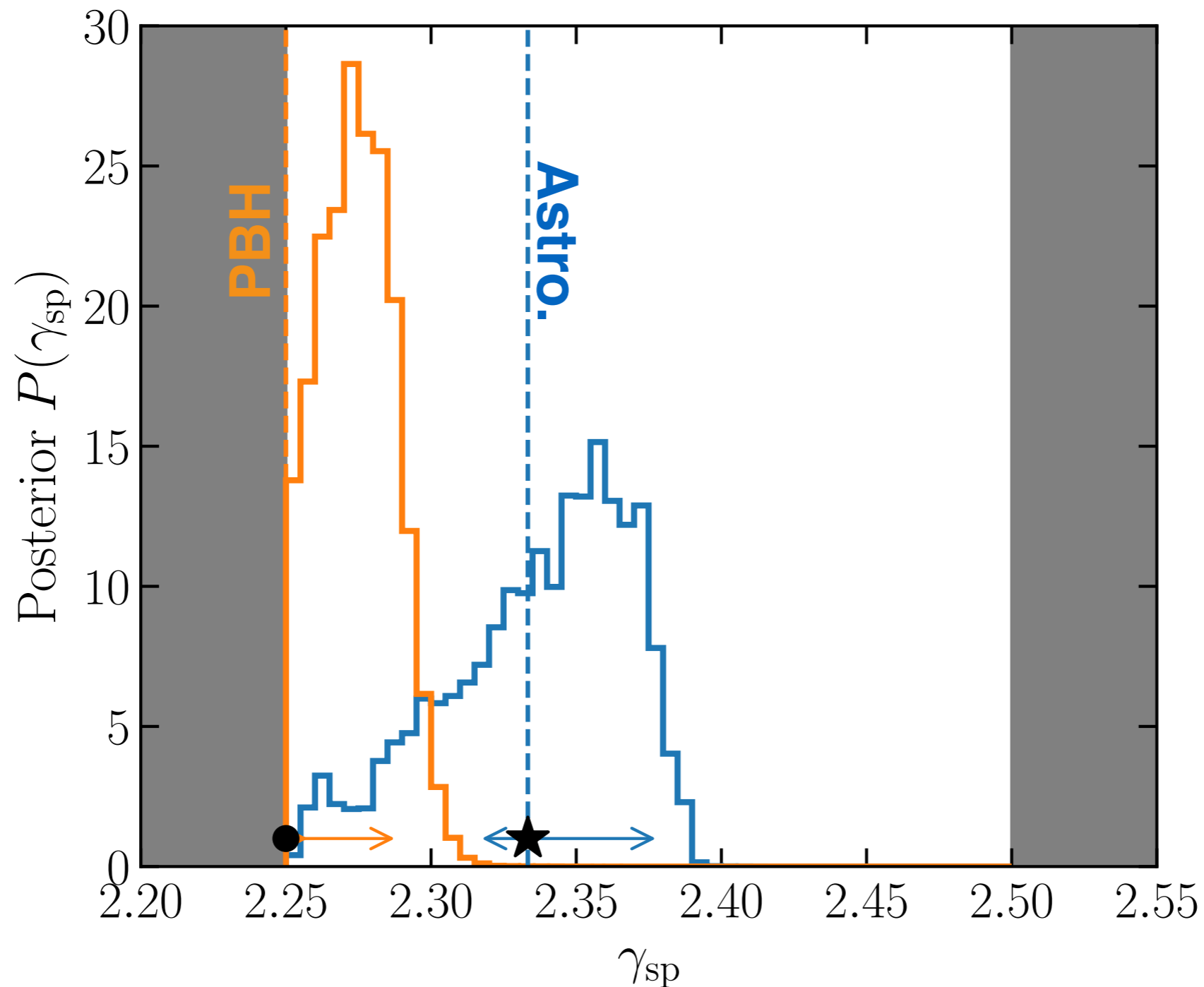
$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

$$\gamma_{\text{sp}} = 7/3 \approx 2.3333 \dots$$

$$\rho_6 \approx 5.45 \times 10^{15} M_\odot \text{pc}^{-3}$$

Fix SNR = 15 ( $\sim 76$  Mpc)



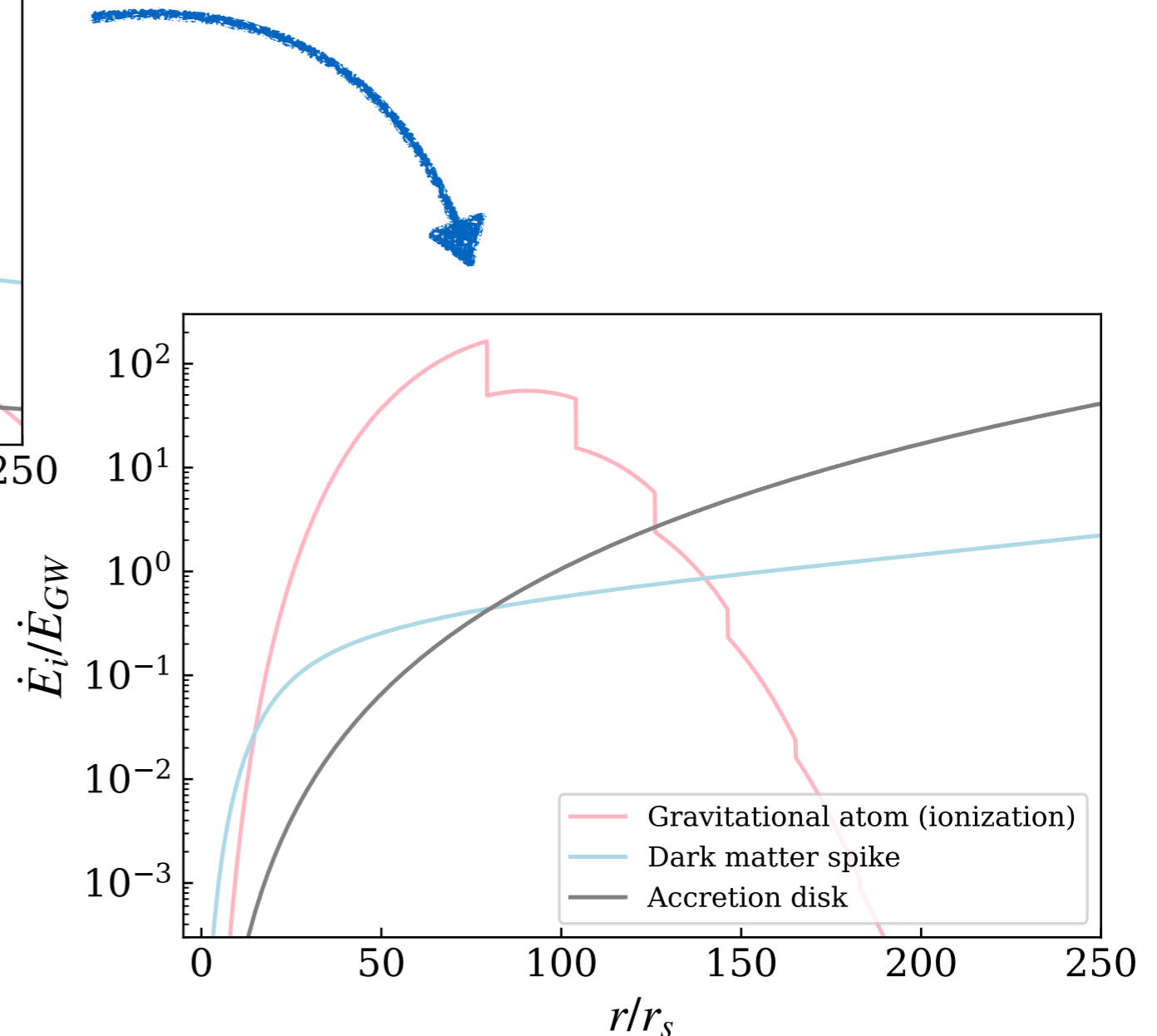
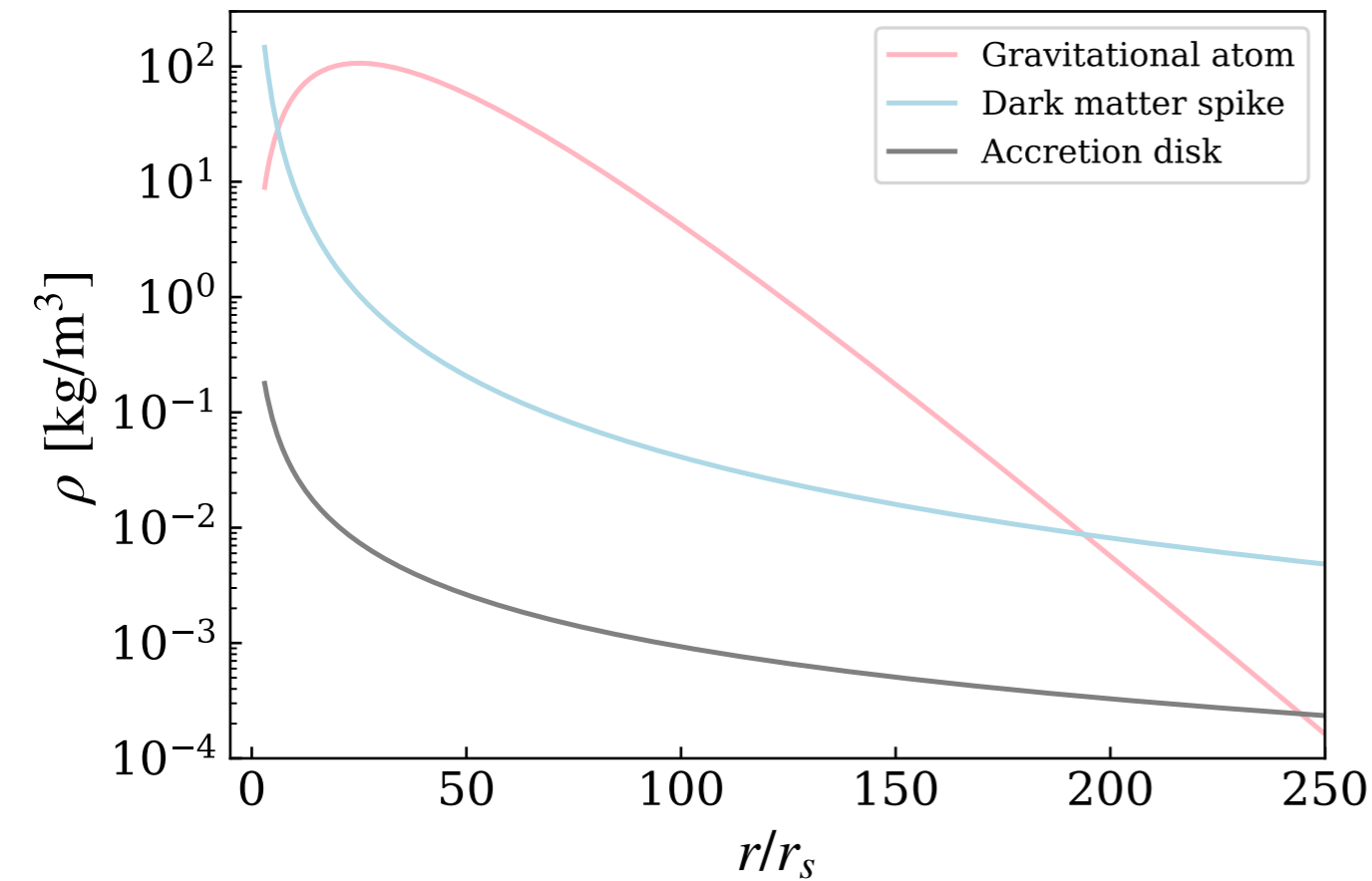
We may be able to distinguish different *shapes* of spike  
 → Different DM models and formation mechanisms!



# Distinguishability

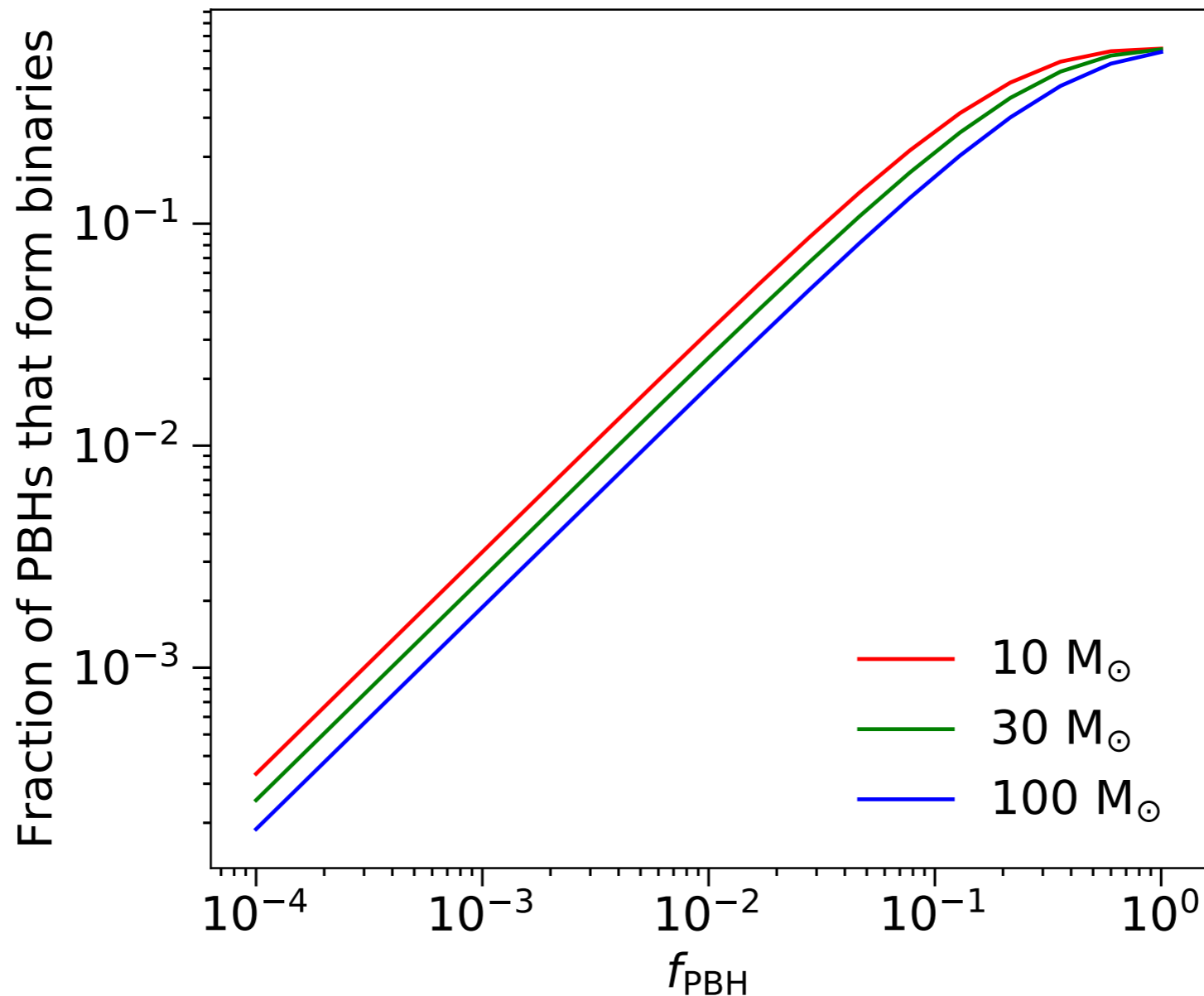
[Cole et al. (including **BJK**), [2211.01362](#)]

Distinguishable from other sources of dephasing:



# Early Universe Binaries

If  $f \sim 1$ , the relative density of PBHs *equals* the background radiation density at matter-radiation equality. All PBHs form binaries...



As  $f$  decreases, only 'nearby' pairs form binaries.

# PBH Binary Population

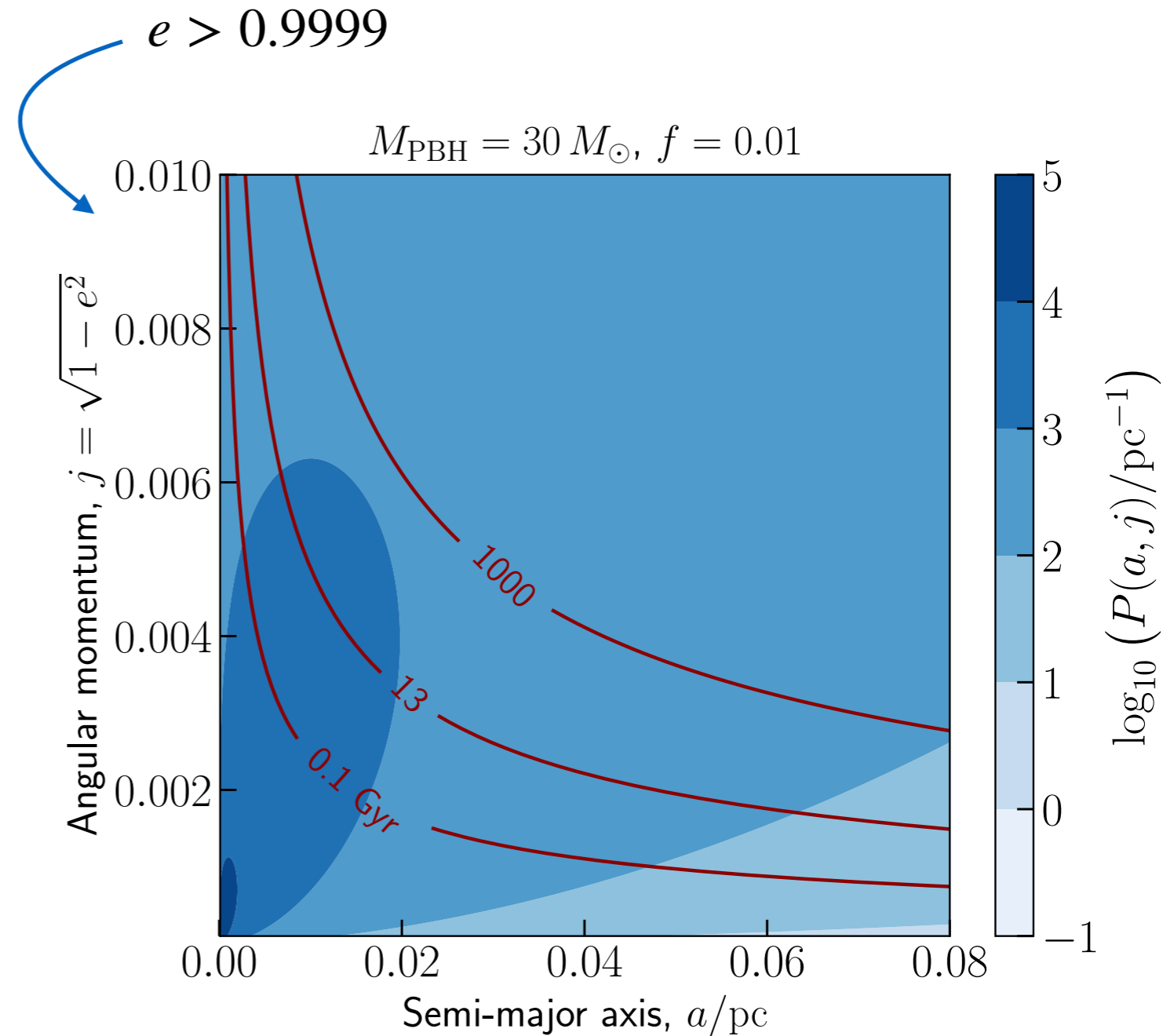
Randomly distributed  
(unclustered) PBHs

Angular momentum set by  
torques from smooth density  
perturbations and *all other PBHs*

Close, eccentric binaries  
merge today:

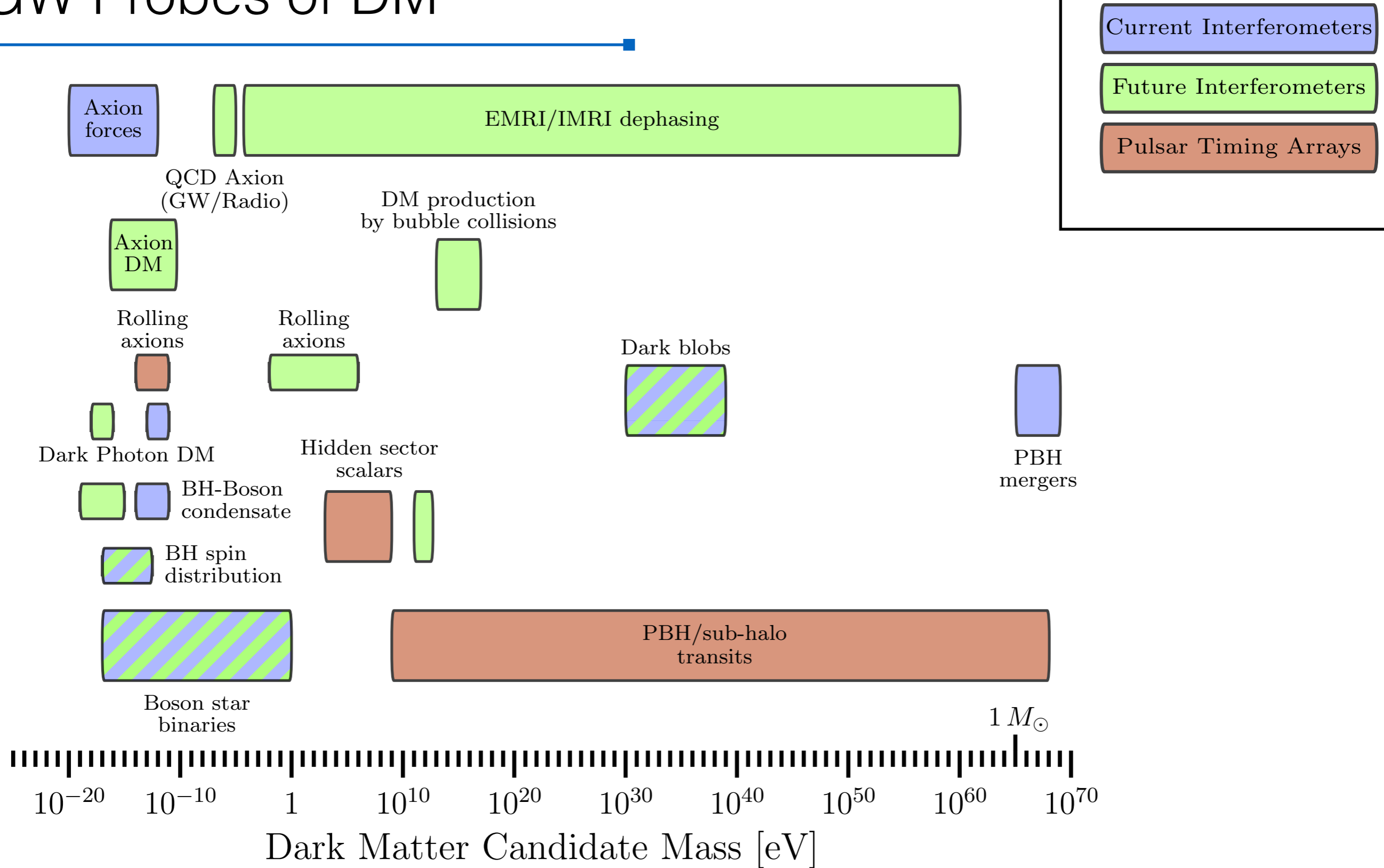
$$t_{\text{merge}} = \frac{3c^5}{170G_N^3} \frac{a^4 j^7}{M_{\text{PBH}}^3}$$

$$j = \sqrt{1 - e^2}$$



[Ali-Haïmoud et al., 1709.06576,  
**BJK**, Gaggero & Bertone, 1805.09034]

# GW Probes of DM



[Bertone, Croon, Amin, Boddy, **BJK**, Mack, Natarajan, Opferkuch, Schutz, Takhistov, Weniger, Yu, SciPost Phys. Core 3, 007 (2020), [1907.10610](https://arxiv.org/abs/1907.10610)]