

Dark Matter in Extreme Environments

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INFN, Pisa
26 October 2023



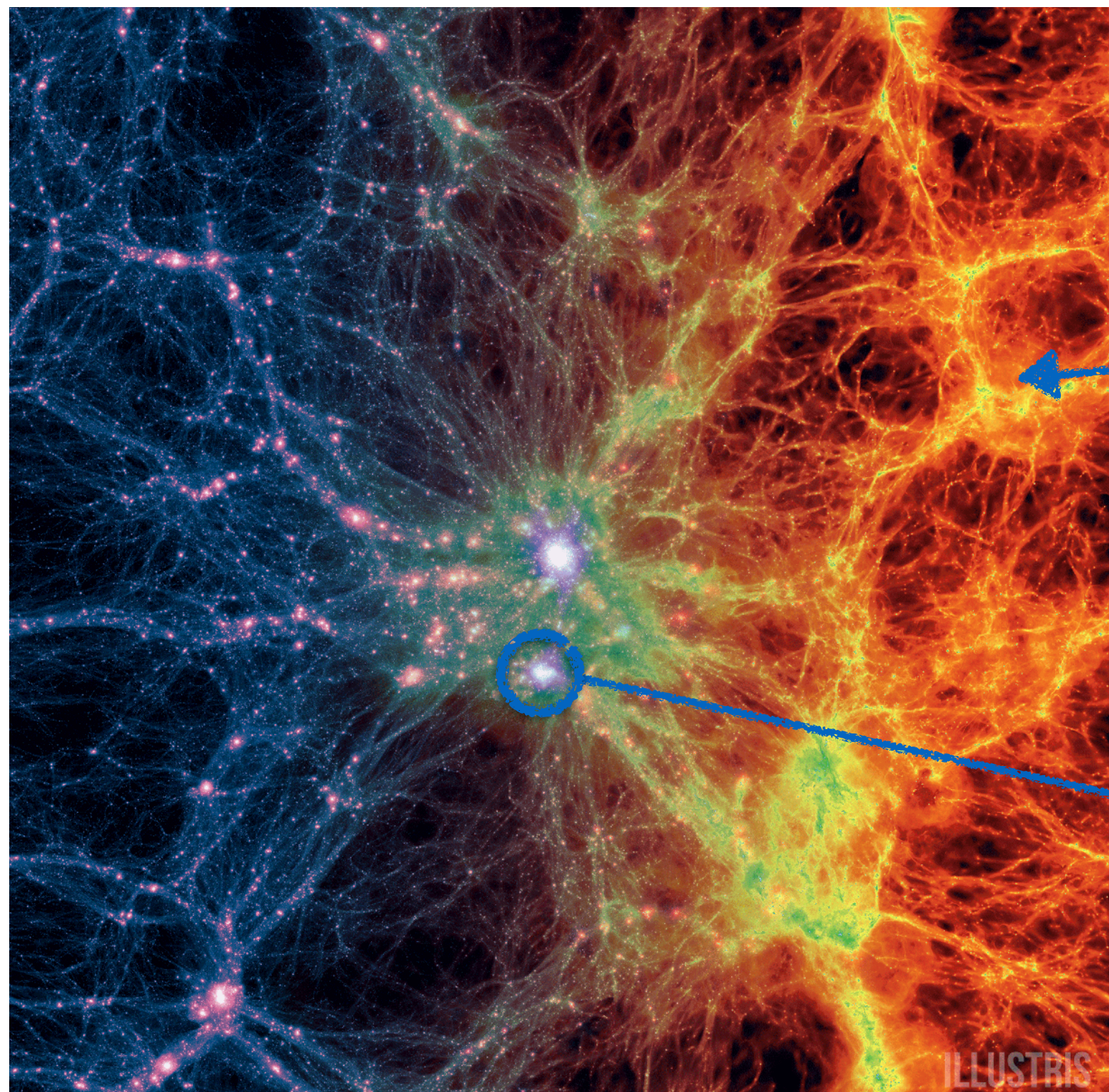
kavanagh@ifca.unican.es



@BradleyKavanagh



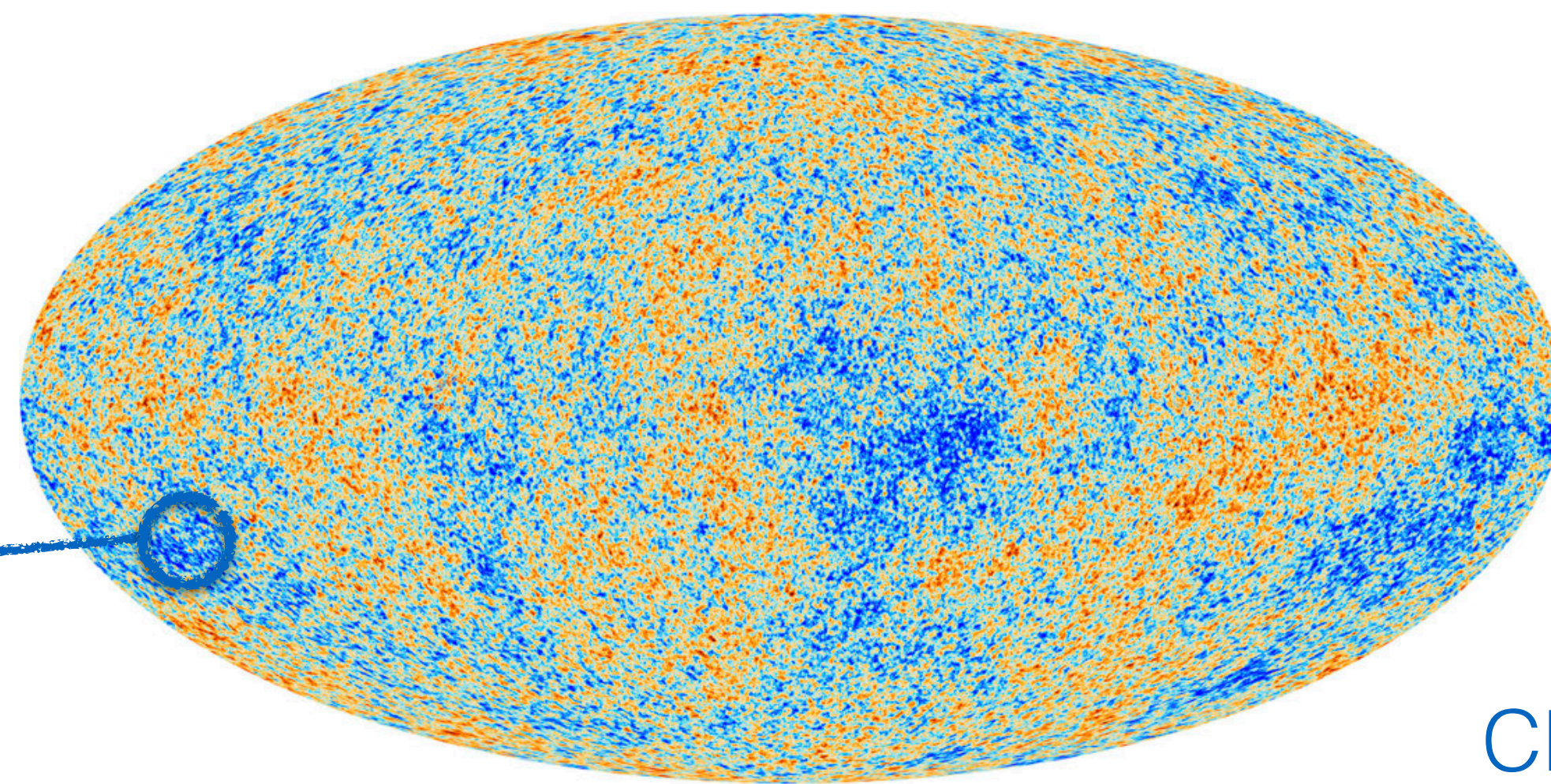
Evidence for Dark Matter



Galaxy clusters

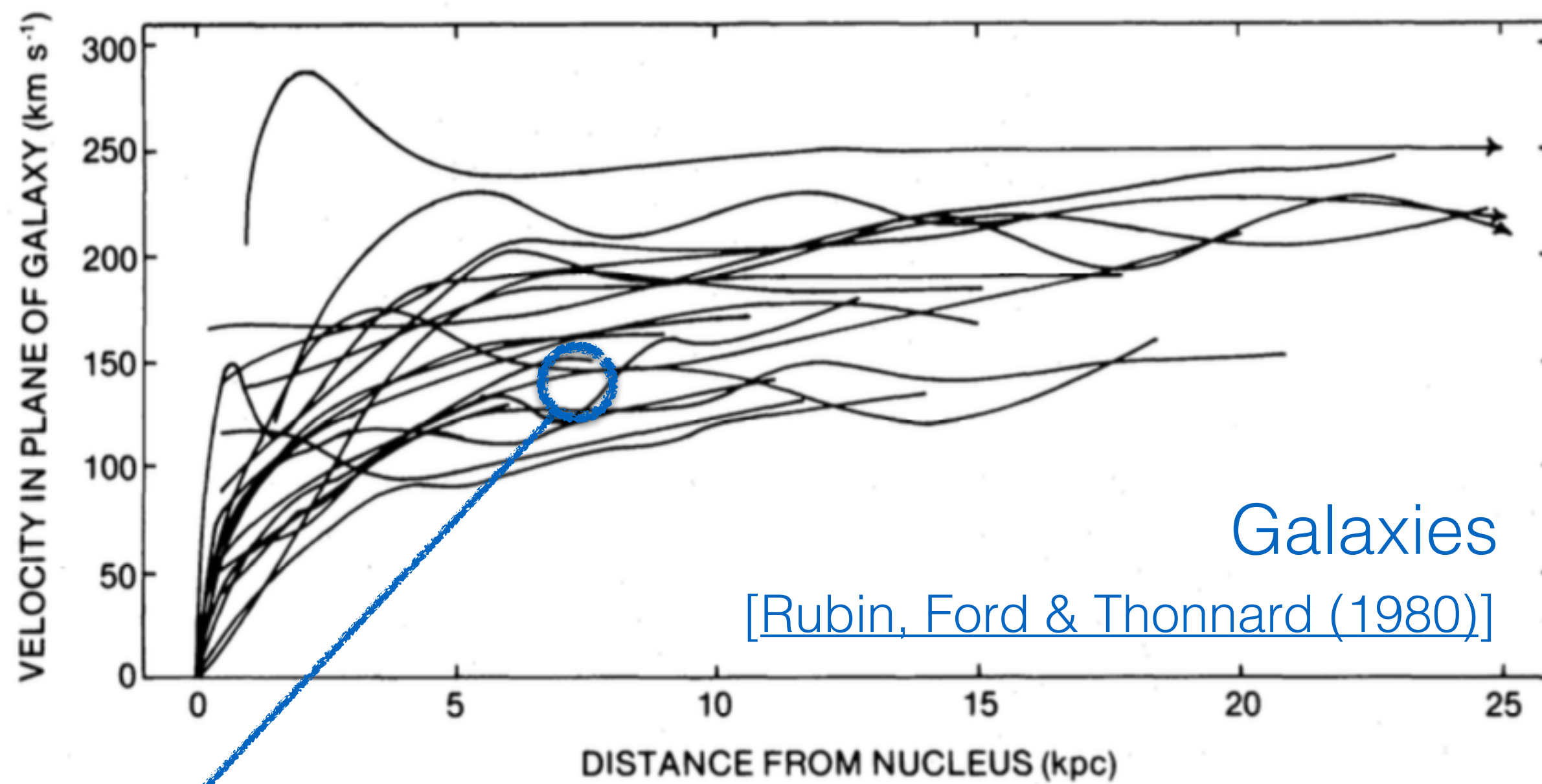
[Illustris, [1405.2921](#)]

[[astro-ph/0006397](#)]



CMB

[Planck, [1502.01589](#)]

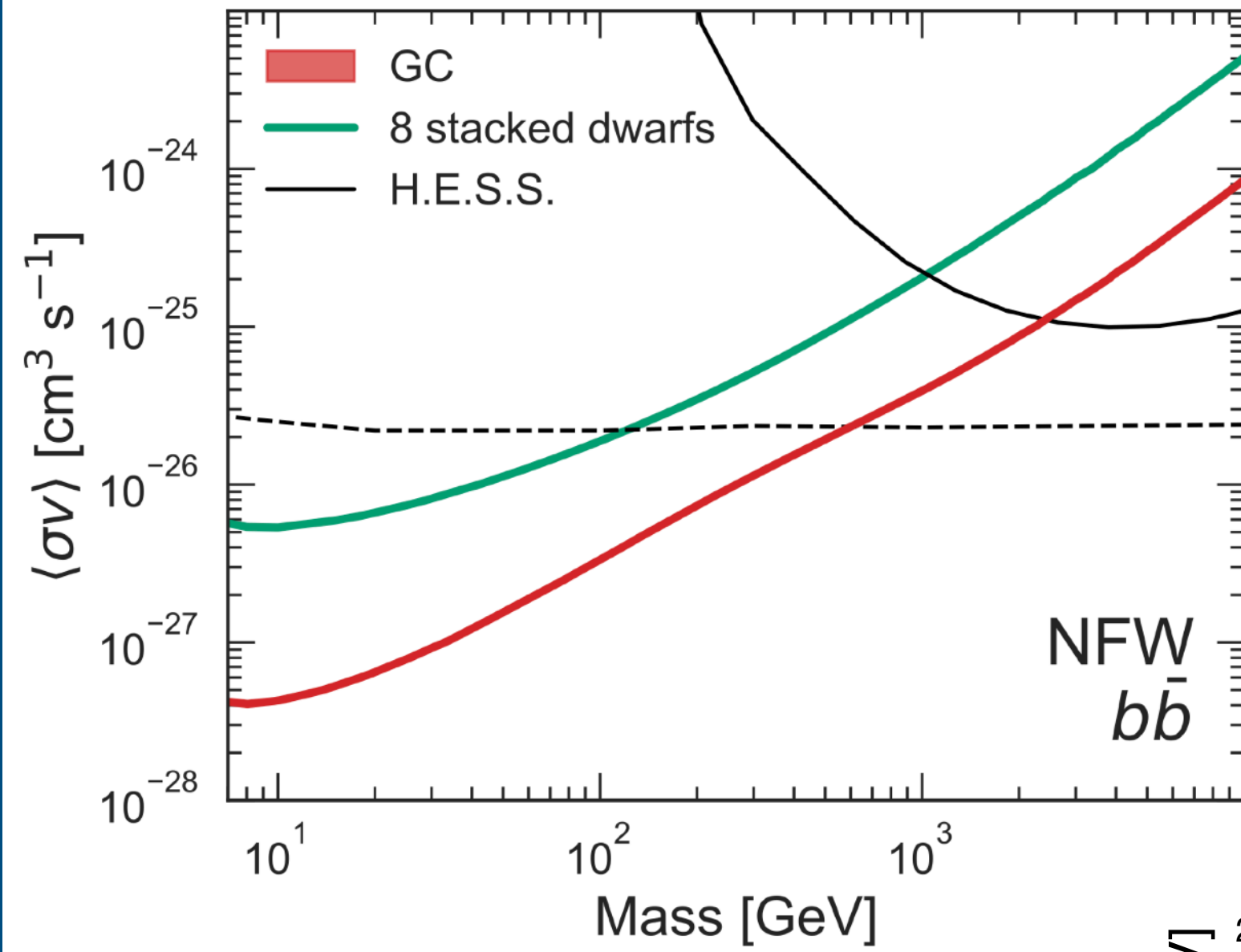


Galaxies

[Rubin, Ford & Thonnard (1980)]

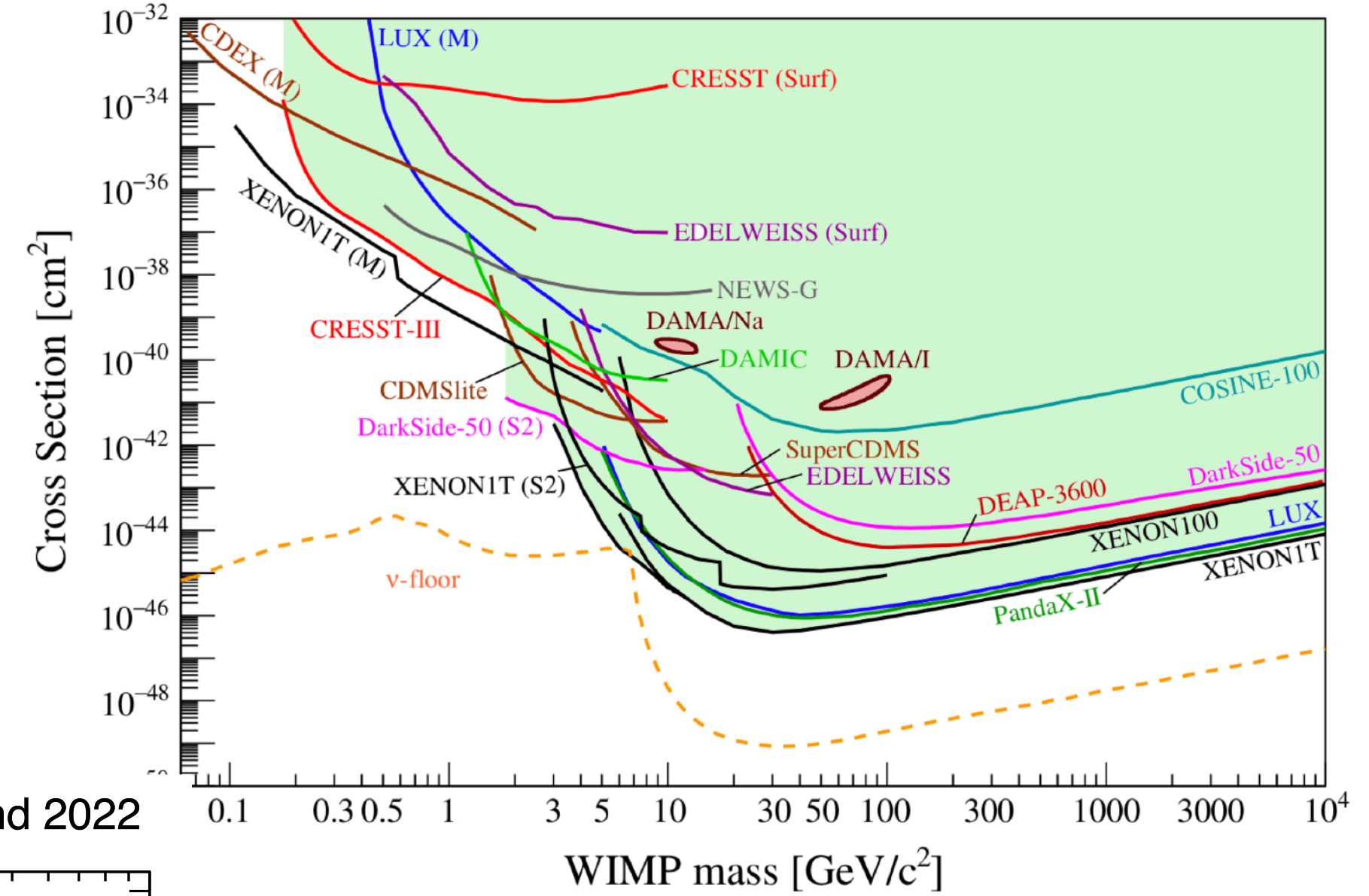
Indirect searches

[Abazajian et al., 2003.10416]



Direct Searches

[APPEC, 2104.07634]

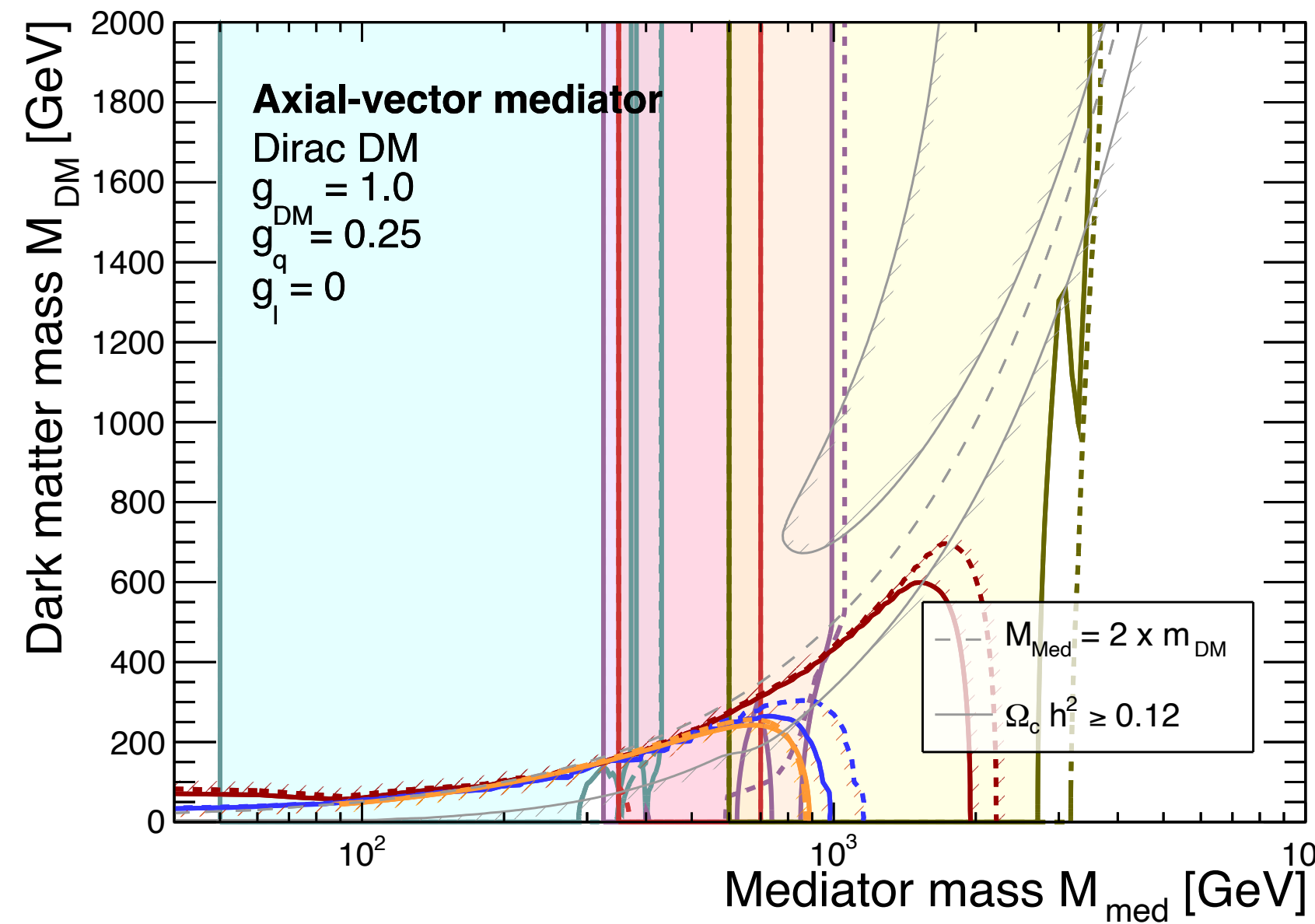


Collider Searches

[CMS, DM Summary Plots]

CMS Preliminary

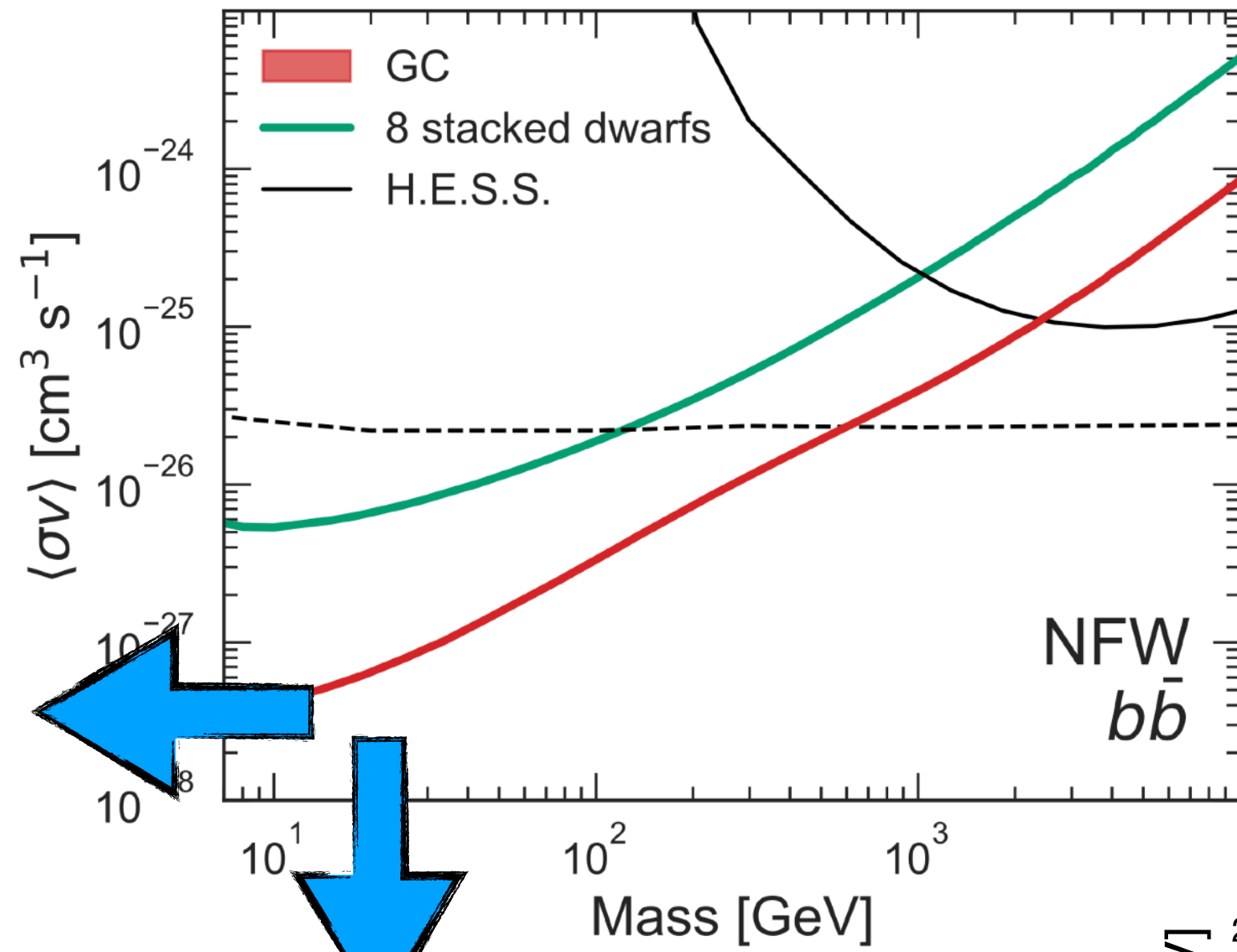
Moriond 2022



New technologies, lower thresholds, larger exposures, higher energies...

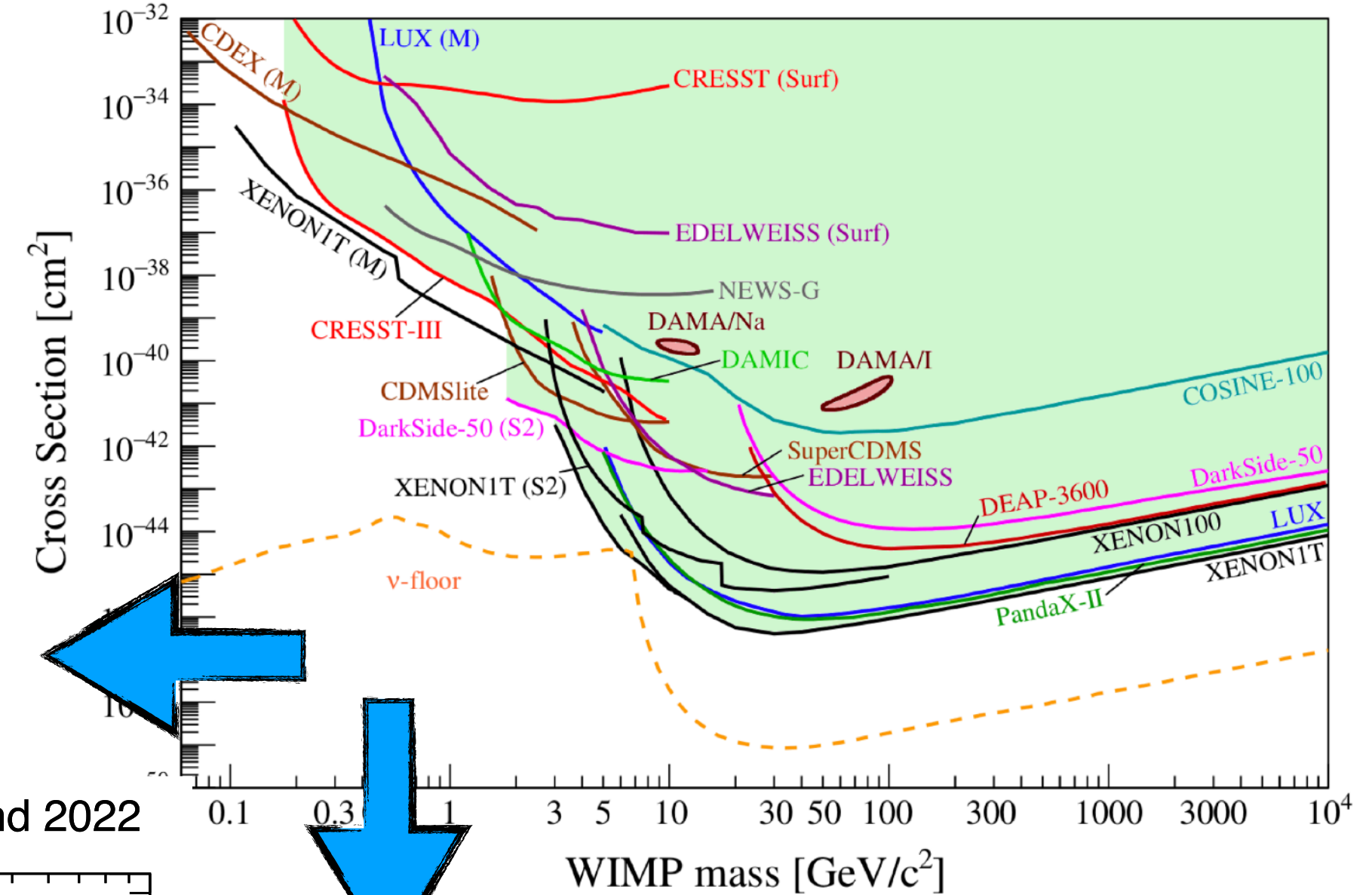
Indirect searches

[Abazajian et al., 2003.10416]



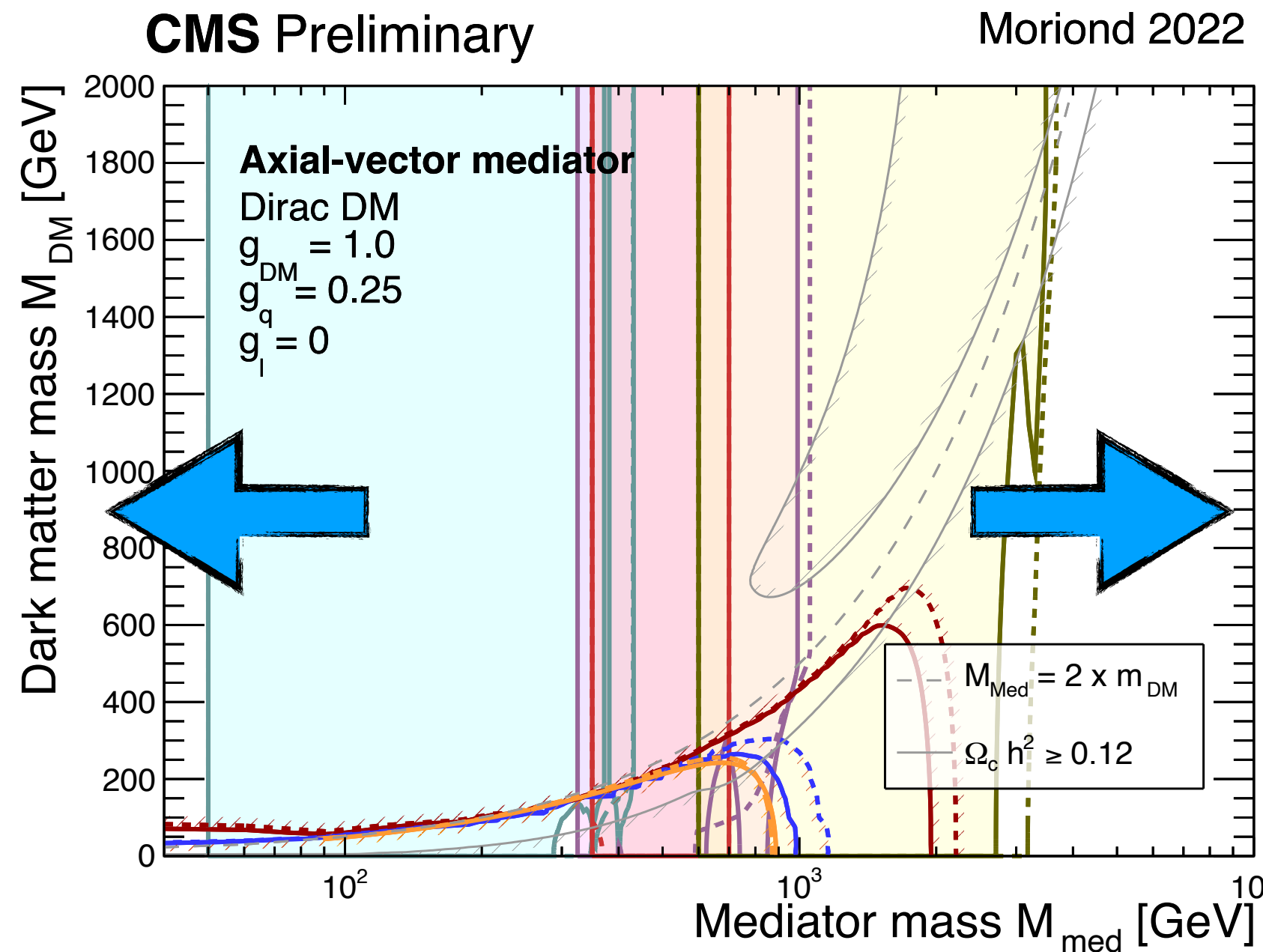
Direct Searches

[APPEC, 2104.07634]



Collider Searches

[CMS, DM Summary Plots]

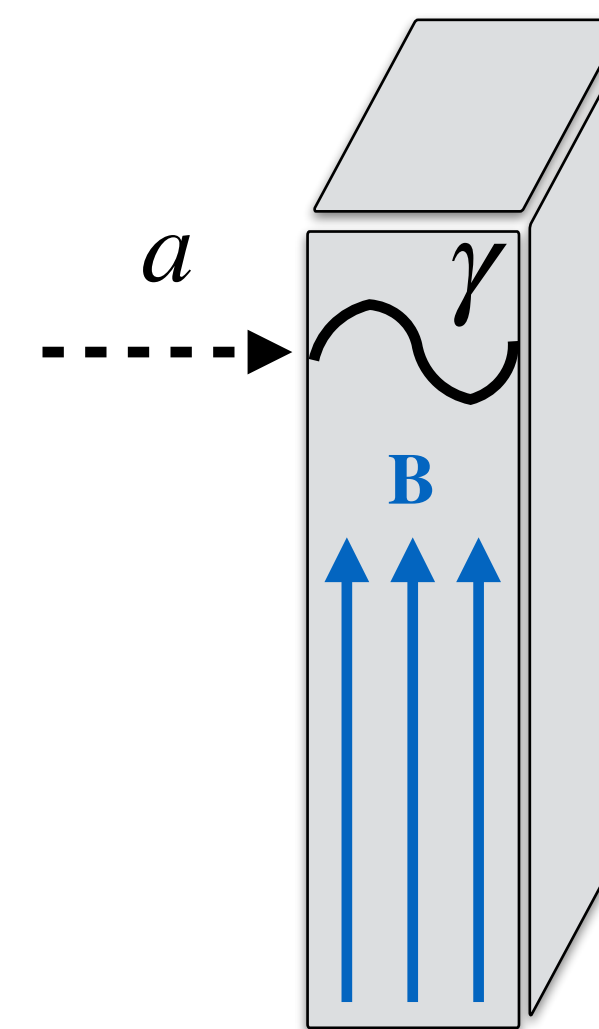
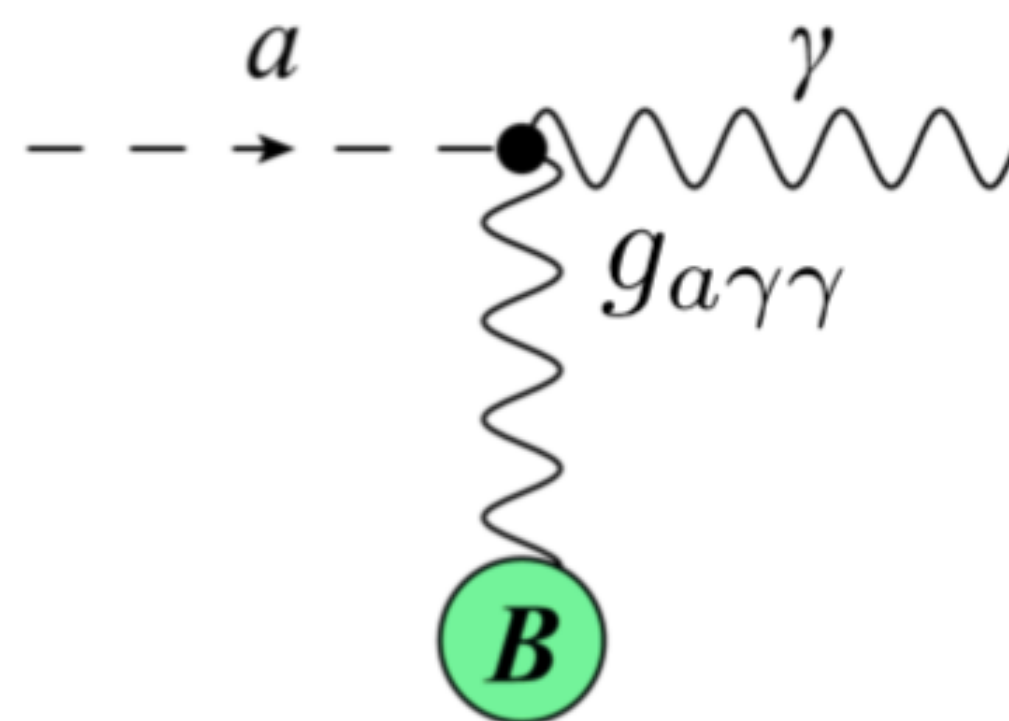


New technologies, lower thresholds, larger exposures, higher energies...

Dark Matter could be in the form of light **pseudo-scalar 'axions'**, which may arise as psuedo-Goldstone bosons of a spontaneously broken Peccei-Quinn symmetry $U(1)_{PQ}$.

Axions can **convert to photons** (and vice versa) in an external magnetic field:

$$\begin{aligned} \mathcal{L} &\supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &= -\frac{1}{4} g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B} \end{aligned}$$



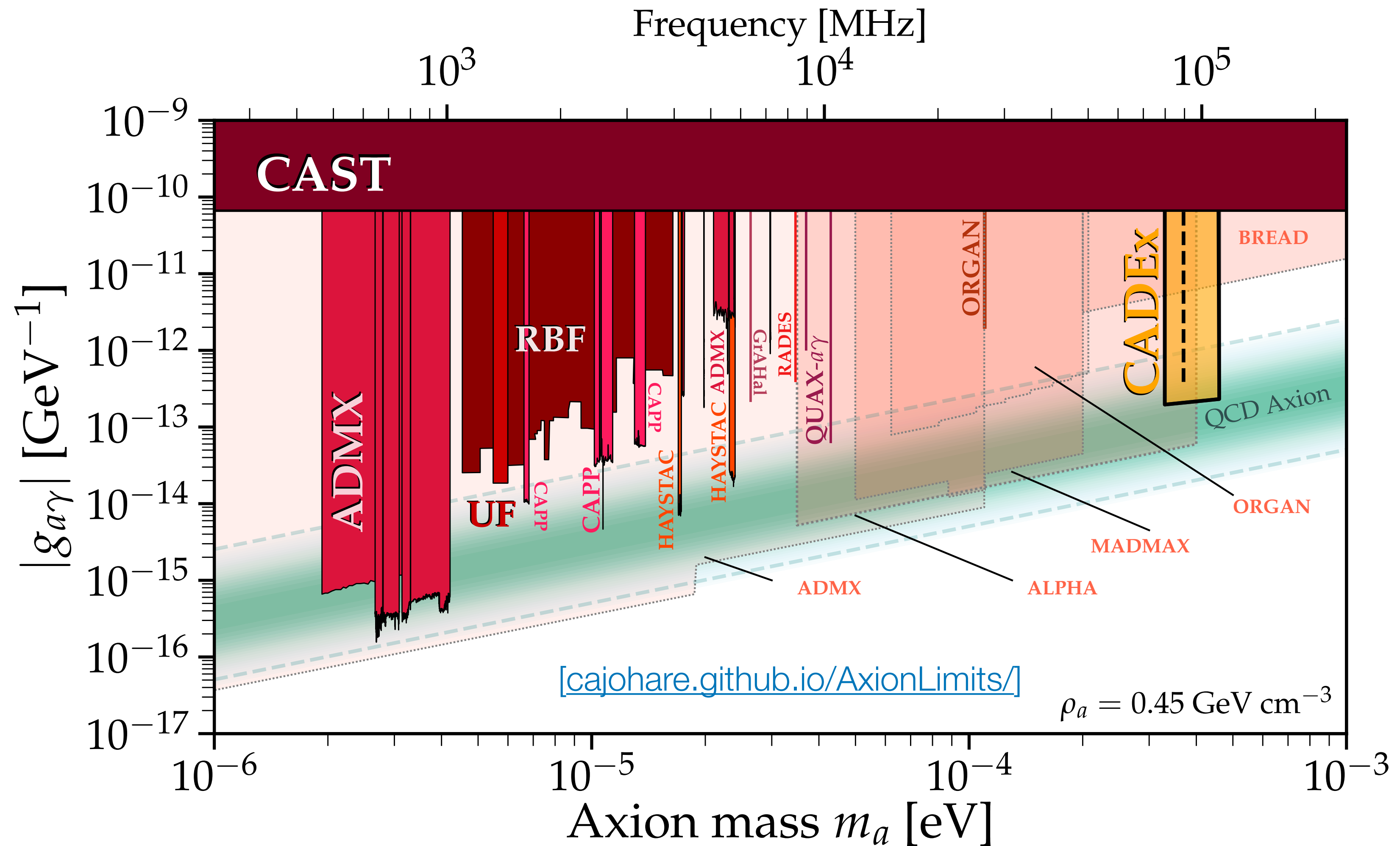
$$\omega = m_a$$

$$\text{Power} \sim g_{a\gamma}^2 \frac{\rho_a}{m_a} B^2 Q_0 V$$

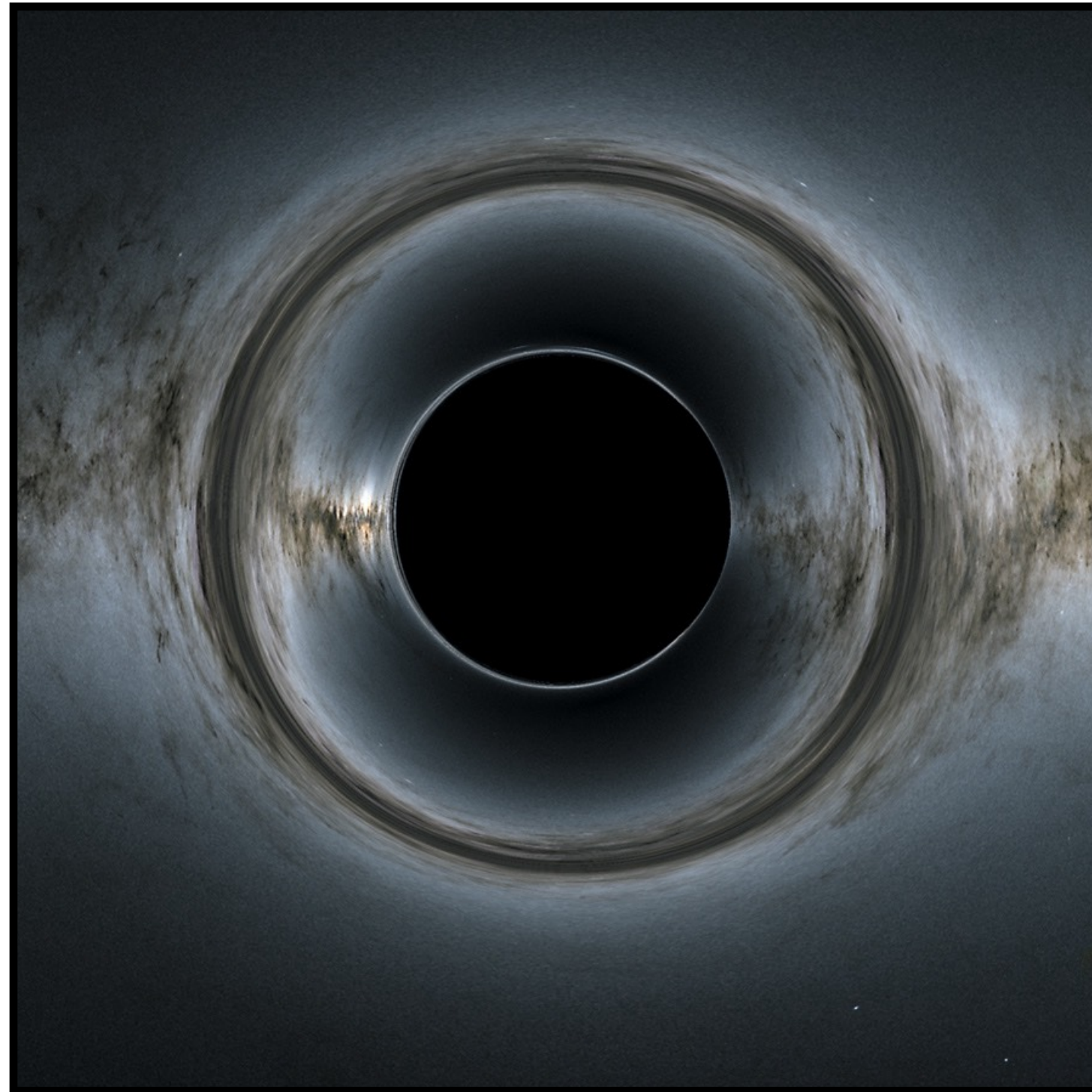
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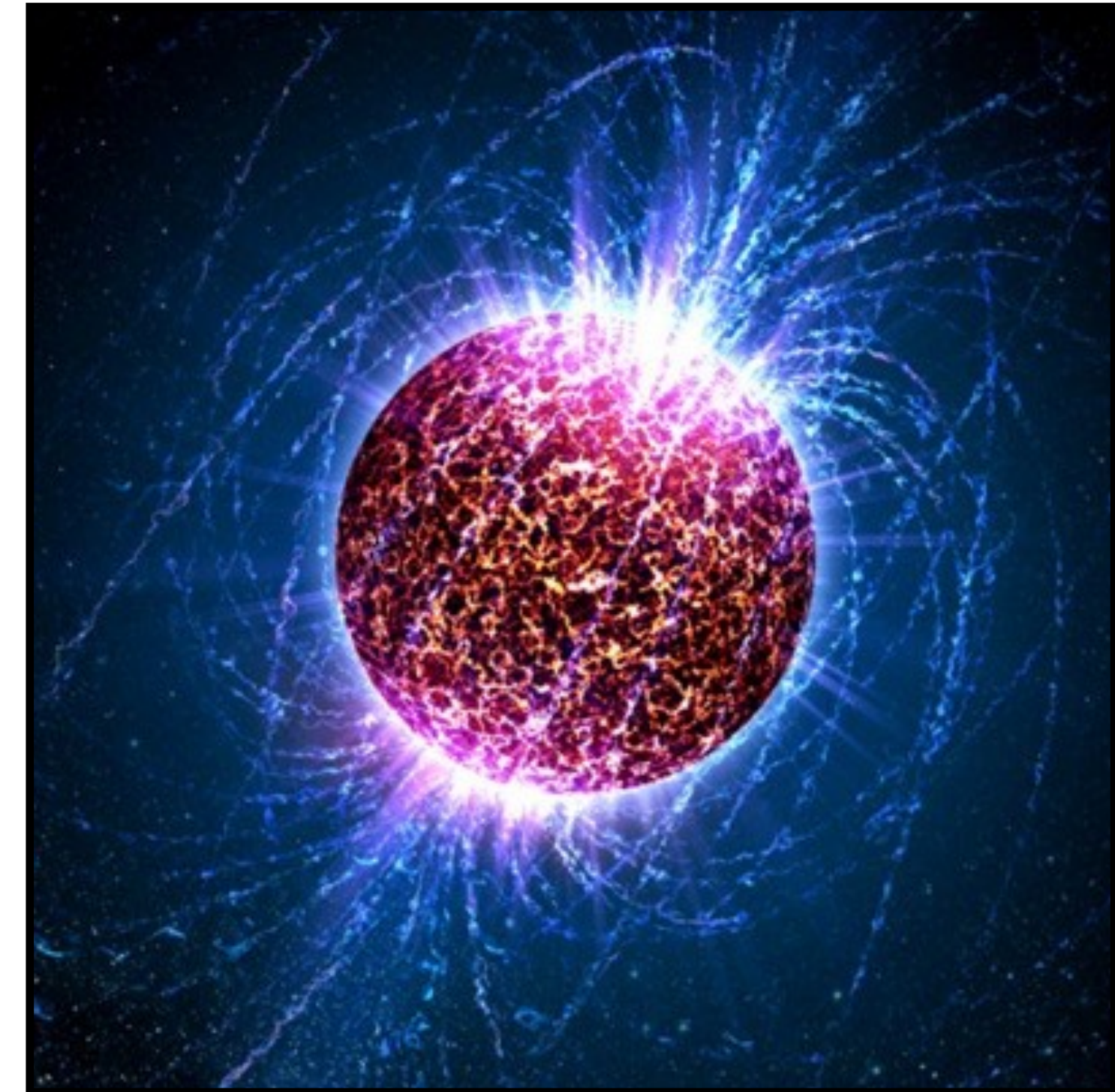


Black Holes



[Credit: NASA's Goddard Space Flight Center; background, ESA/Gaia/DPAC]

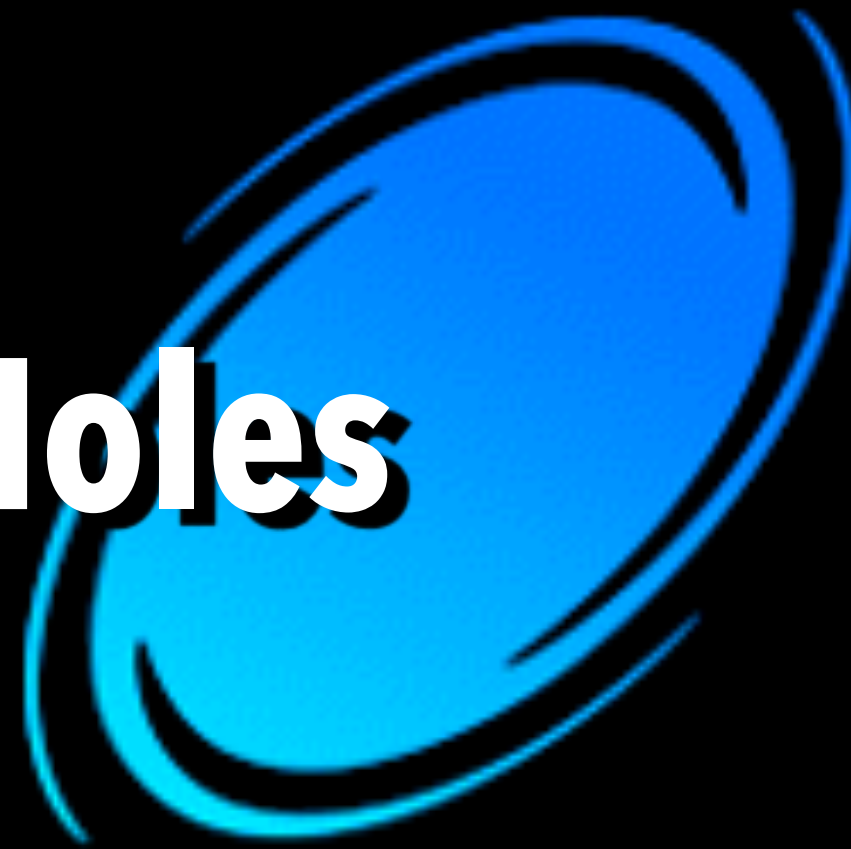
Neutron Stars



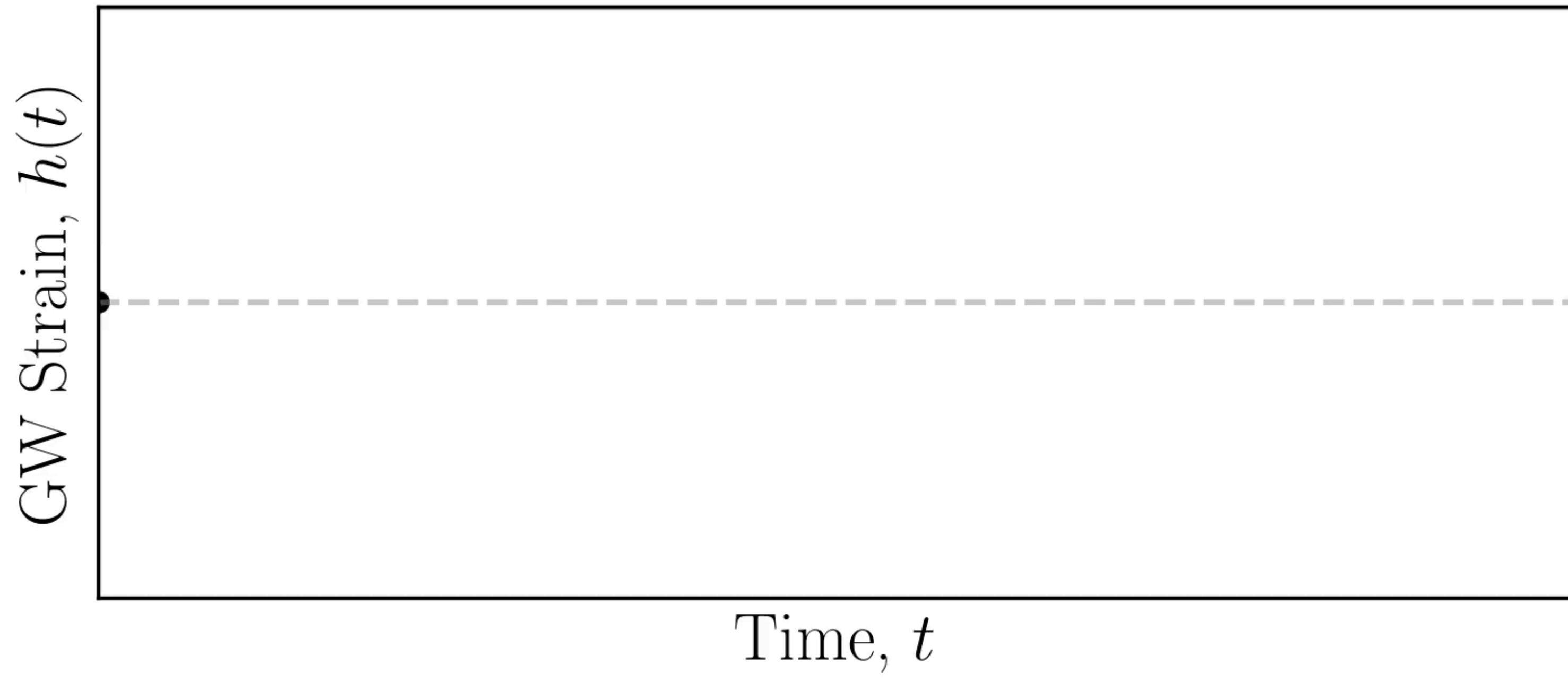
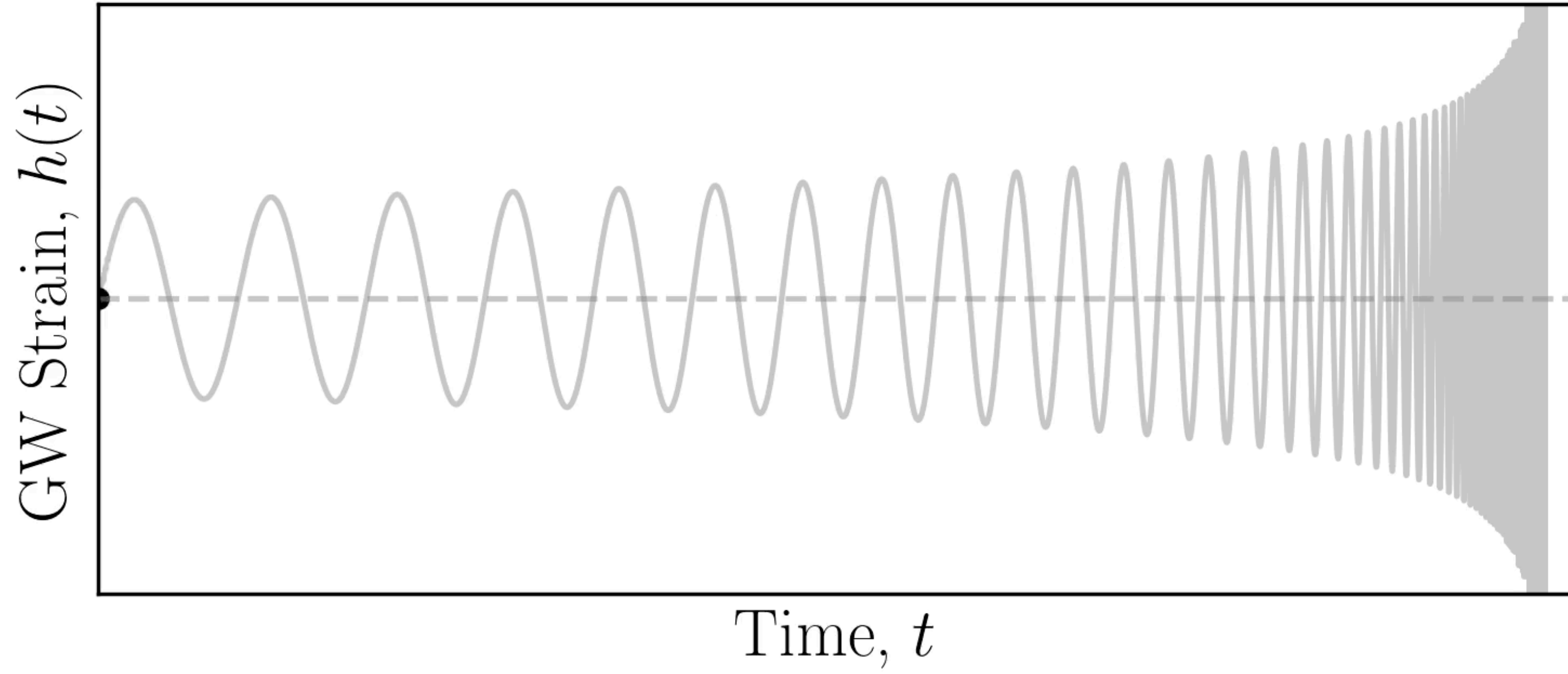
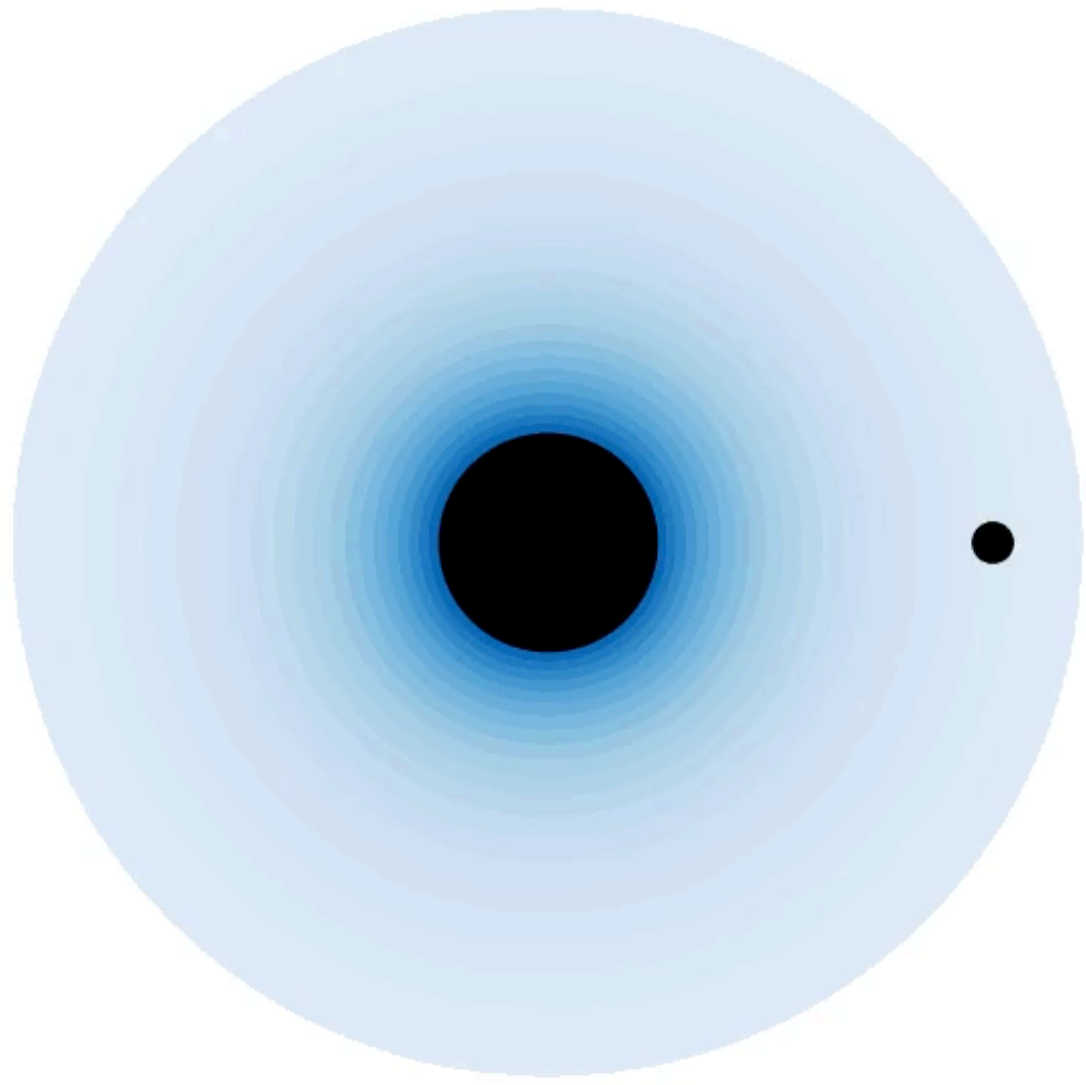
[Credit: Casey Reed (Penn State University), Wikimedia Commons]

Higher densities, larger magnetic fields, longer timescales...

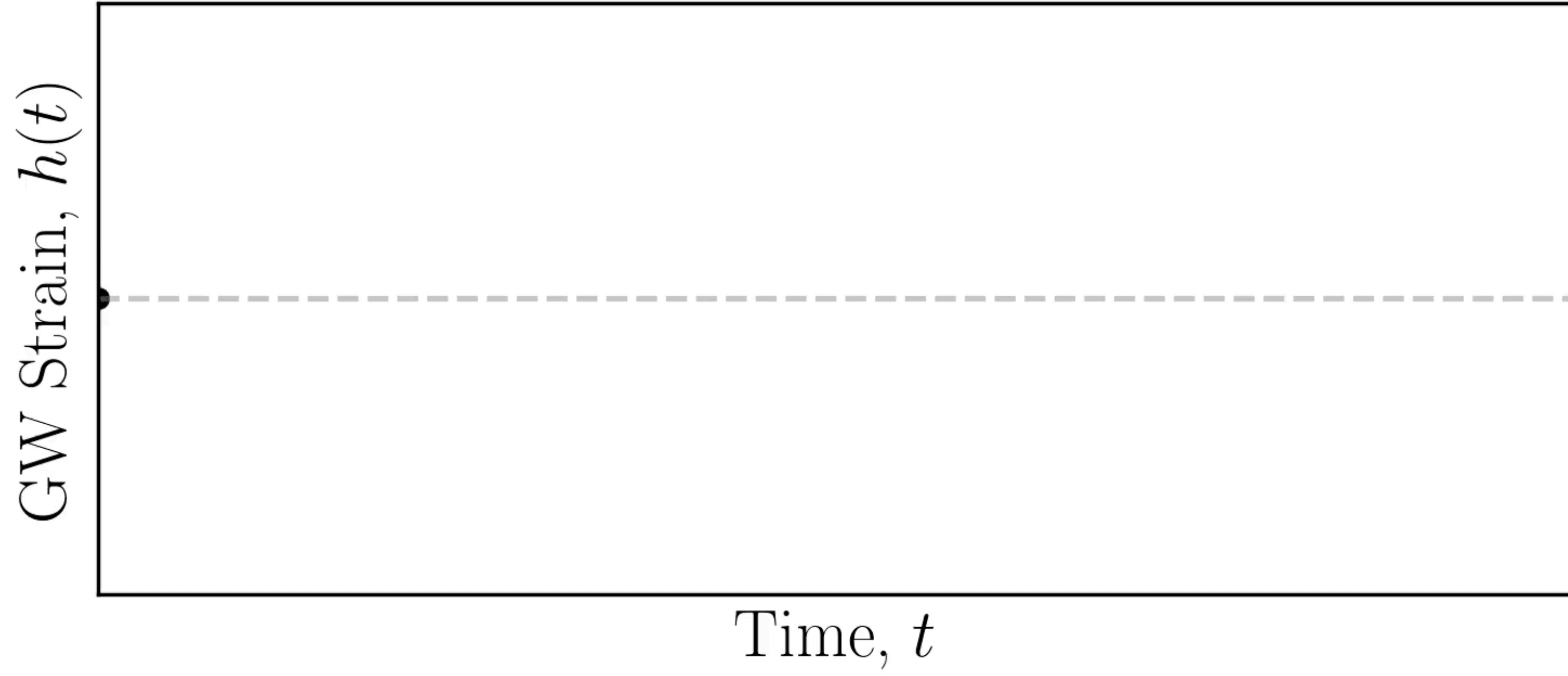
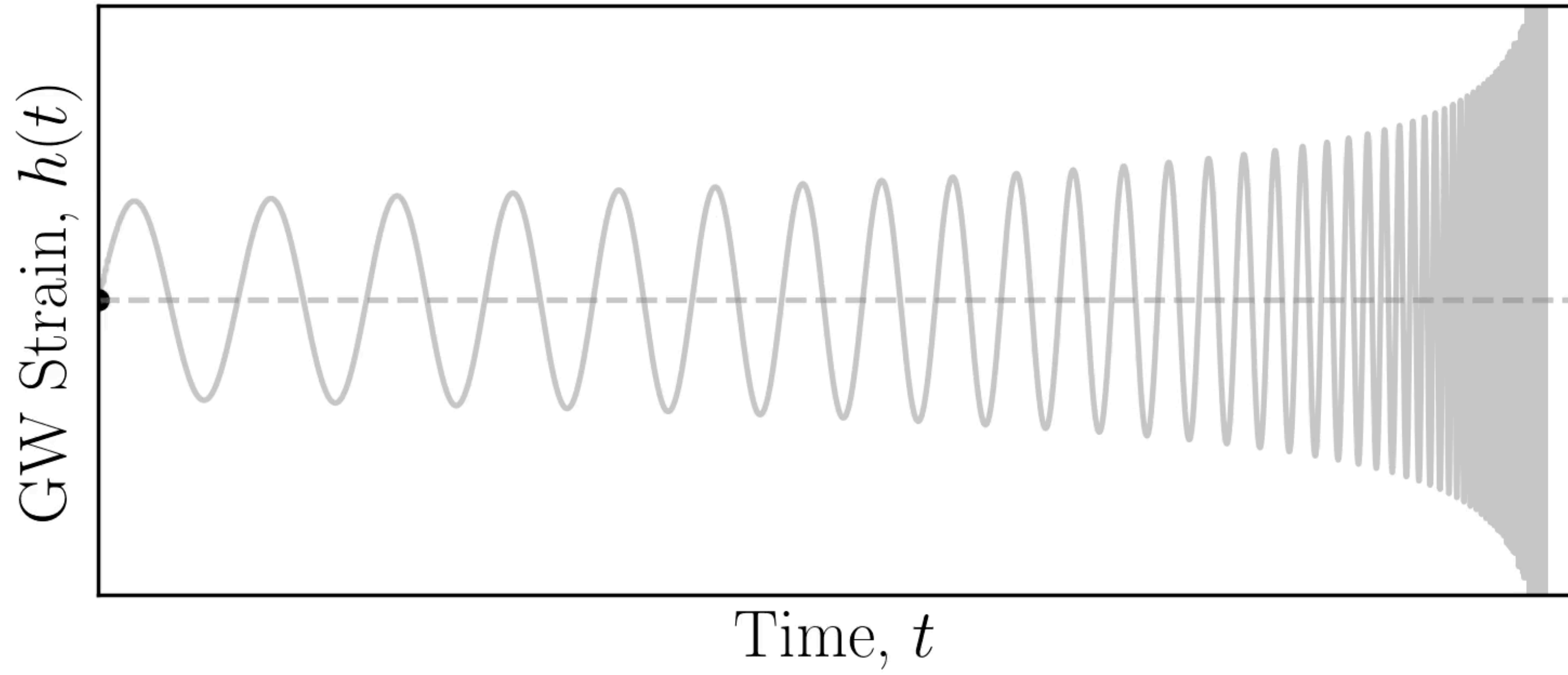
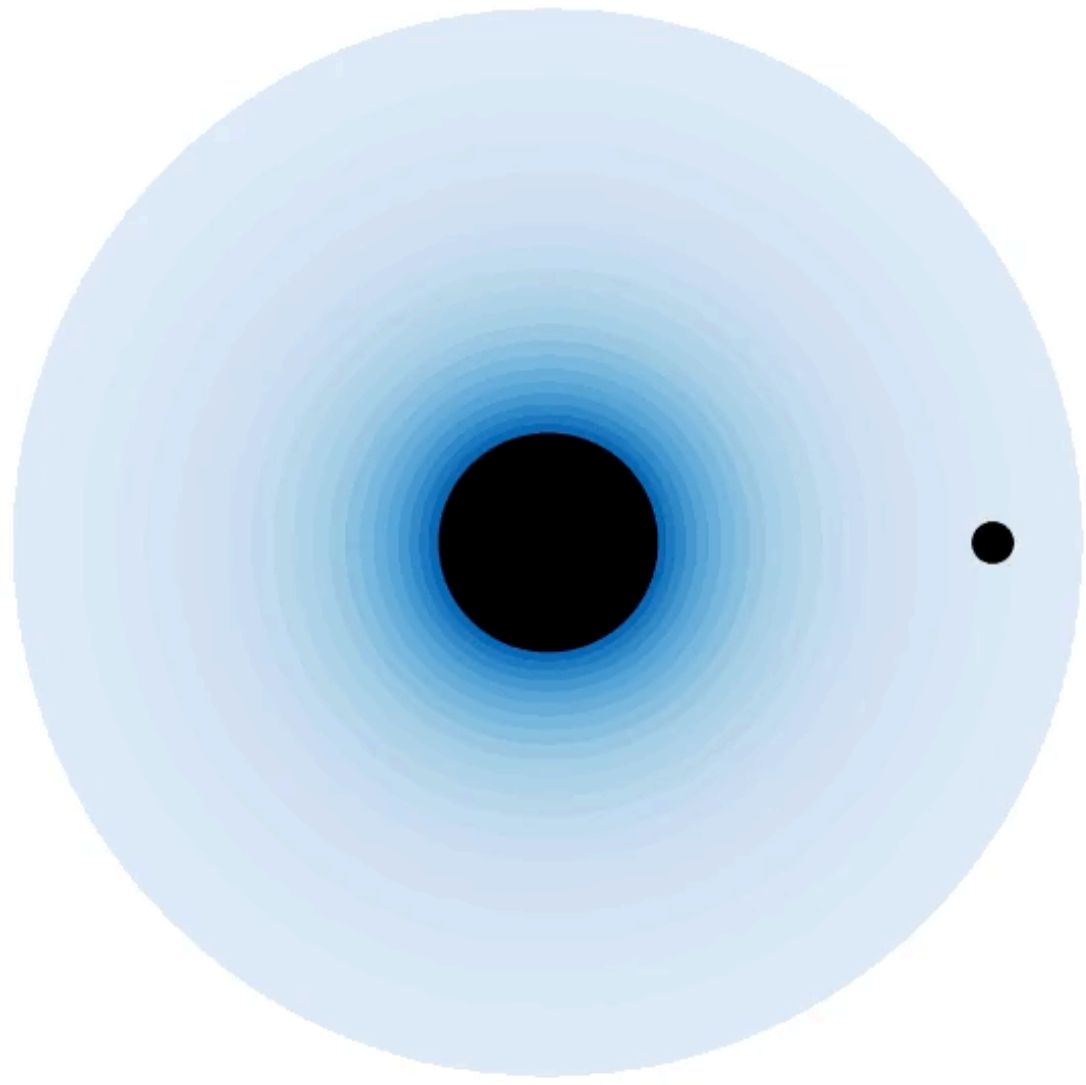
Part 1:
Black Holes



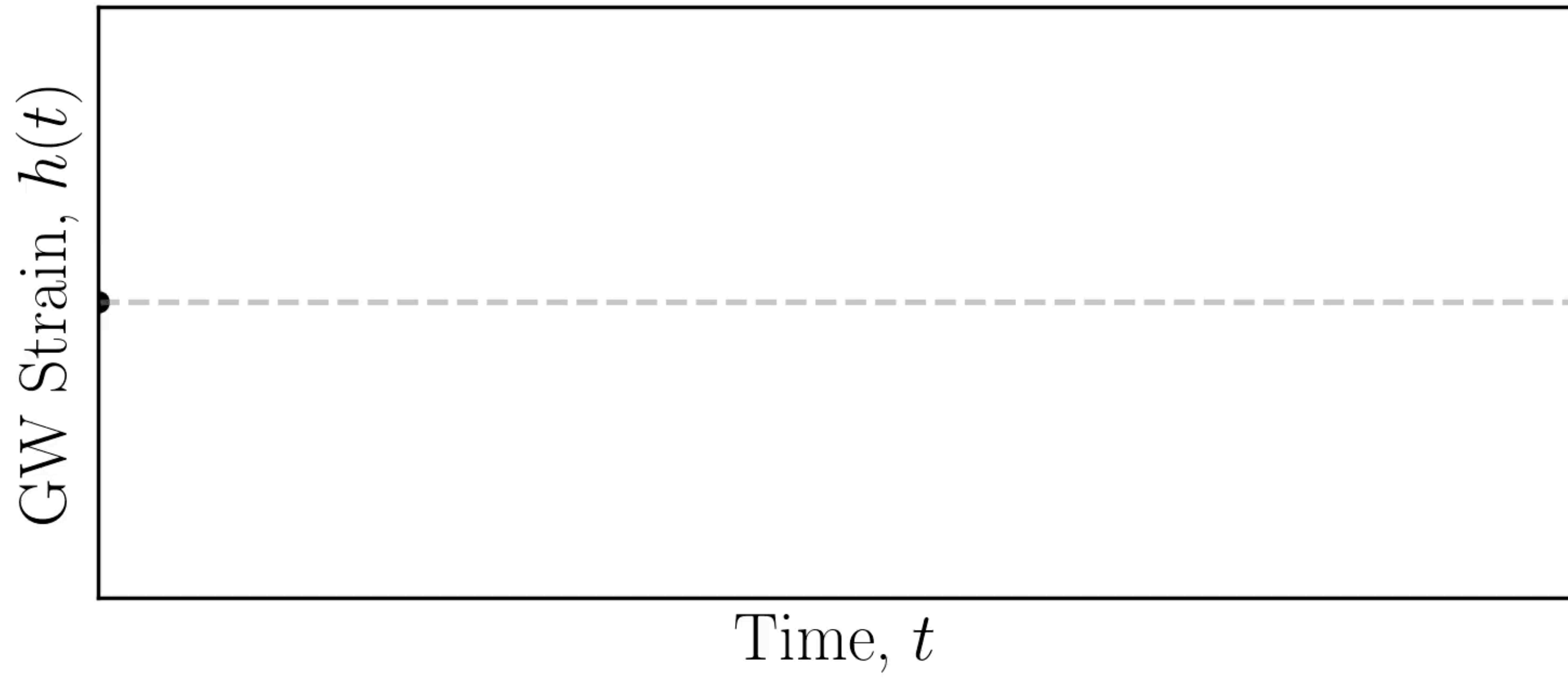
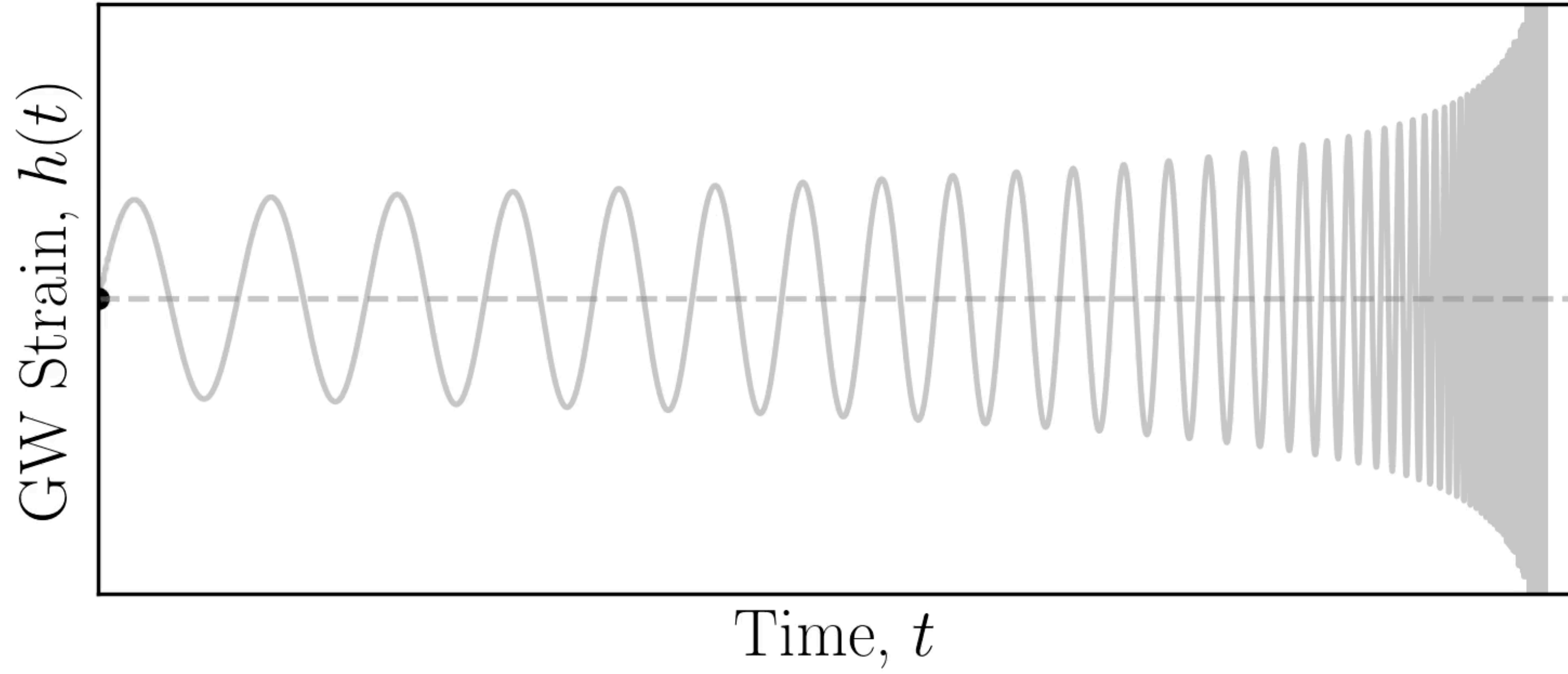
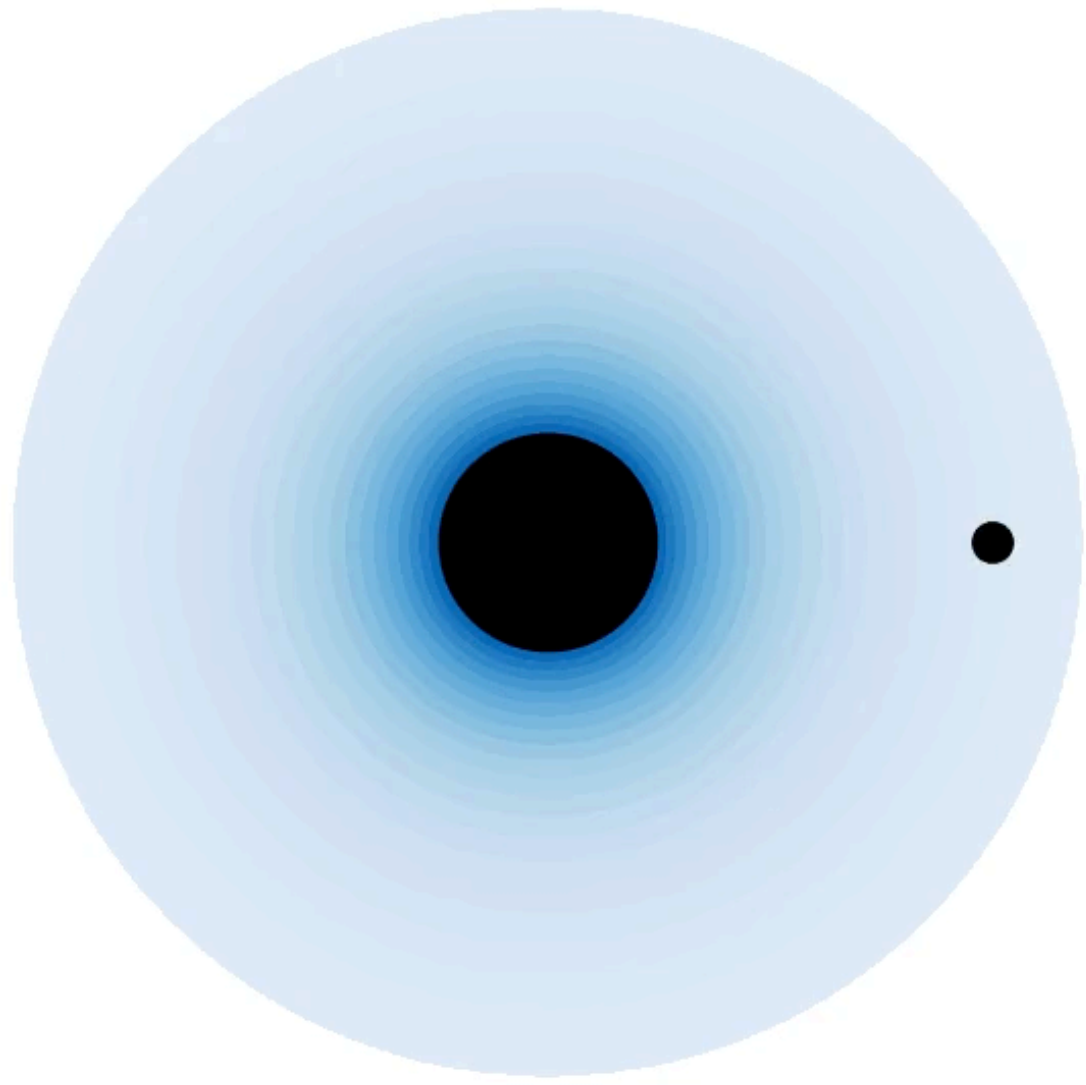
Gravitational Wave Dephasing



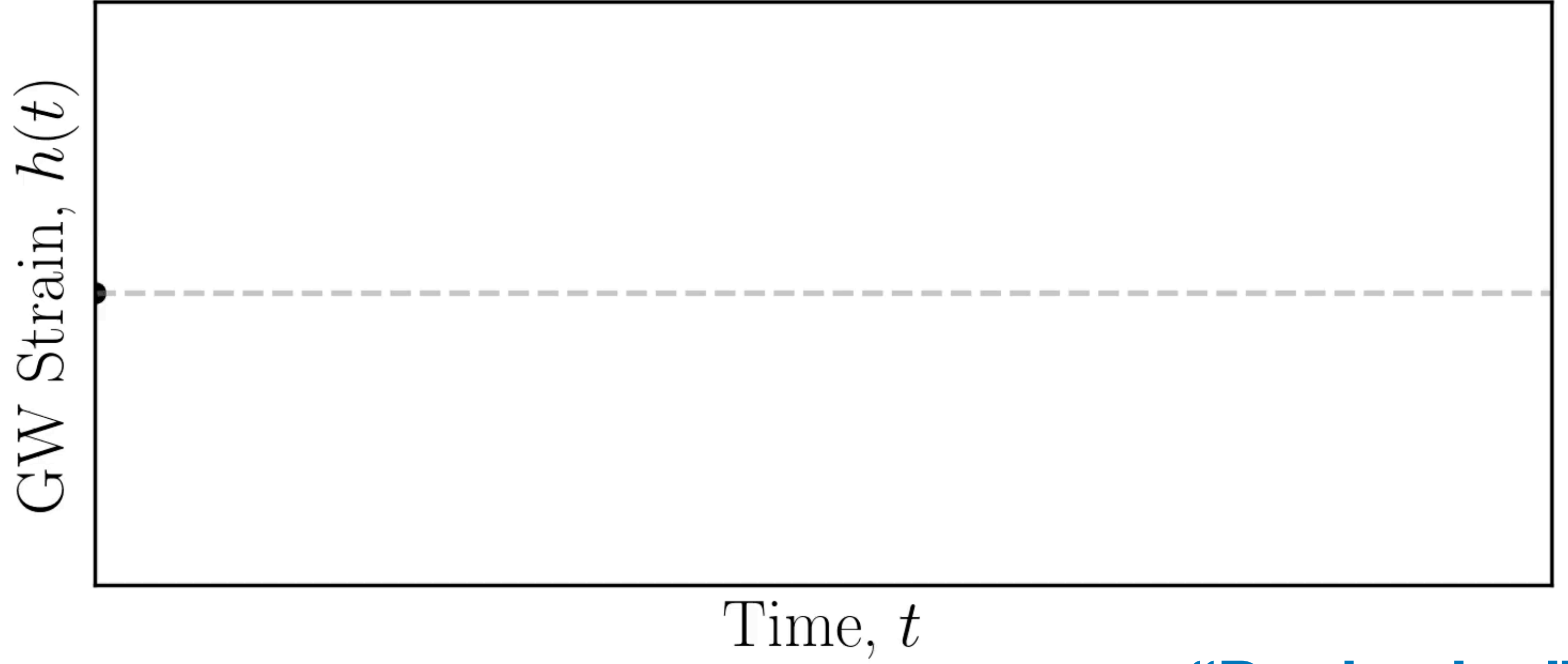
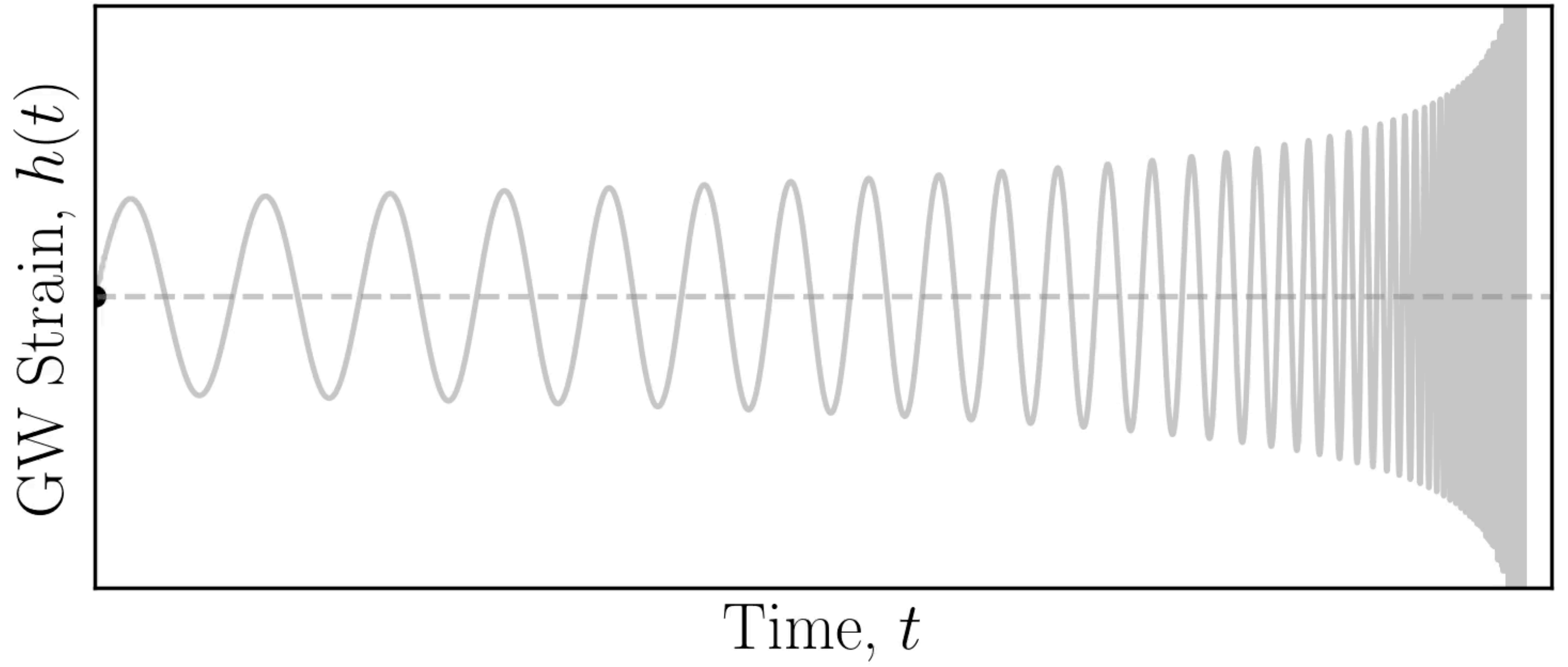
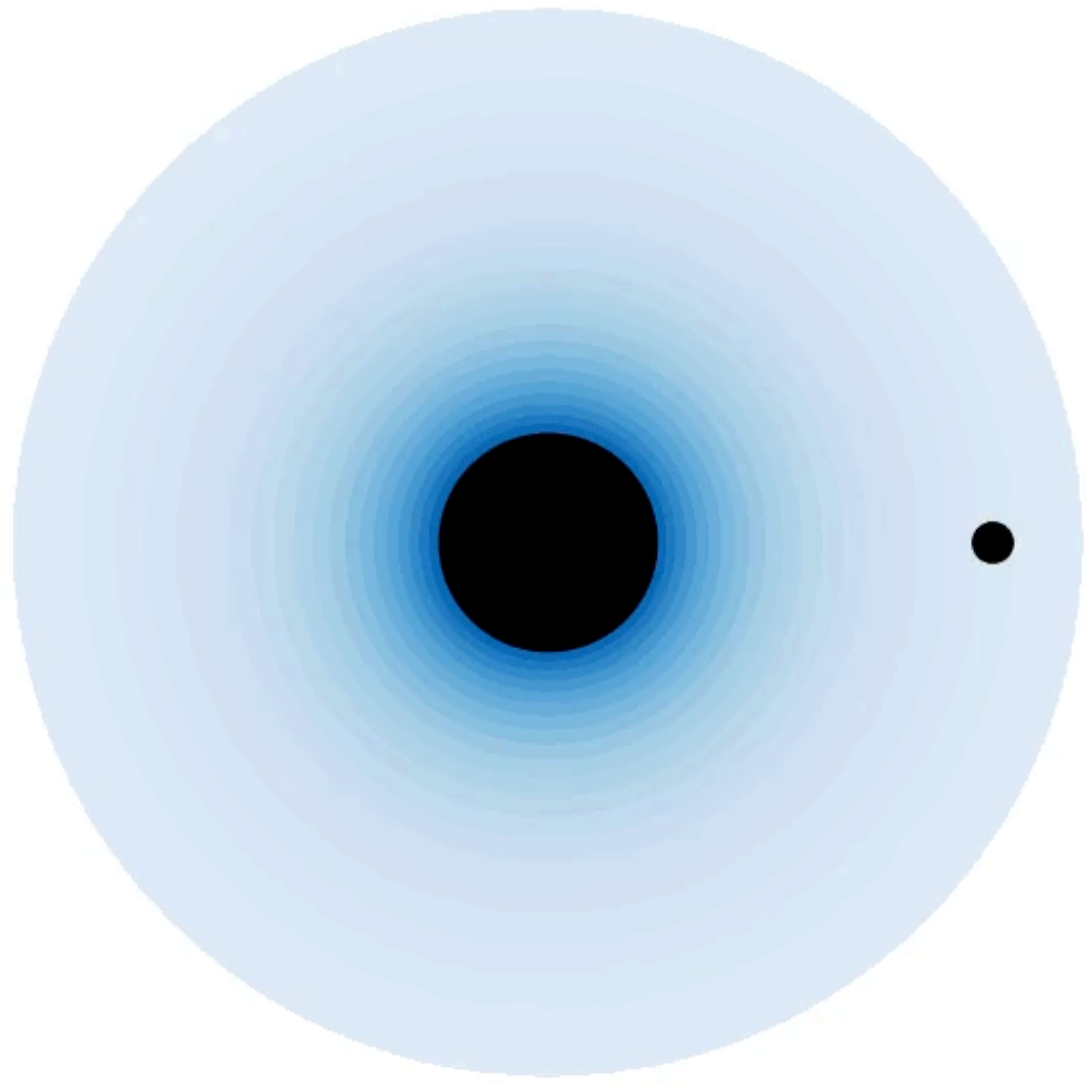
Gravitational Wave Dephasing

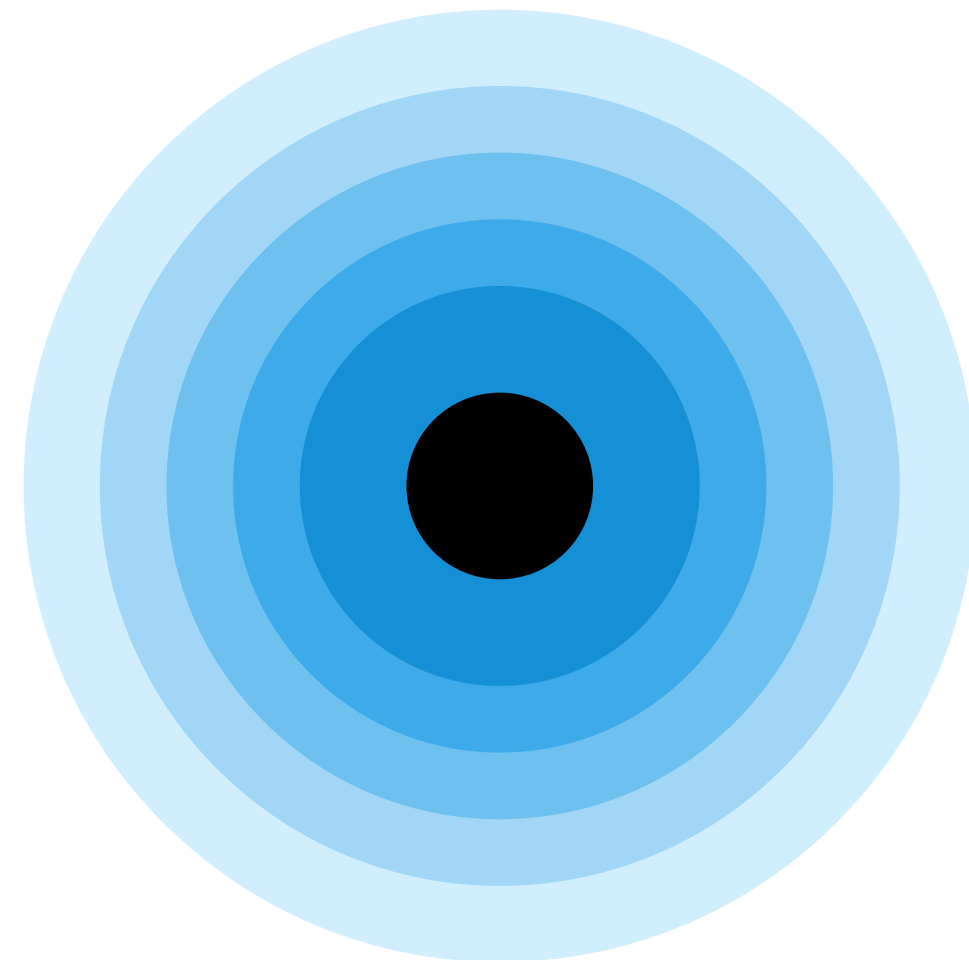
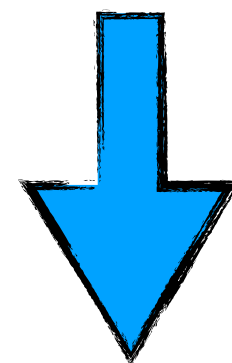
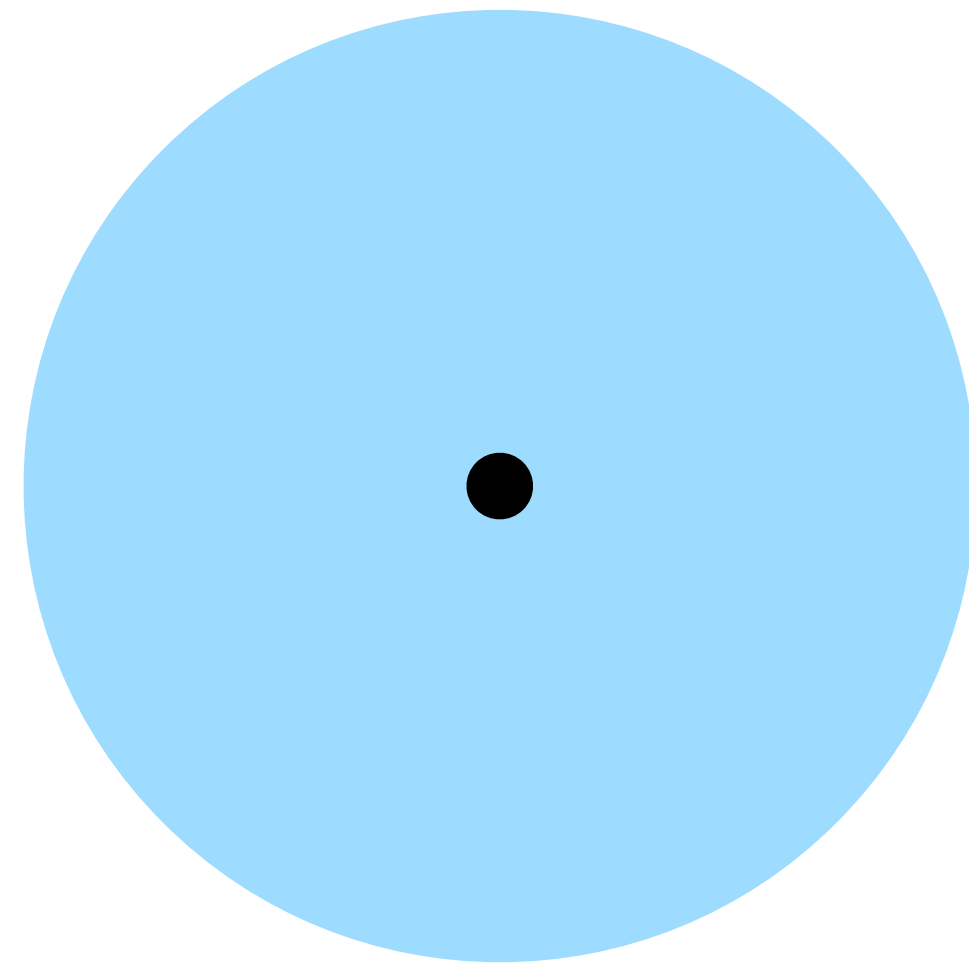


Gravitational Wave Dephasing



Gravitational Wave Dephasing





‘**Spikes**’ or ‘**dresses**’ of cold, particle-like DM may form around BHs:

From the slow (‘adiabatic’) growth of a BH at the centre of a DM halo

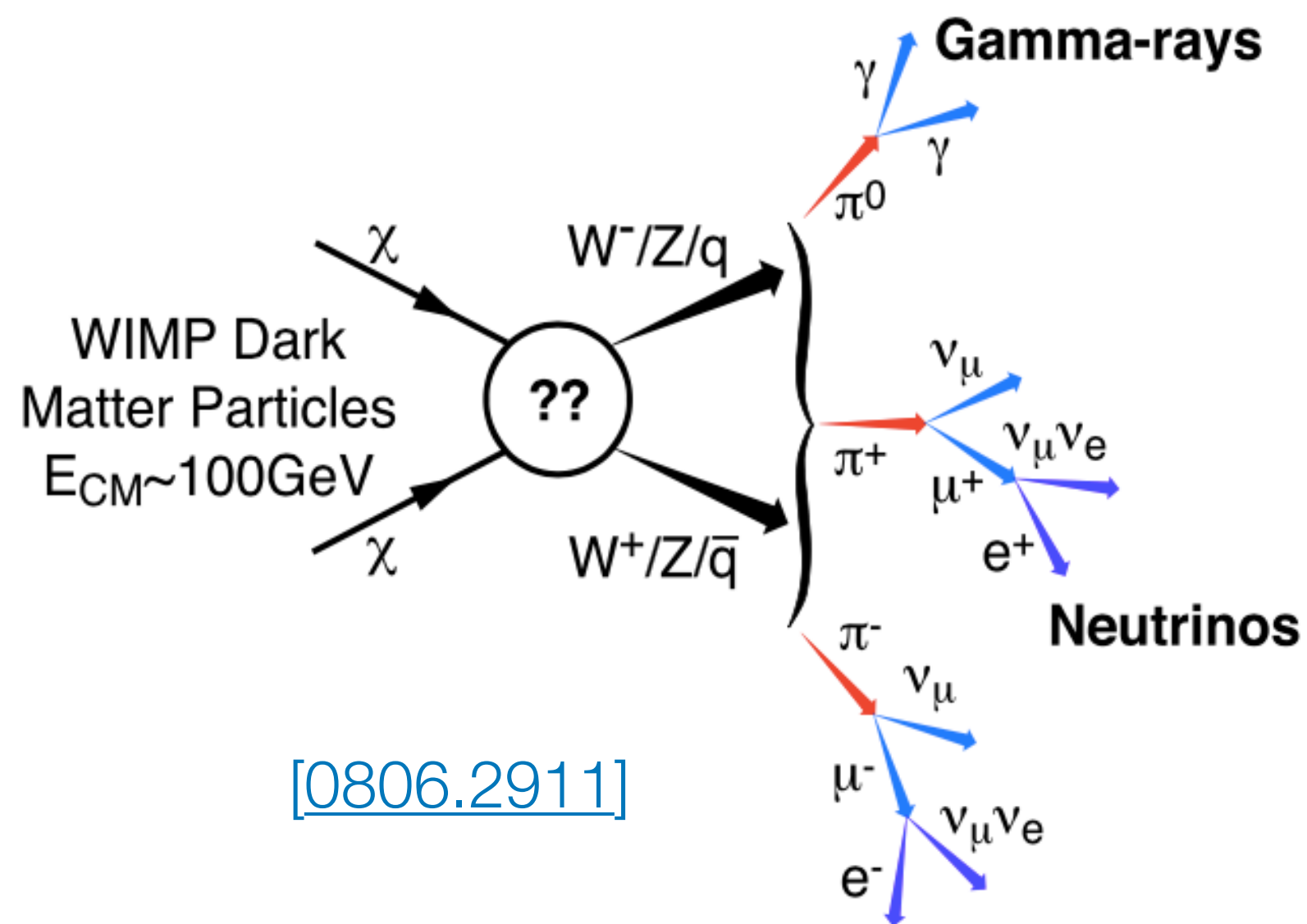
“**Astrophysical scenario**”

[[astro-ph/9906391](#), [astro-ph/0509565](#), [1305.2619](#), ...]

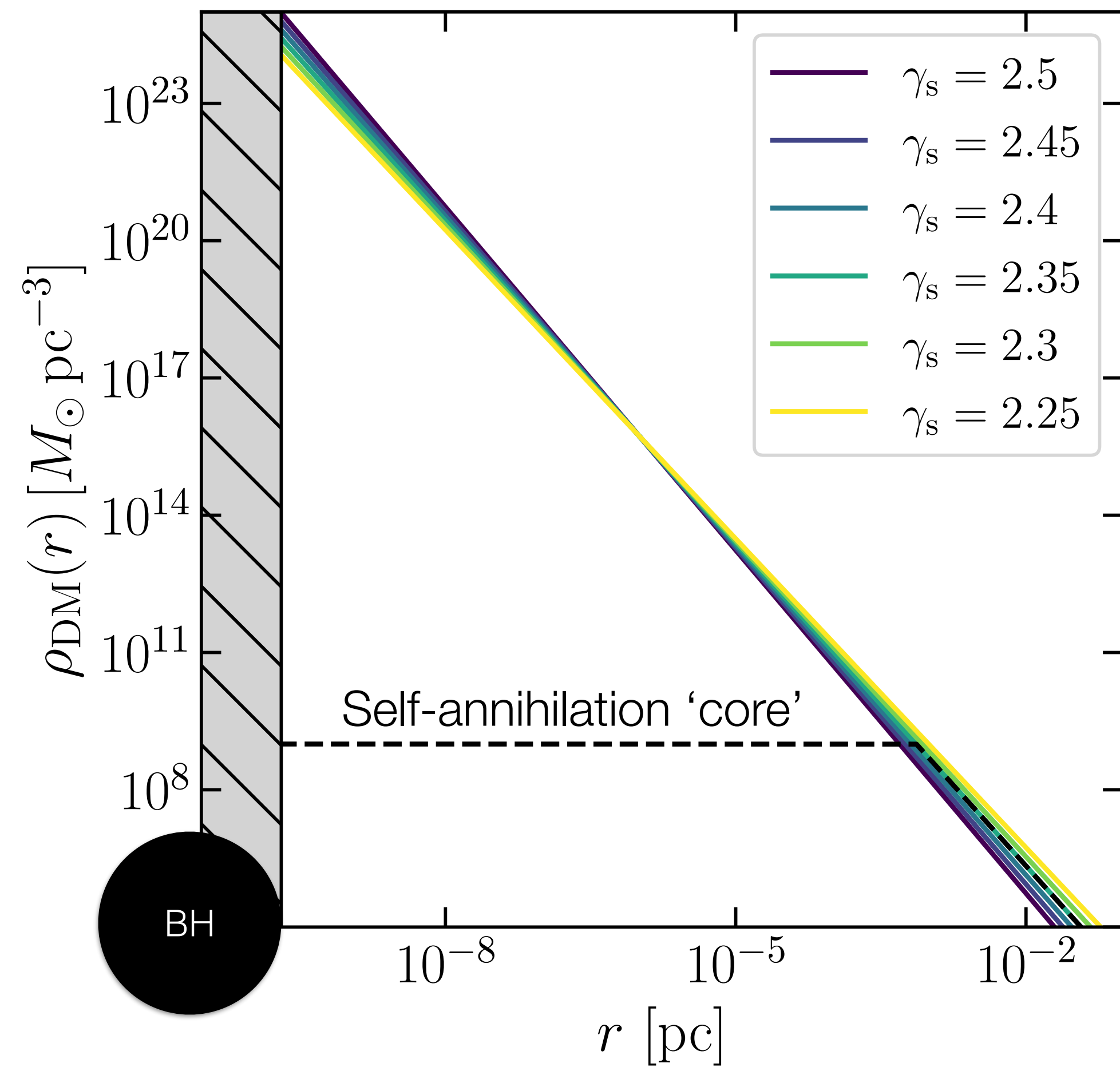
Around BHs which form from large density fluctuations in the early Universe (i.e. Primordial Black Holes)

“**PBH scenario**”

[[Bertschinger \(1985\)](#), [astro-ph/0608642](#), [1901.08528](#), ...]



$$\rho_{\text{DM}} = \rho_6 \left(\frac{10^{-6} \text{ pc}}{r} \right)^{\gamma_{\text{sp}}}$$

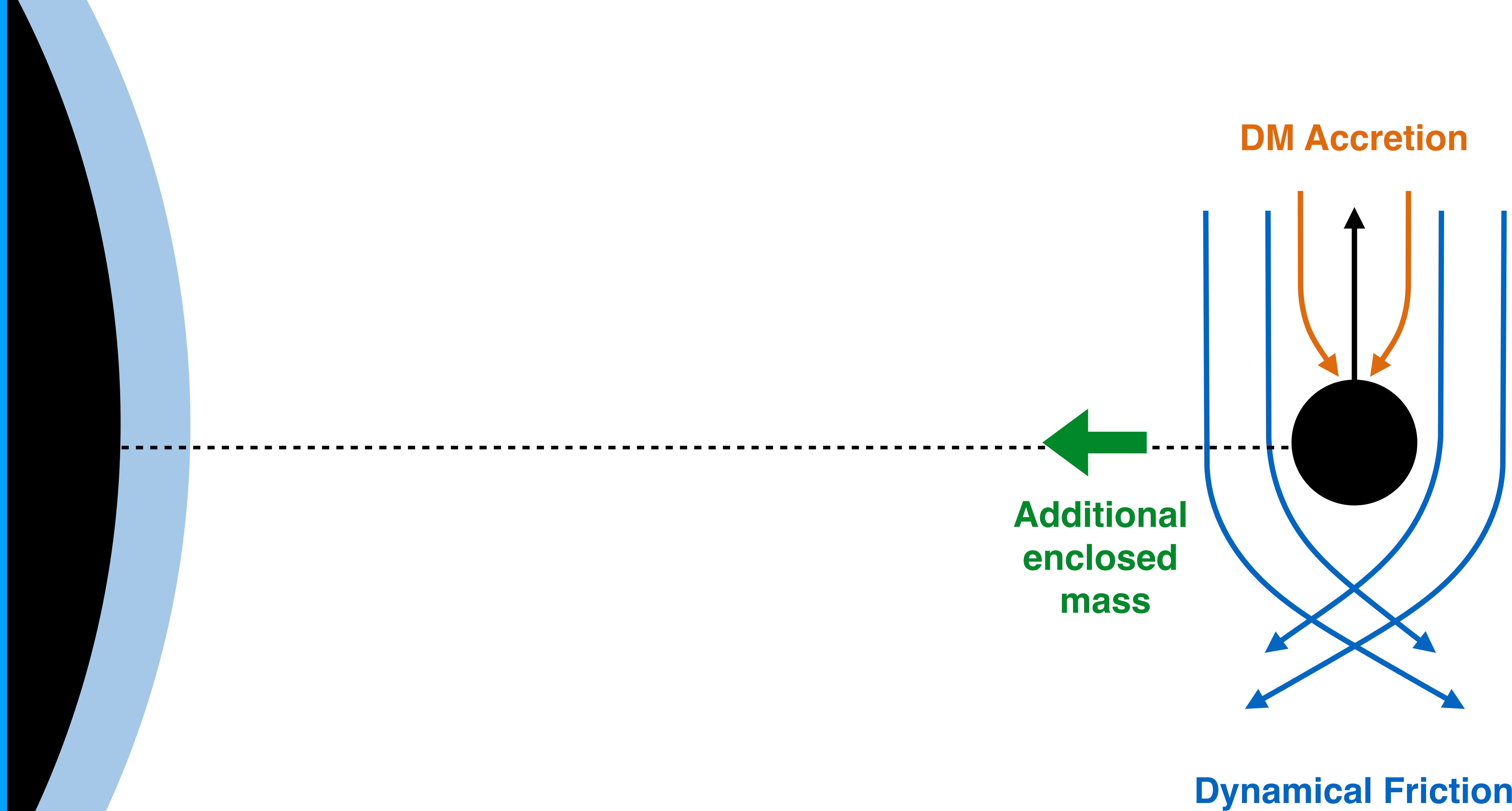


DM self-annihilation can suppress the spike density, but can still lead to large (diffuse and point source) fluxes of gamma-rays and neutrinos

[E.g. Lacroix & Silk, [1712.00452](#), Bertone et al., [1905.01238](#), Freese et al., [2202.01126](#)]

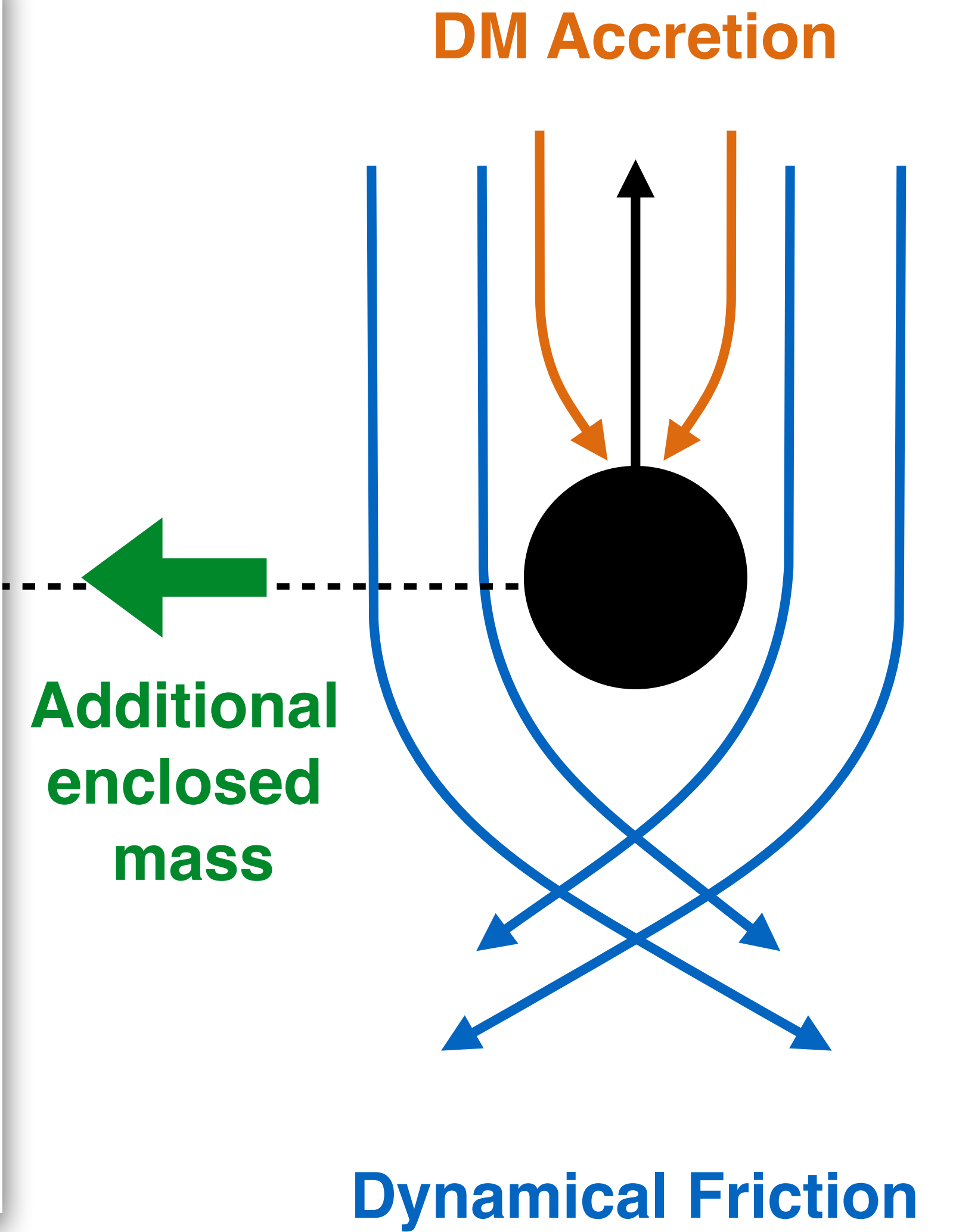
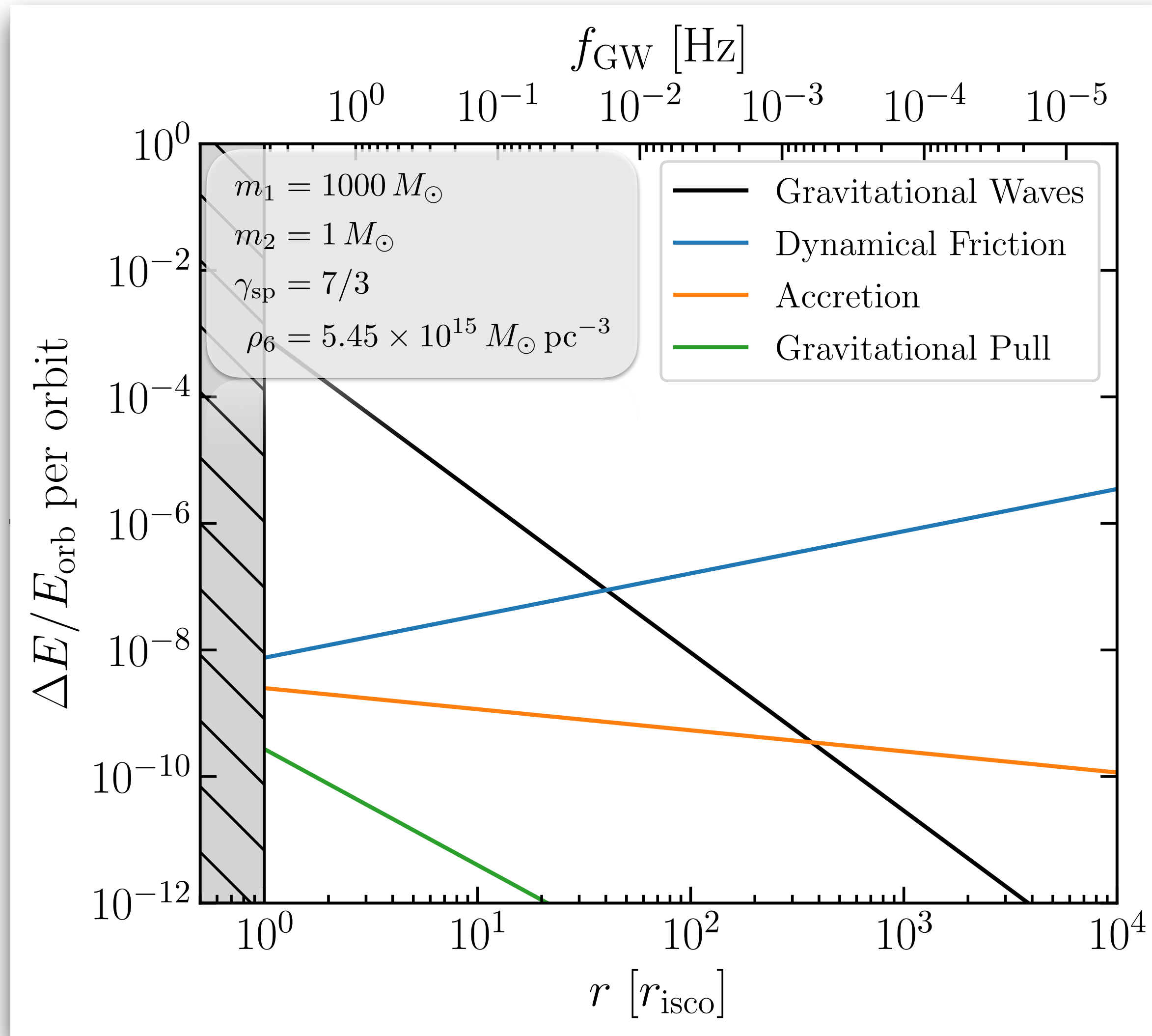
What about **non-annihilating DM**?

$$\rho_{\text{DM, local}} \sim 10^{-2} M_\odot / \text{pc}^3$$



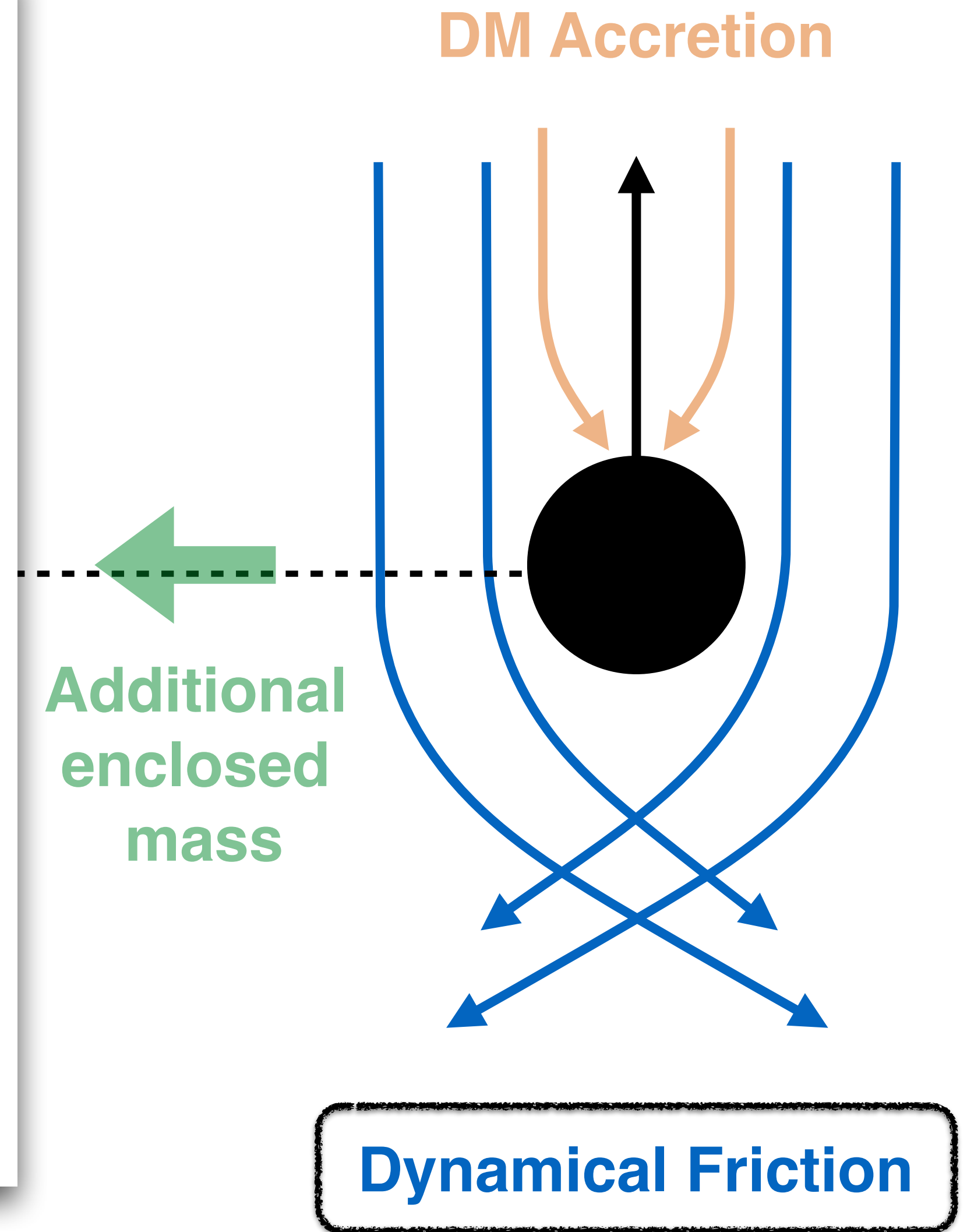
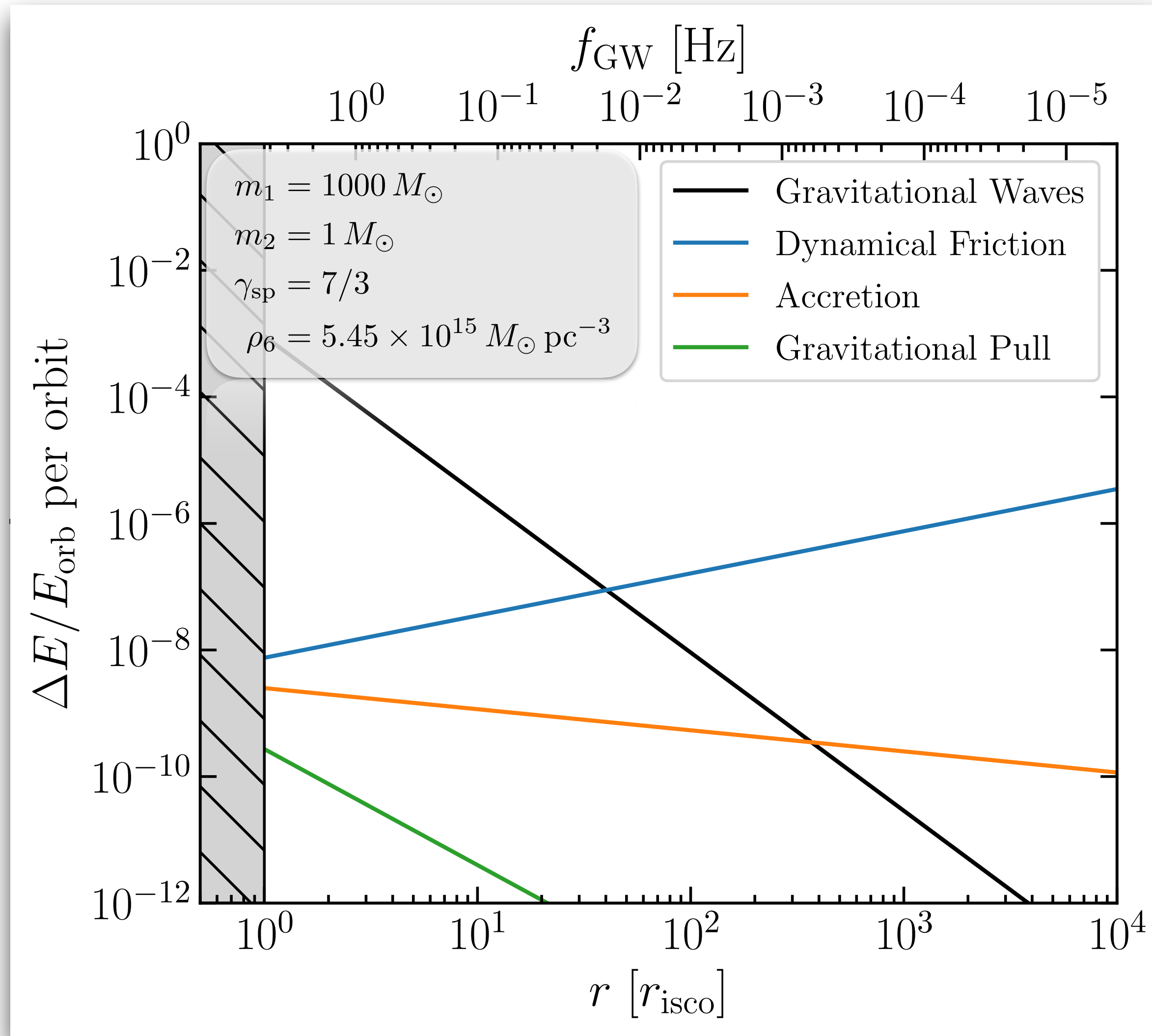
[See e.g. Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]

Impact of DM Spikes



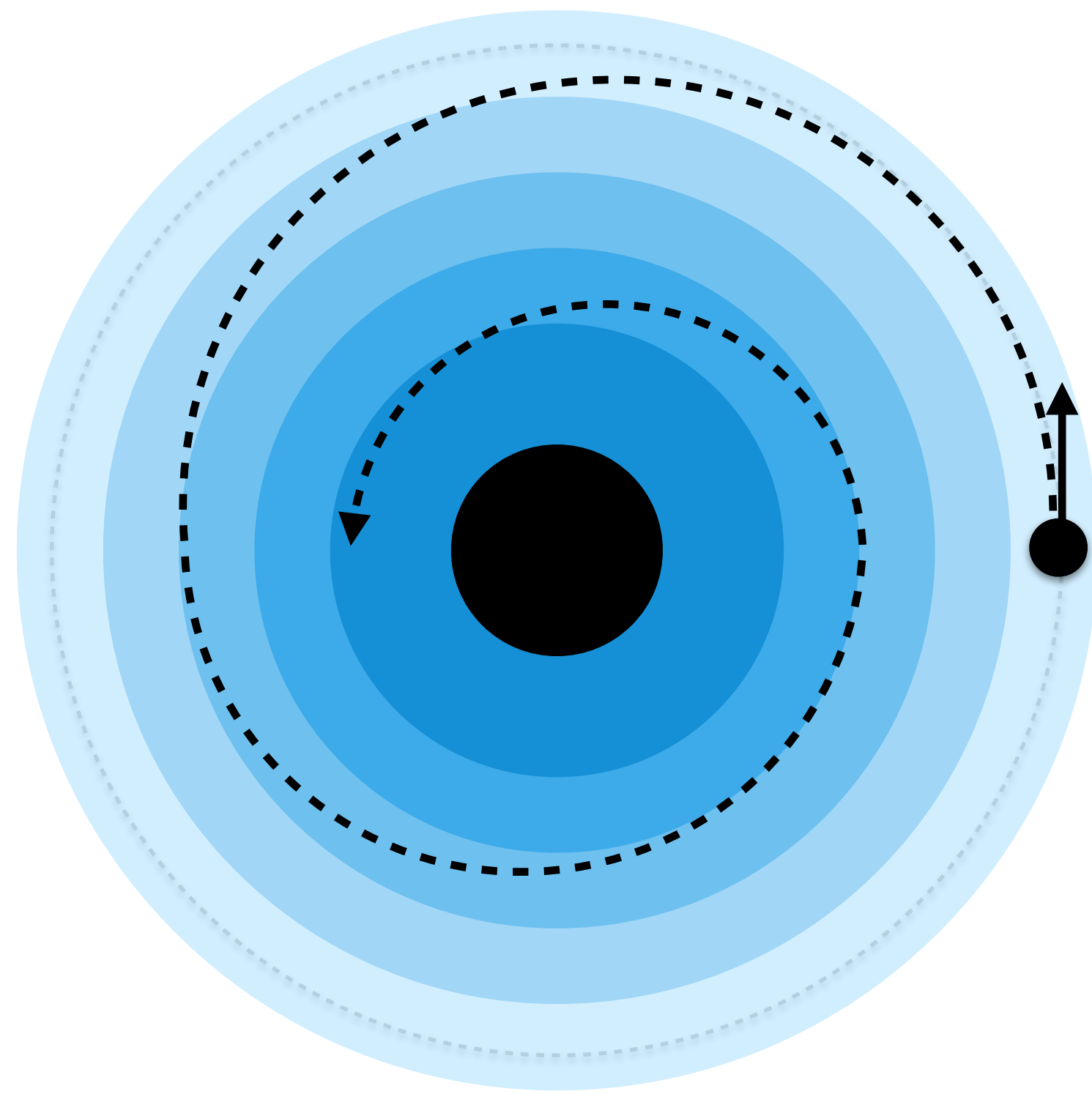
[See e.g. Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]

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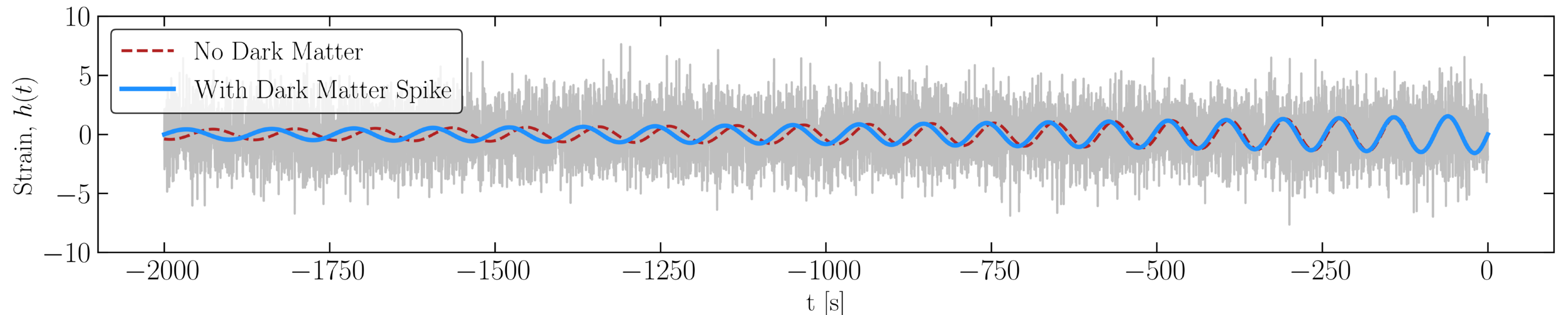
$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$



$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$

Solve for Newtonian motion of the binary, taking into account:

- GW emission
- Dynamical Friction
- **DM Halo Feedback**



[See e.g. Eda et al. [1301.5971](#), [1408.3534](#), Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

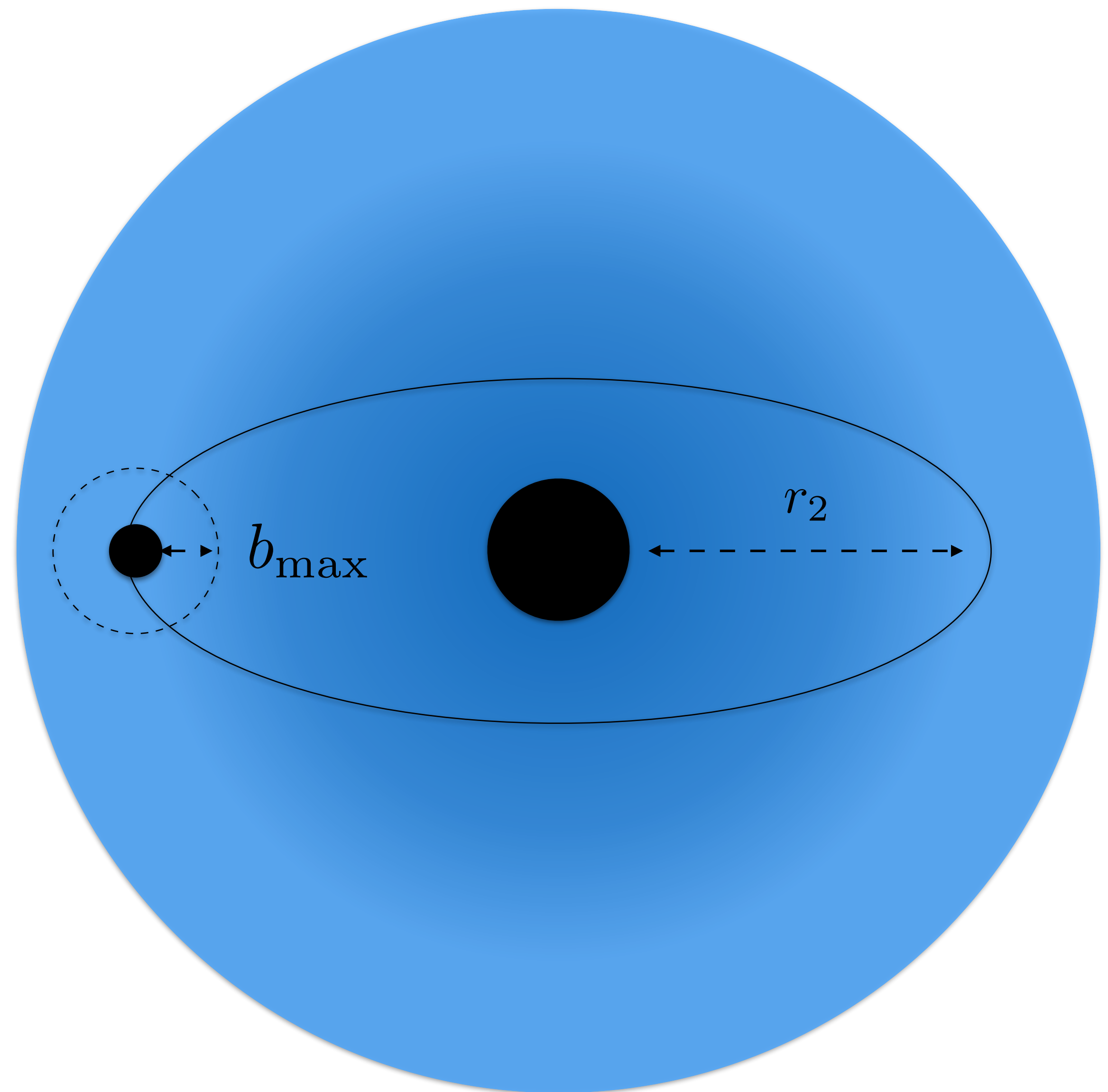
Each particle receives a 'kick' through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]



Compact object scatters with all DM particles within 'torus' of influence over one orbit

[Code available online: github.com/bradkav/HaloFeedback]

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{df(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E} - \Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy \mathcal{E} to scatter and receive a 'kick' $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E} \quad \text{- total probability for a particle with energy } \mathcal{E} \text{ to scatter}$$

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Particles scattering from
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

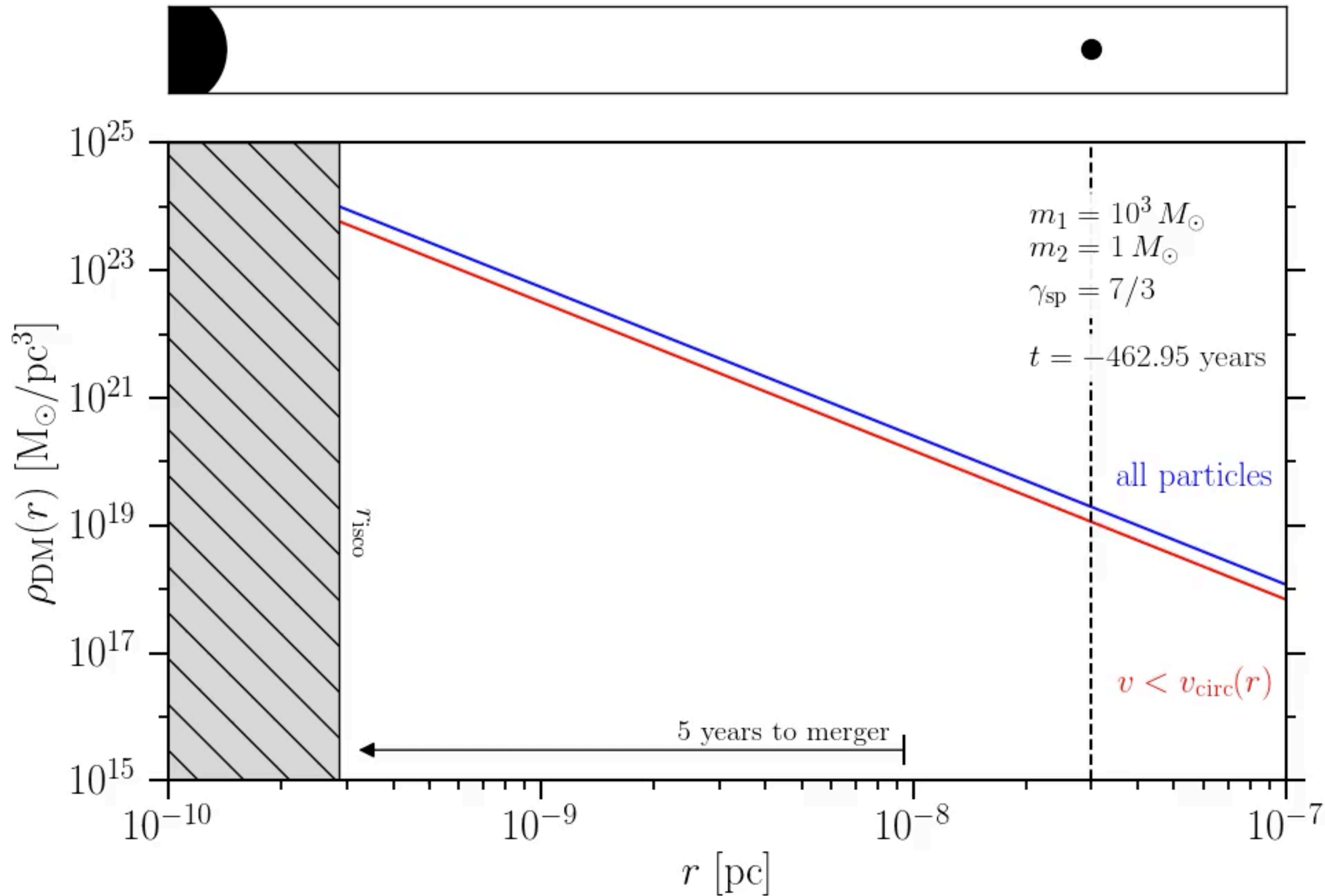
Particles scattering from
 $\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

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Density of the DM spike is depleted (and replenished...)

Size of the dephasing effect is **reduced from $\mathcal{O}(1)$ to $\mathcal{O}(1\%)$** .



This is one of the reasons we want to look at IMRIs/EMRIs...

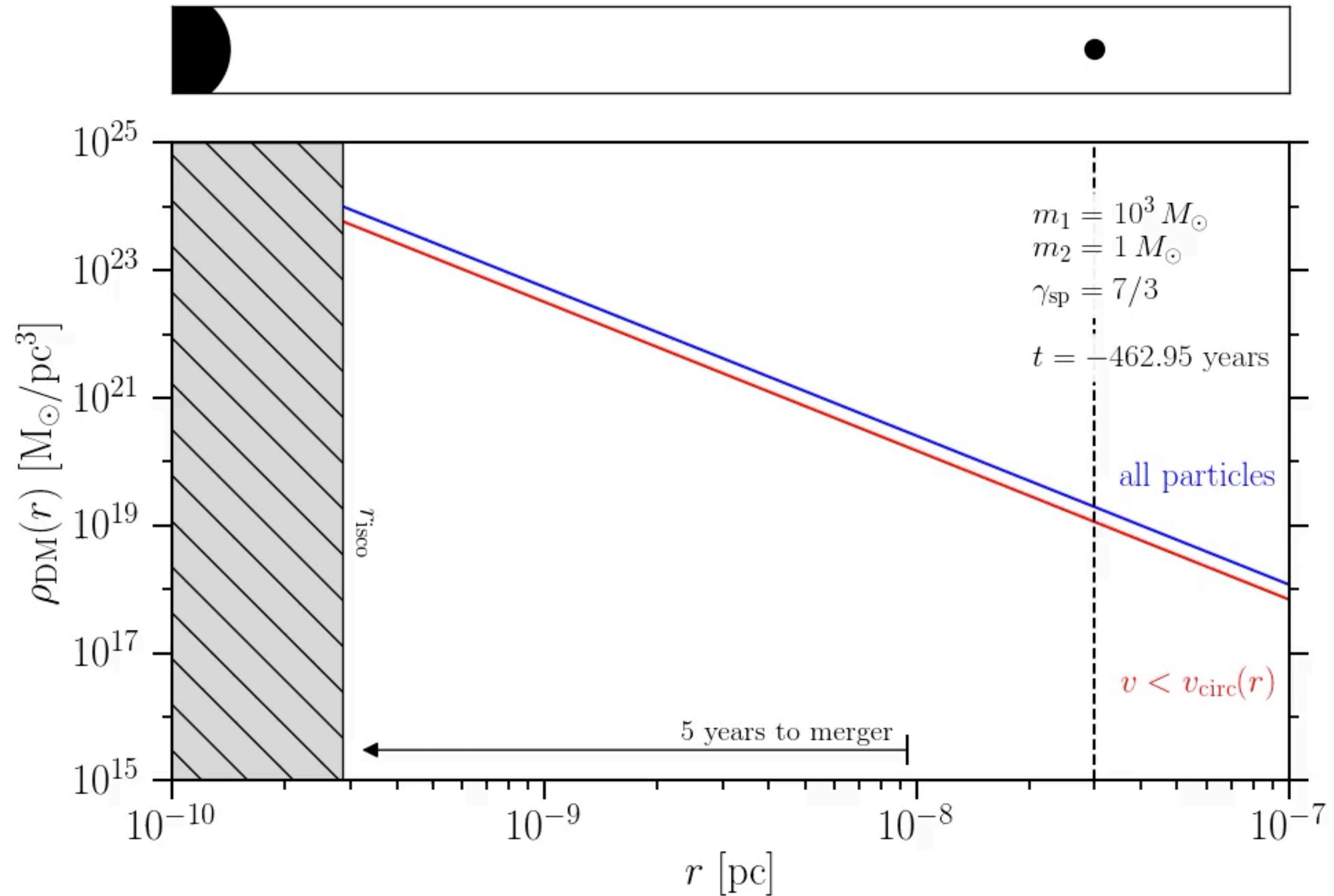
[Movies: tinyurl.com/GW4DM]

[BJK, Nichols, Gaggero, Bertone, [2002.12811](https://arxiv.org/abs/2002.12811)]

[Code: github.com/bradkav/HaloFeedback]

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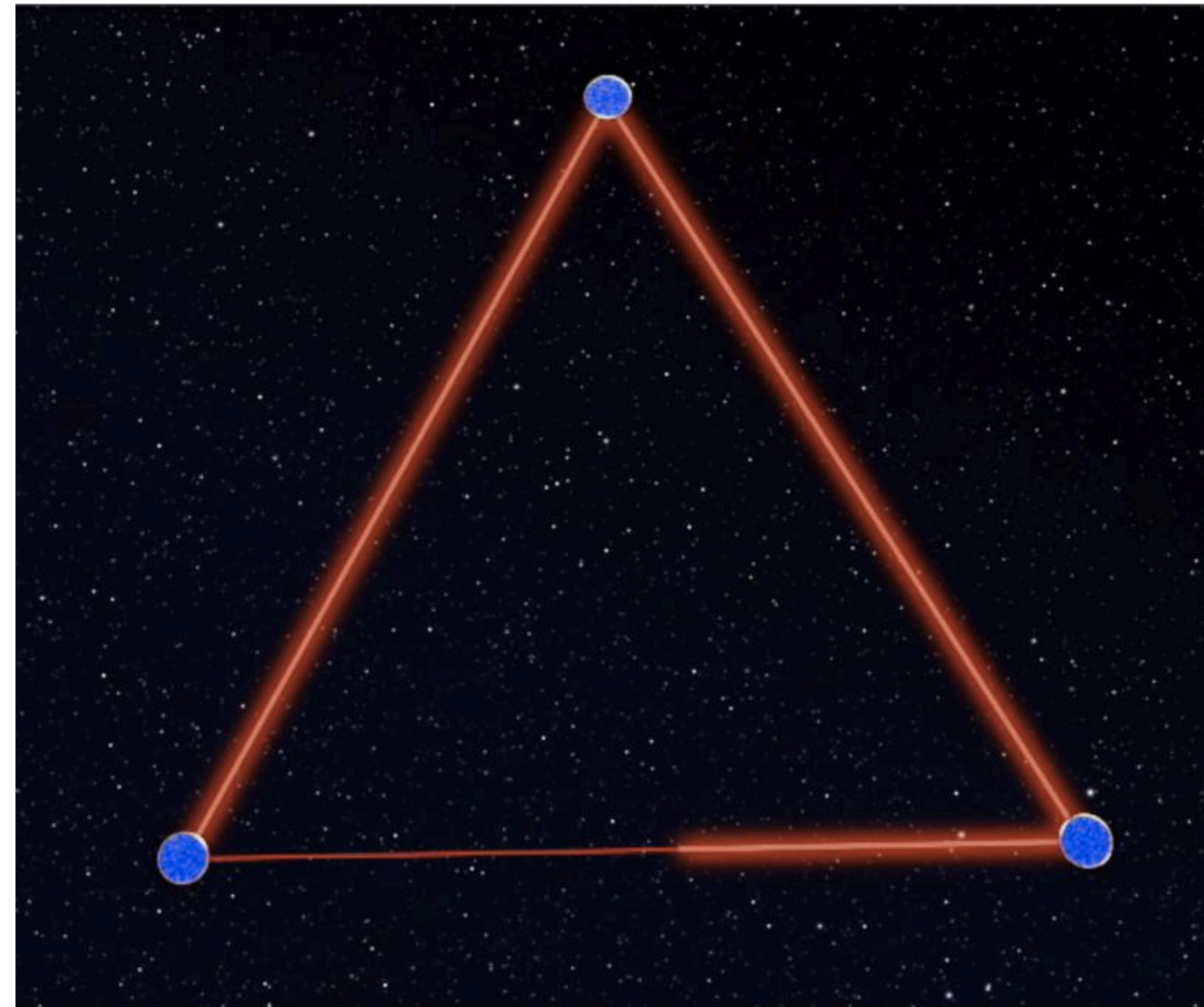
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[Movies: tinyurl.com/GW4DM]

[BJK, Nichols, Gaggero, Bertone, [2002.12811](https://arxiv.org/abs/2002.12811)]

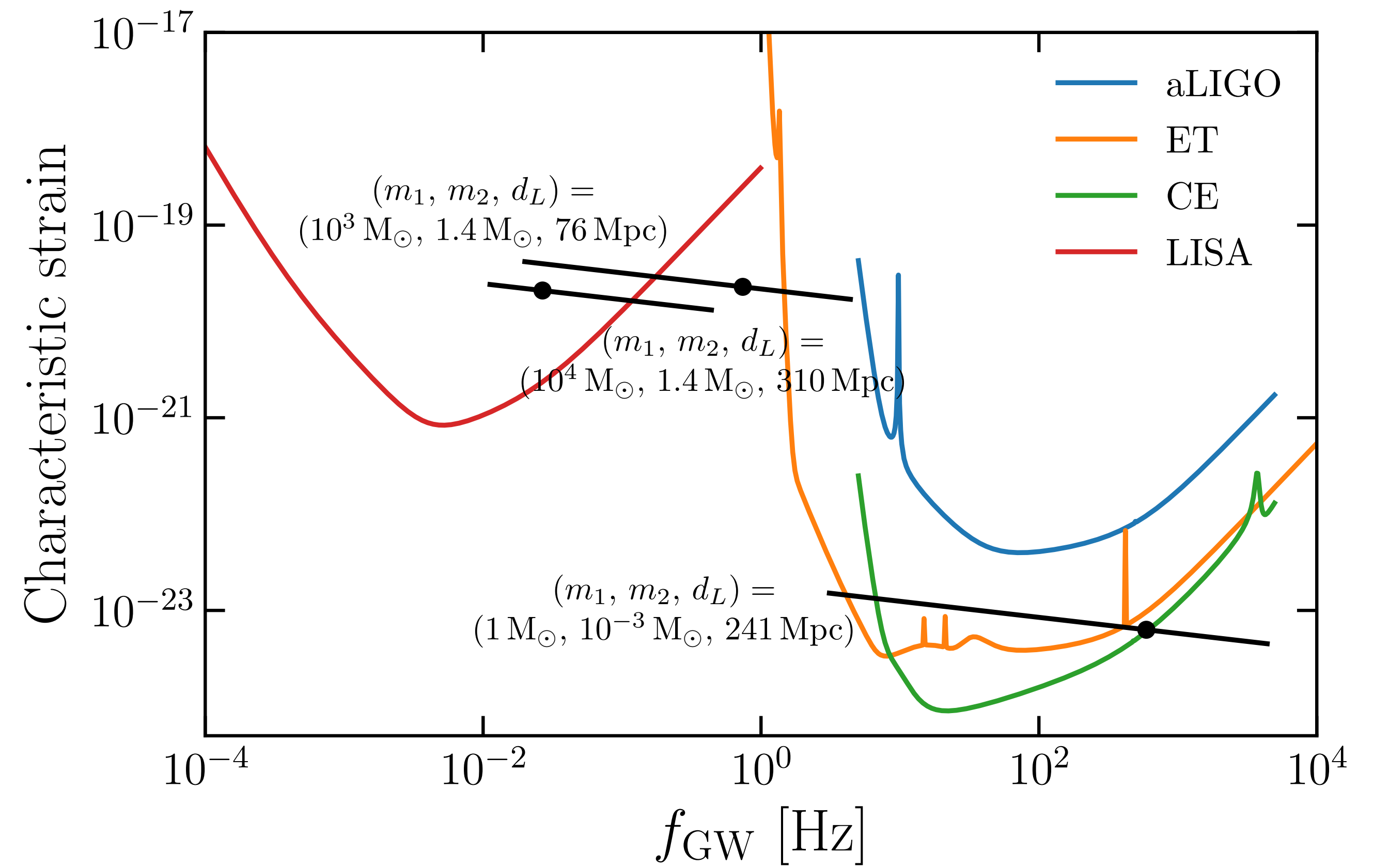
[Code: github.com/bradkav/HaloFeedback]

© AEI / MM / exozet



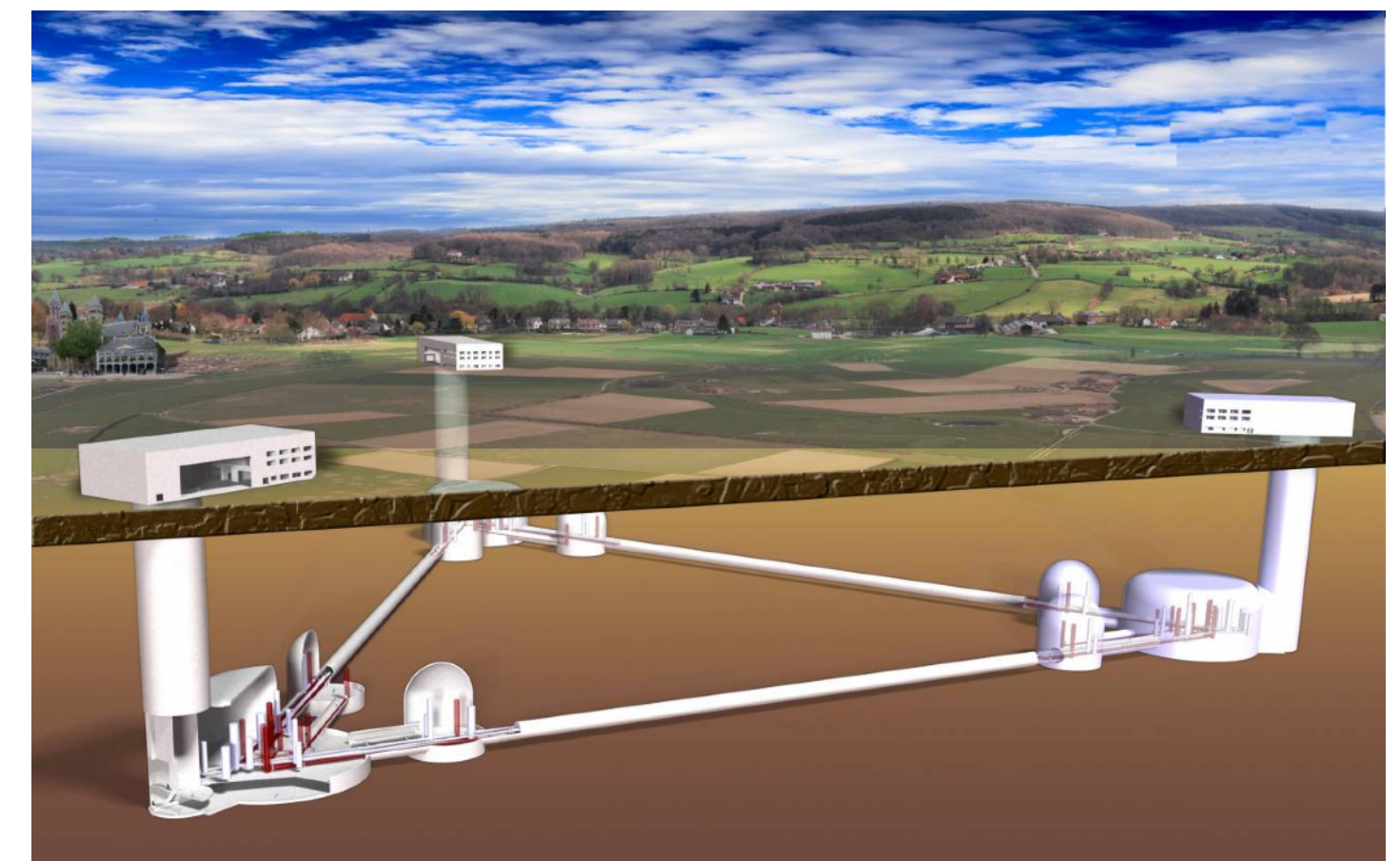
Laser **I**nterferometer **S**pace **A**ntenna
(planned for the 2030s)

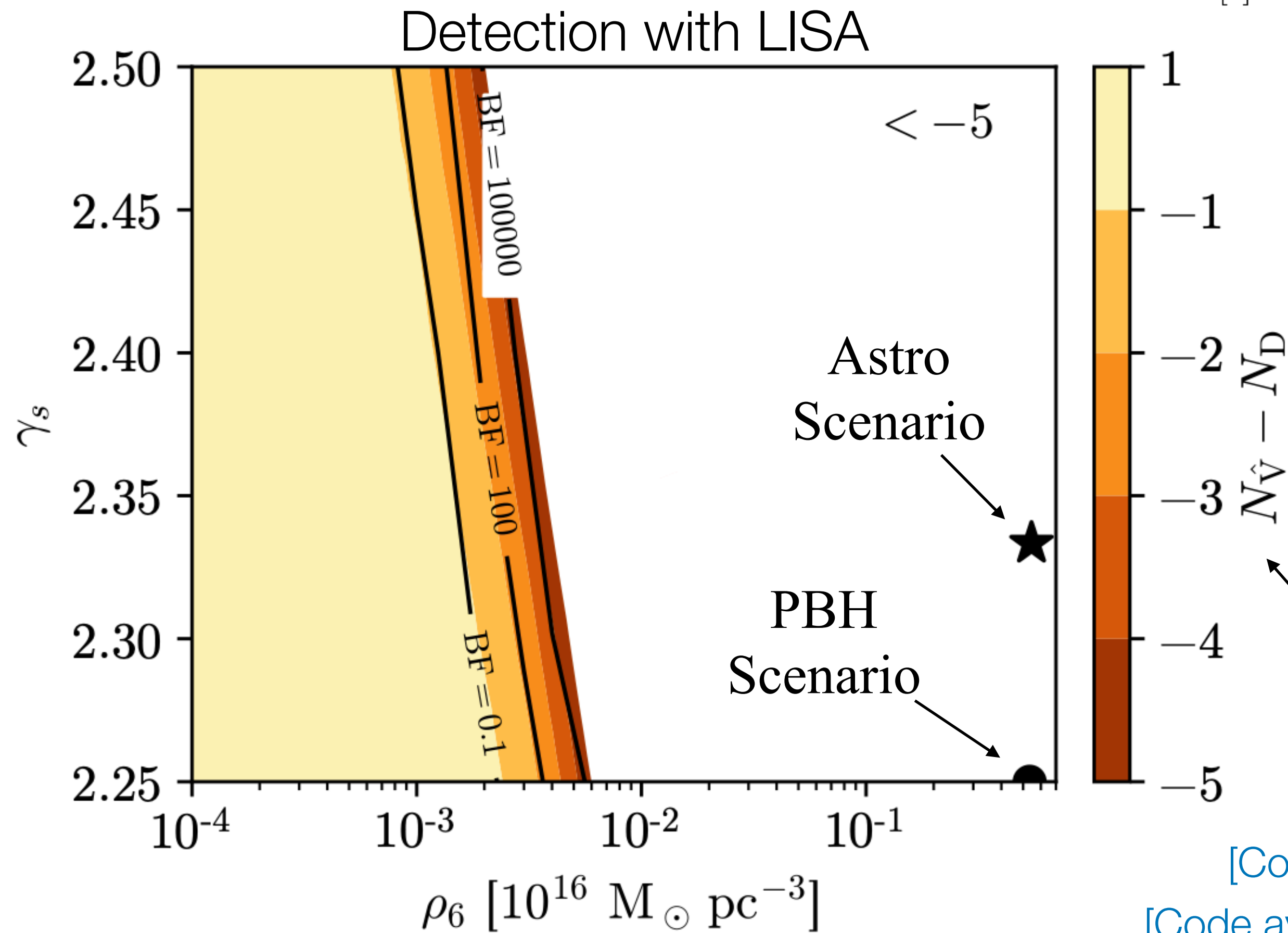
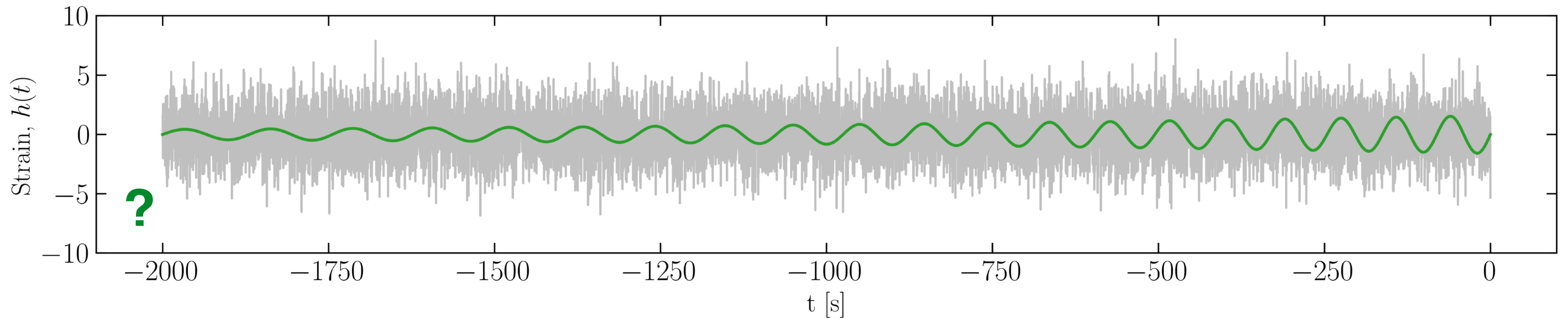
[\[1907.06482\]](#)



Einstein **T**elescope

[\[1912.02622\]](#)





Compare **Bayes factor (BF)** for the vacuum case (V) and the DM dressed case (D)

$$\theta_V = \{\mathcal{M}\}$$

vs.

$$\theta_D = \{\gamma_{sp}, \rho_6, \mathcal{M}, \log_{10} q\}$$

Number of GW cycles of dephasing

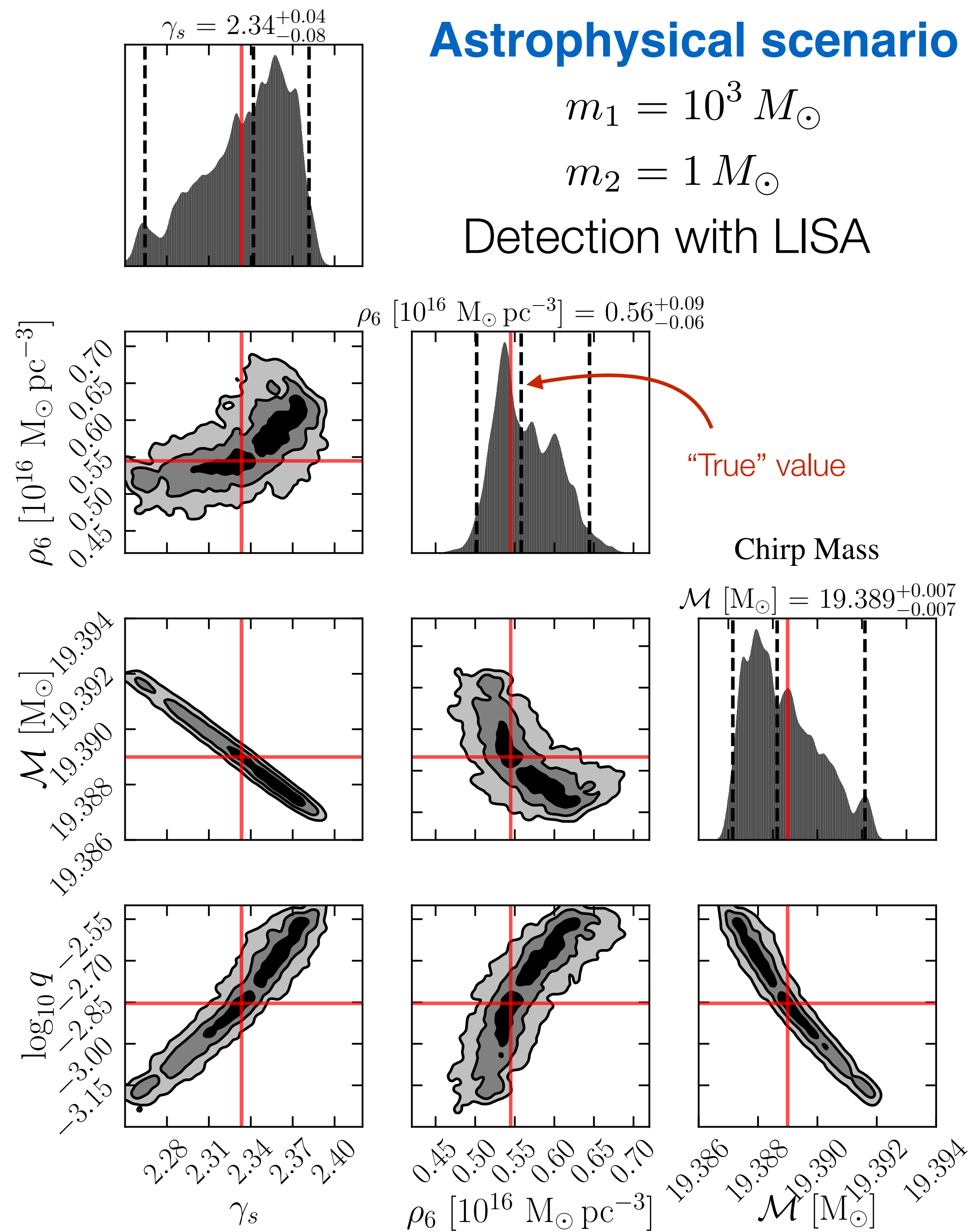
[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](https://arxiv.org/abs/2108.04154)]
 [Code available online: <https://github.com/adam-coogan/pydd>]

Astrophysical scenario

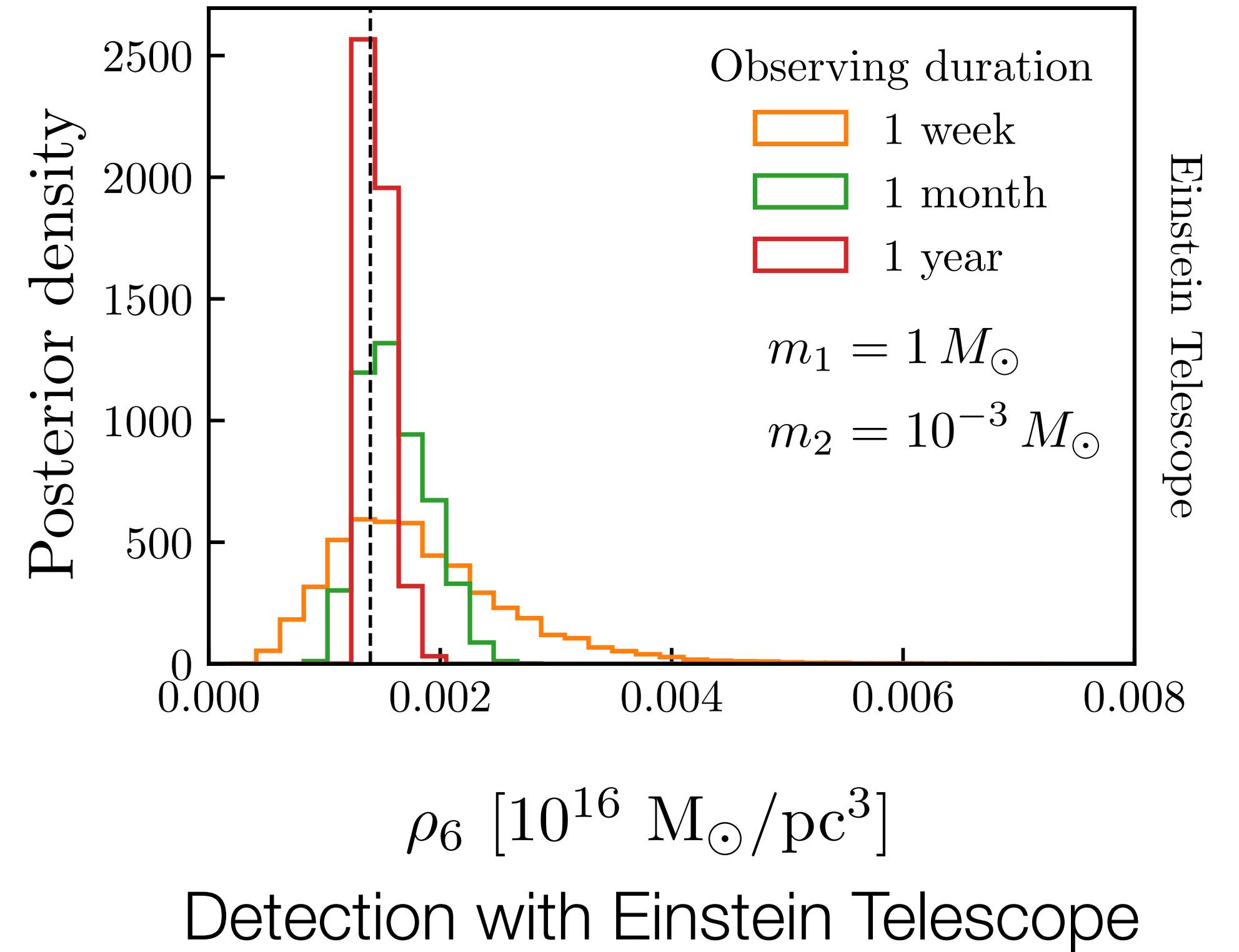
$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

Detection with LISA



PBH scenario



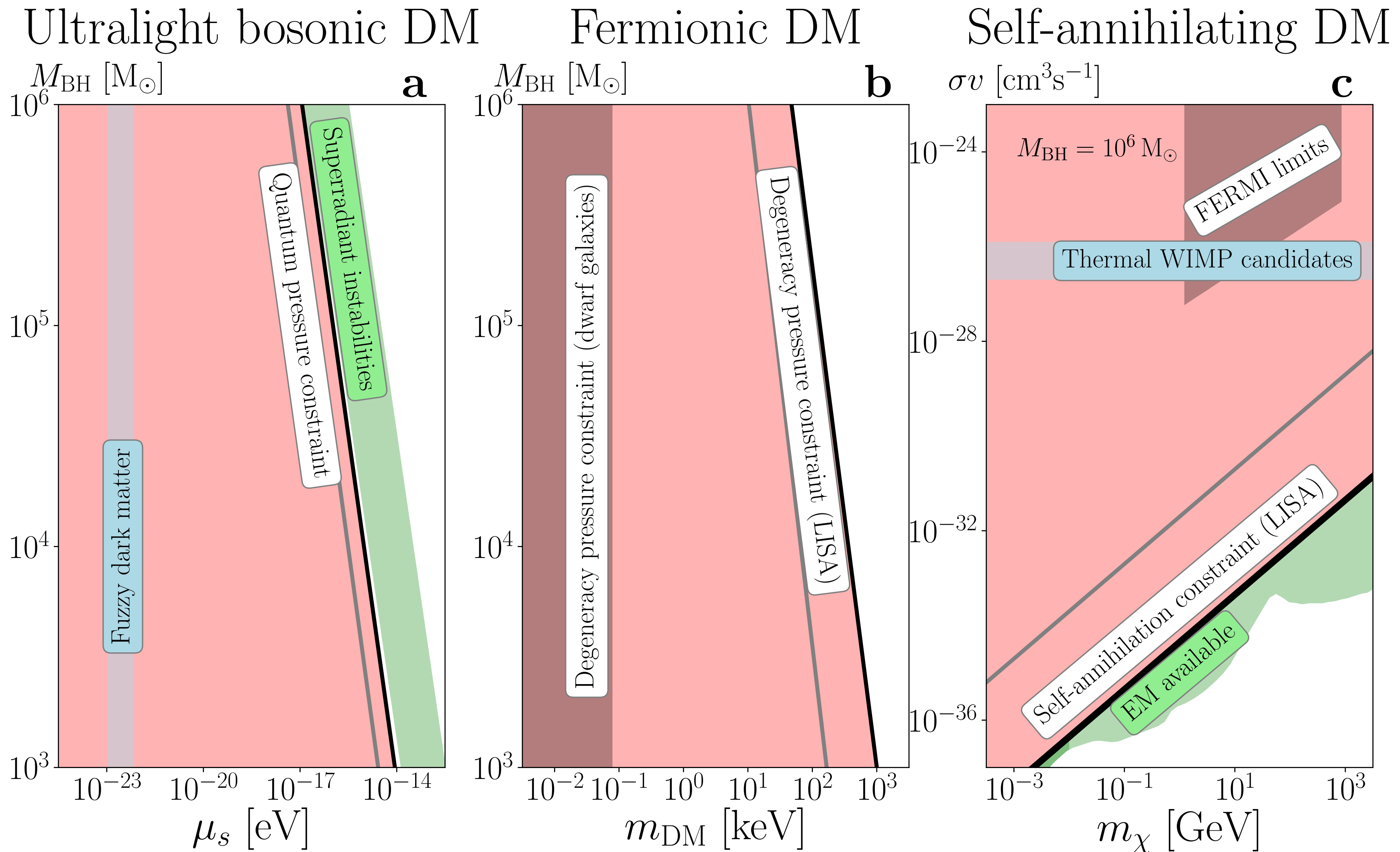
[Cole, Coogan, **BJK**, Bertone, [2207.07576](#)]

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]

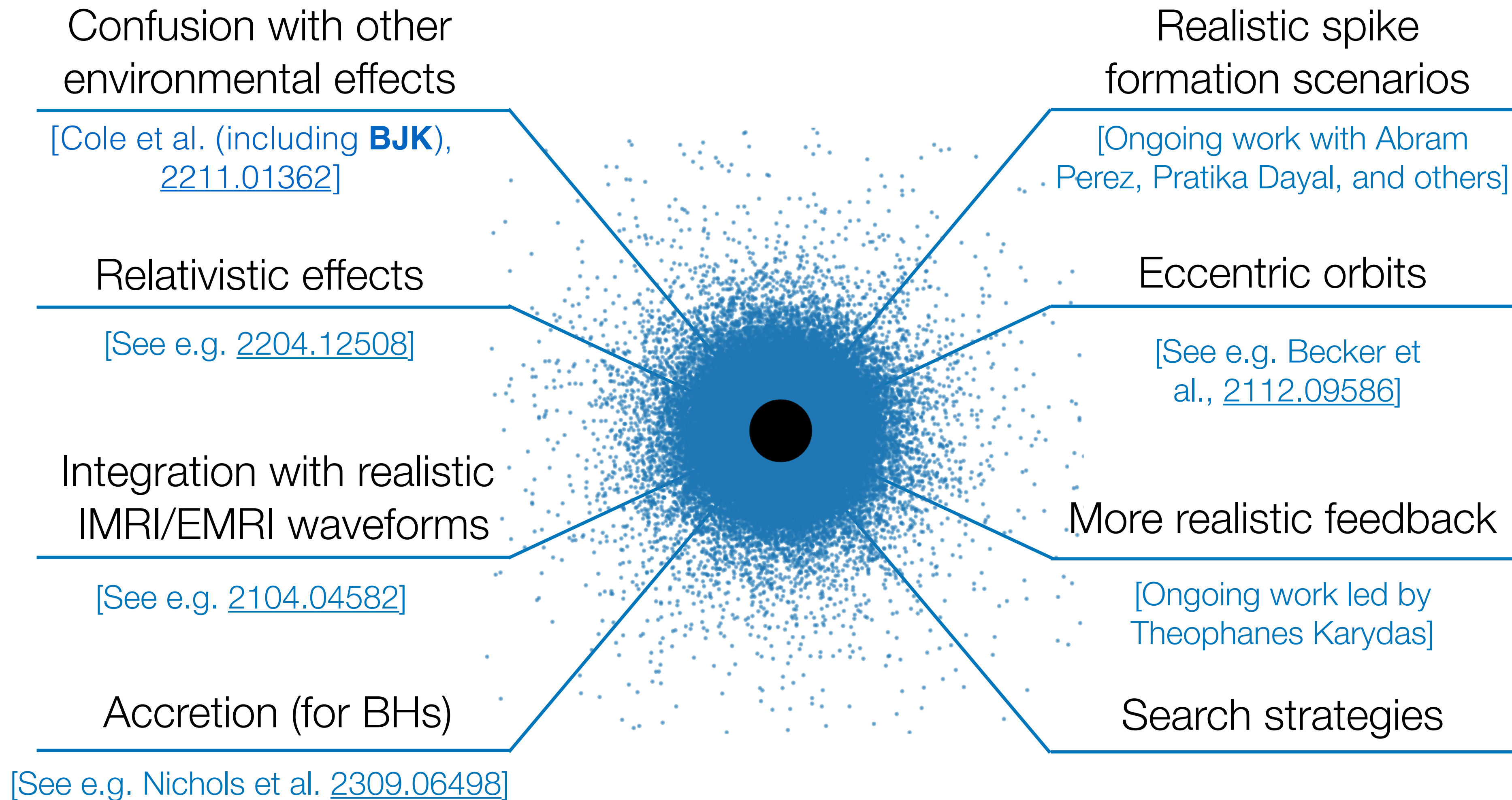
[Code: github.com/adam-coogan/pydd]

Red regions would be ruled out by observation of a DM spike!

[1906.11845]



[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, 1905.01238]



Confusion with other environmental effects
[Cole et al. (including **BJK**), [2211.01362](#)]

Relativistic effects
[See e.g. [2204.12508](#)]

Integration with realistic IMRI/EMRI waveforms
[See e.g. [2104.04582](#)]

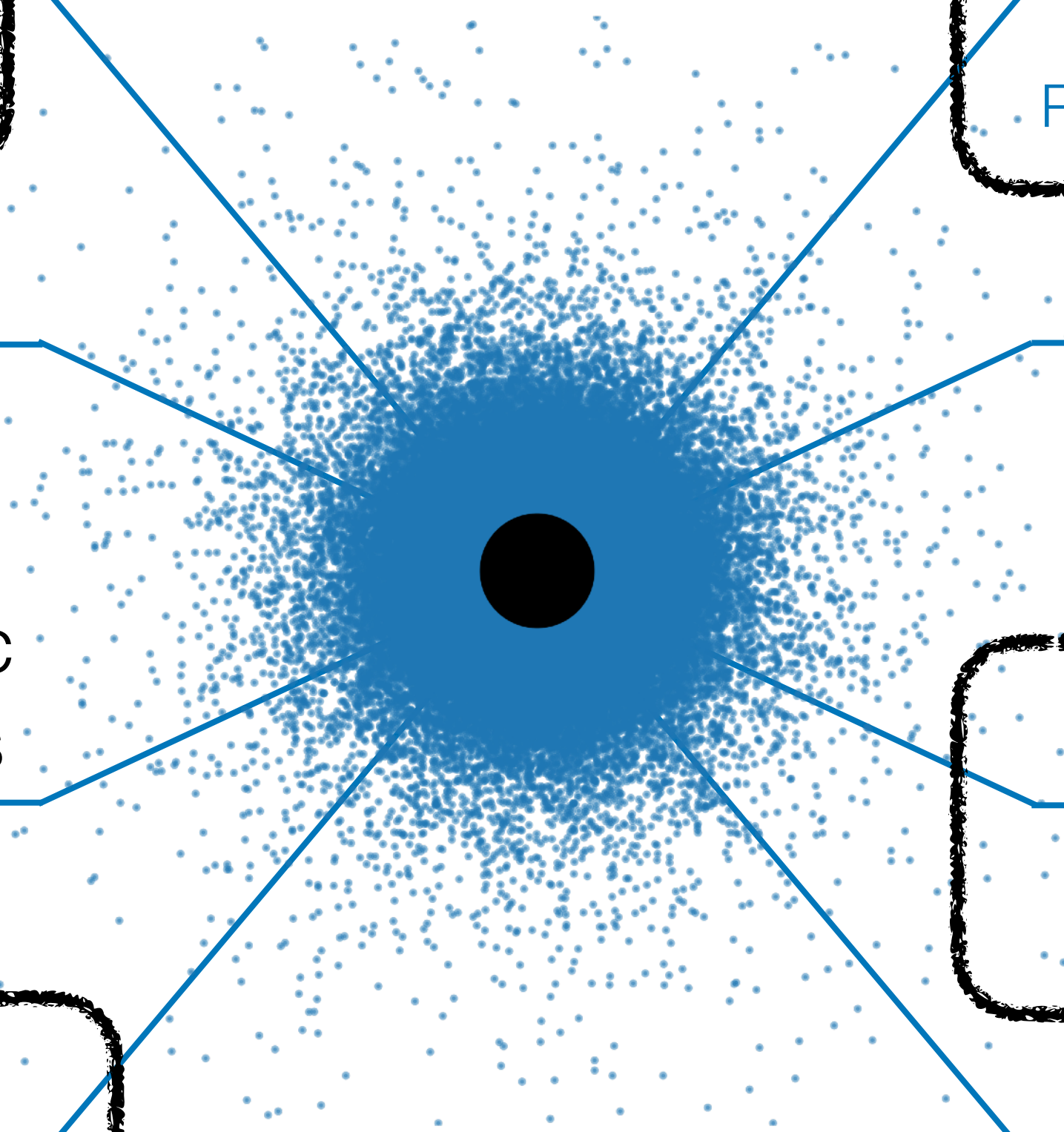
Accretion (for BHs)
[See e.g. Nichols et al. [2309.06498](#)]

Realistic spike formation scenarios
[Ongoing work with Abram Perez, Pratika Dayal, and others]

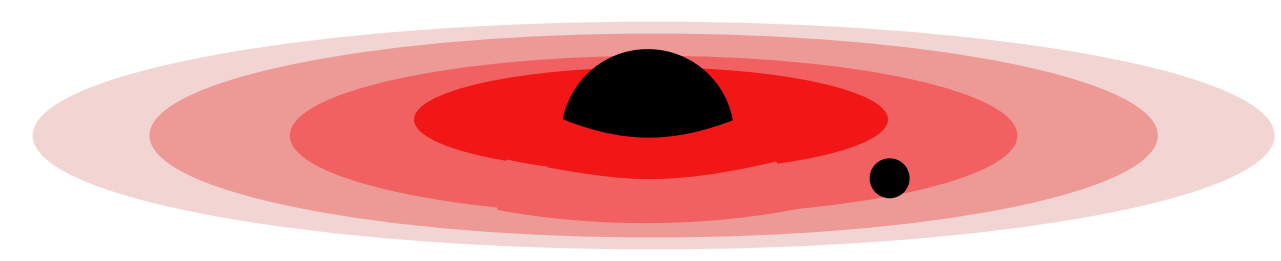
Eccentric orbits
[See e.g. Becker et al., [2112.09586](#)]

More realistic feedback
[Ongoing work led by Theophanes Karydas]

Search strategies

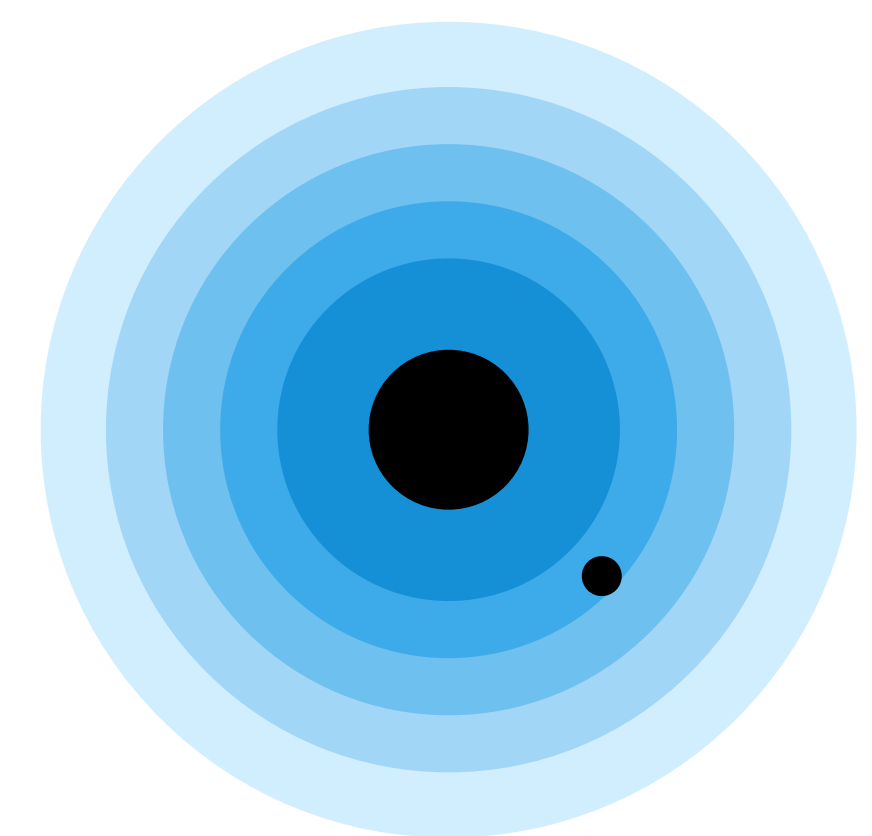


Generate waveform assuming:



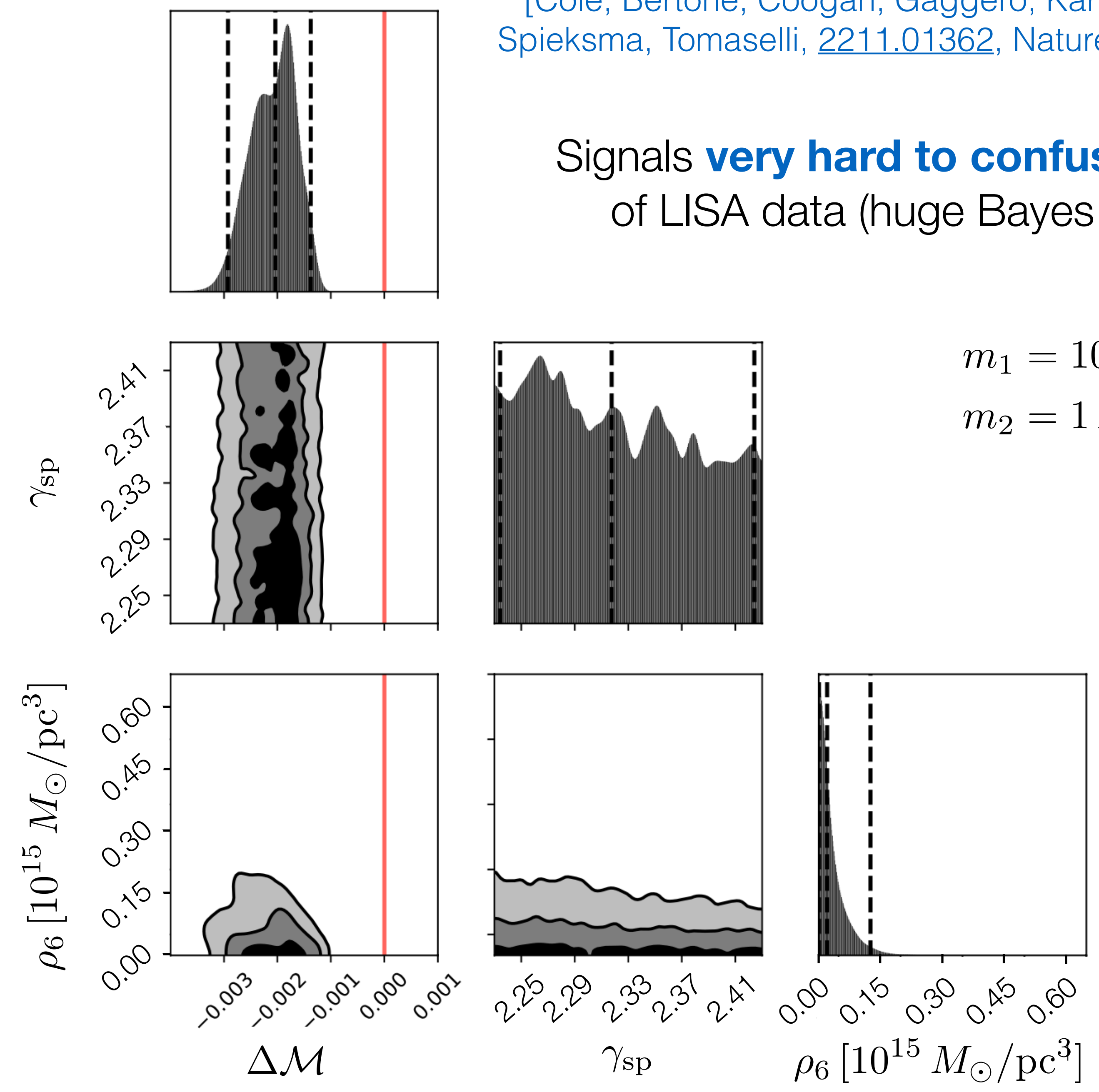
$$\Sigma(r) = \Sigma_0 \left(\frac{r}{r_0} \right)^{-1/2}$$

Fit signal assuming:

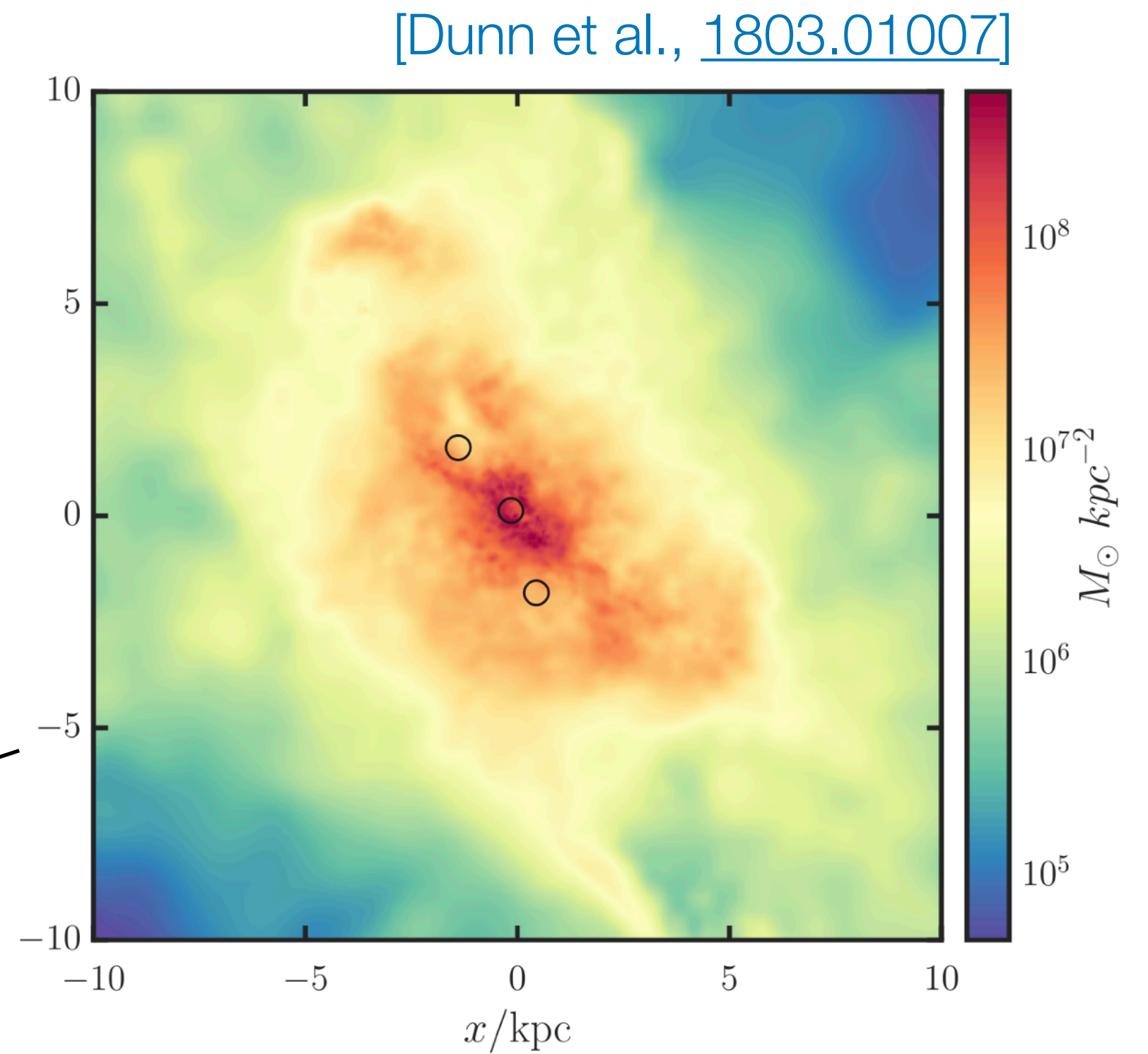
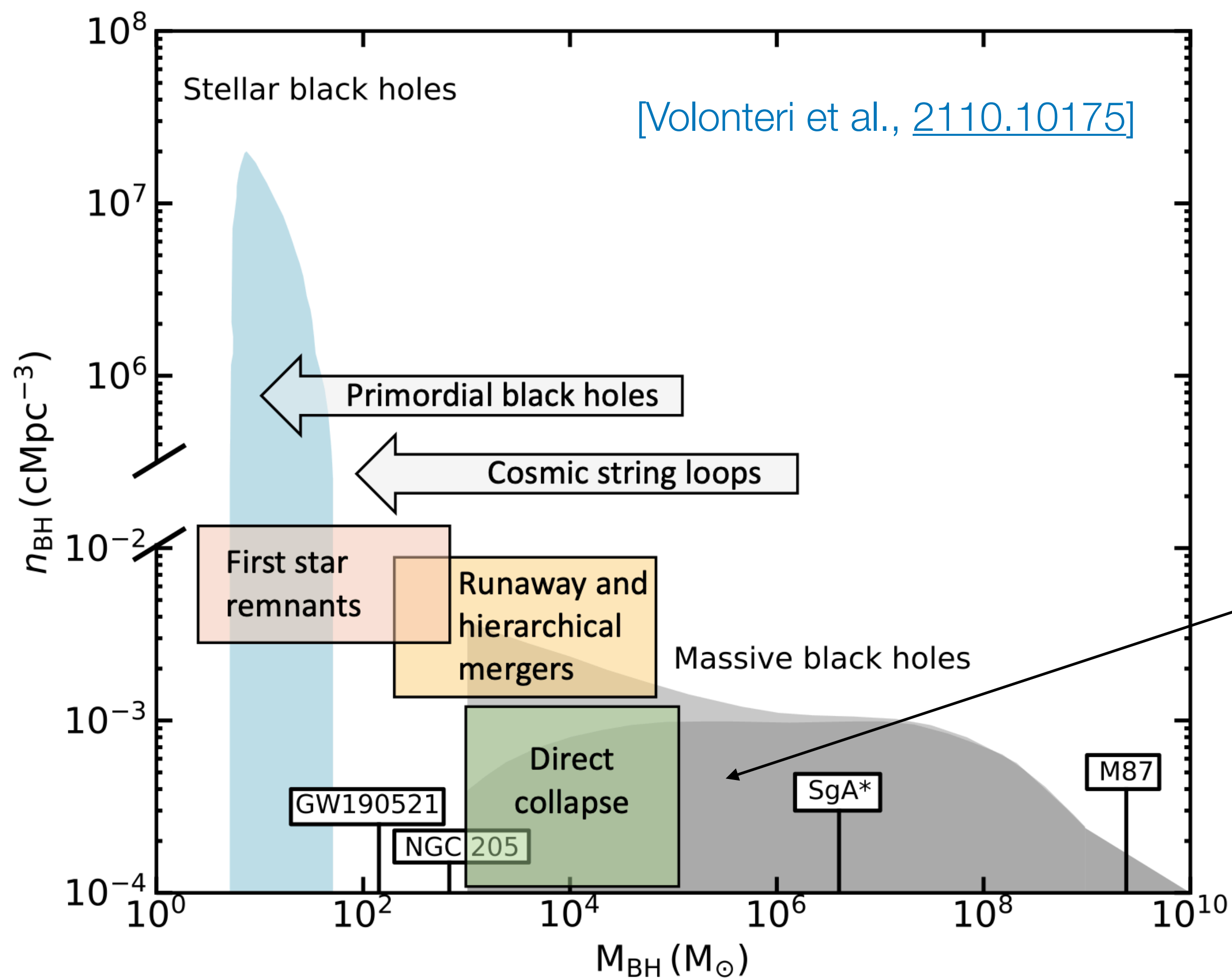


[Cole, Bertone, Coogan, Gaggero, Karydas, **BJK**, Spieksma, Tomaselli, [2211.01362](#), Nature Astronomy]

Signals **very hard to confuse** in 1 year of LISA data (huge Bayes factors!)



$m_1 = 10^3 M_\odot$
 $m_2 = 1 M_\odot$



Use semi-analytic galaxy formation models to study the properties of Direct Collapse Black Holes and the halos they form in.

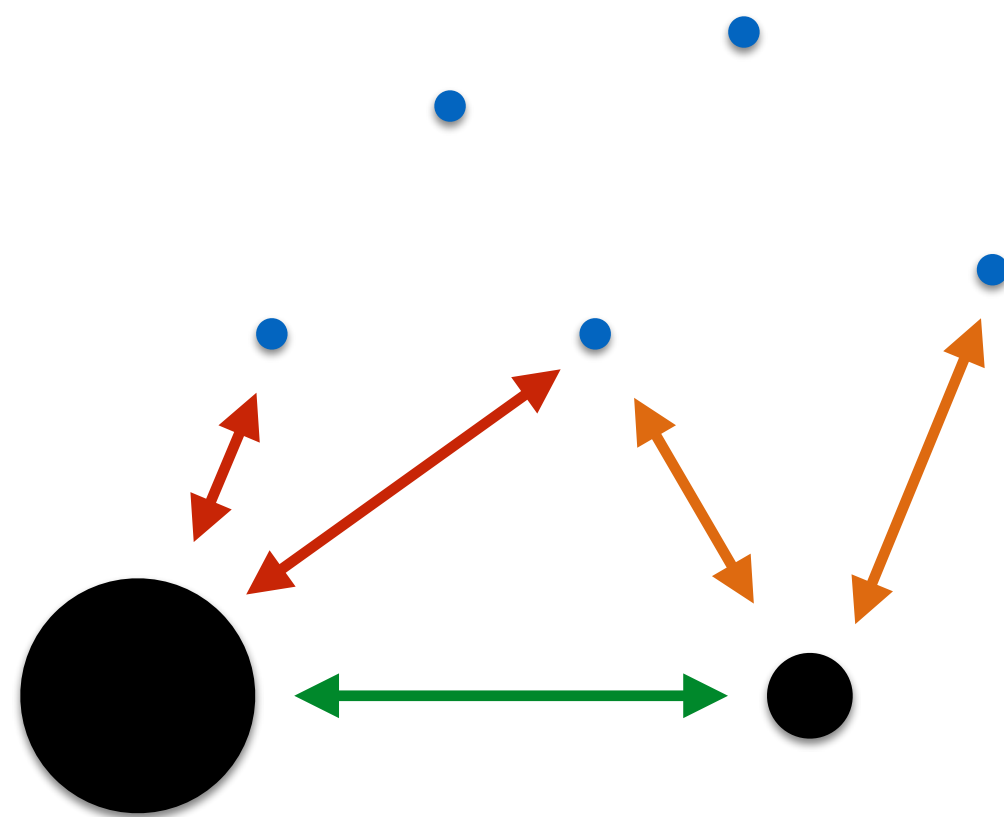
Preliminary results suggest that large densities are possible but do these systems survive, and are they common?

$$\rho_6 \gtrsim 10^{16} M_{\odot} \text{pc}^{-3}$$

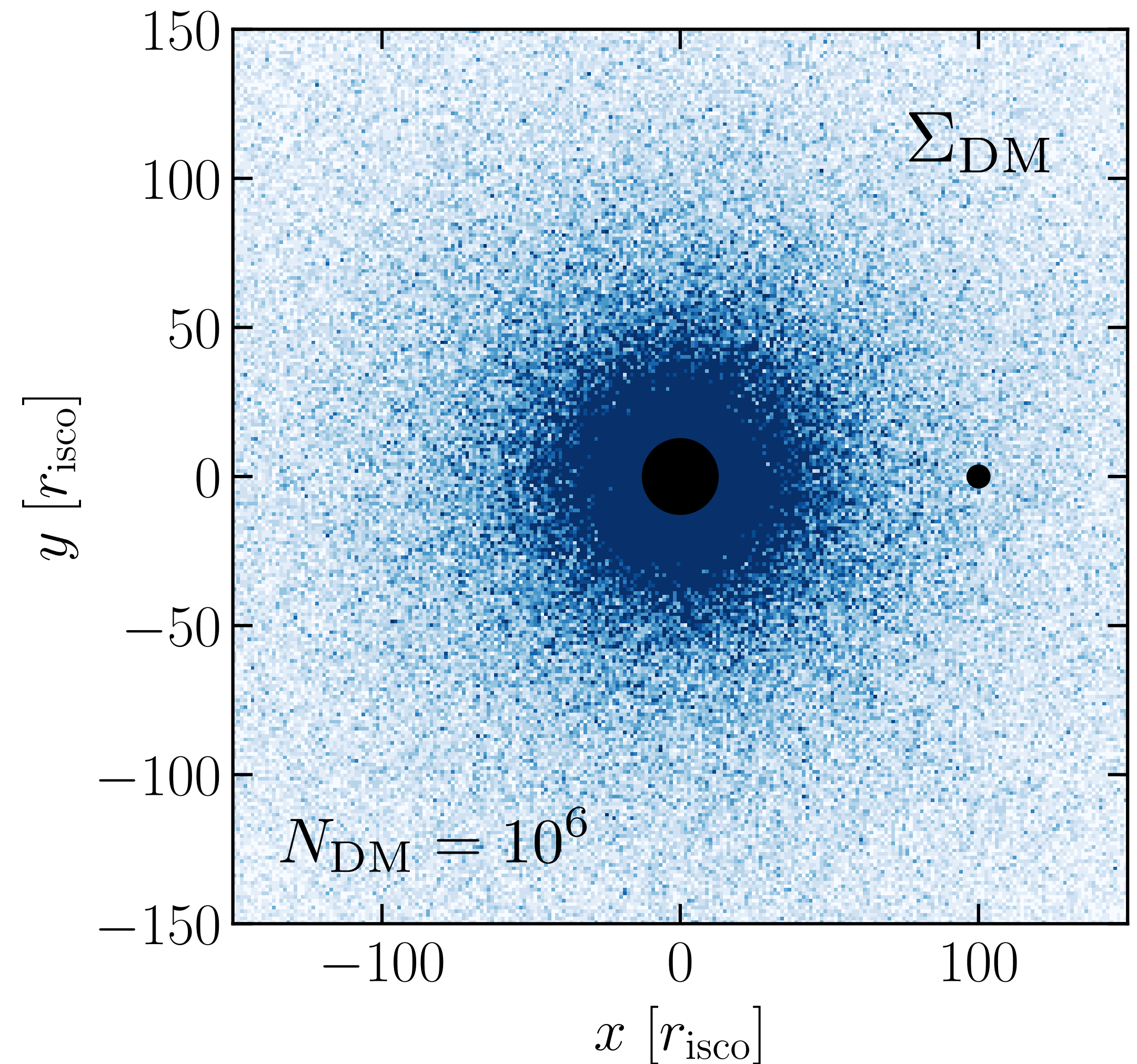
Eventually need to expand and verify our description of dynamical friction and feedback in the DM spike (also include anisotropy, accretion, post-Newtonian corrections...)

↪ **NbodyIMRI**: N-body solver tailored to DM spikes

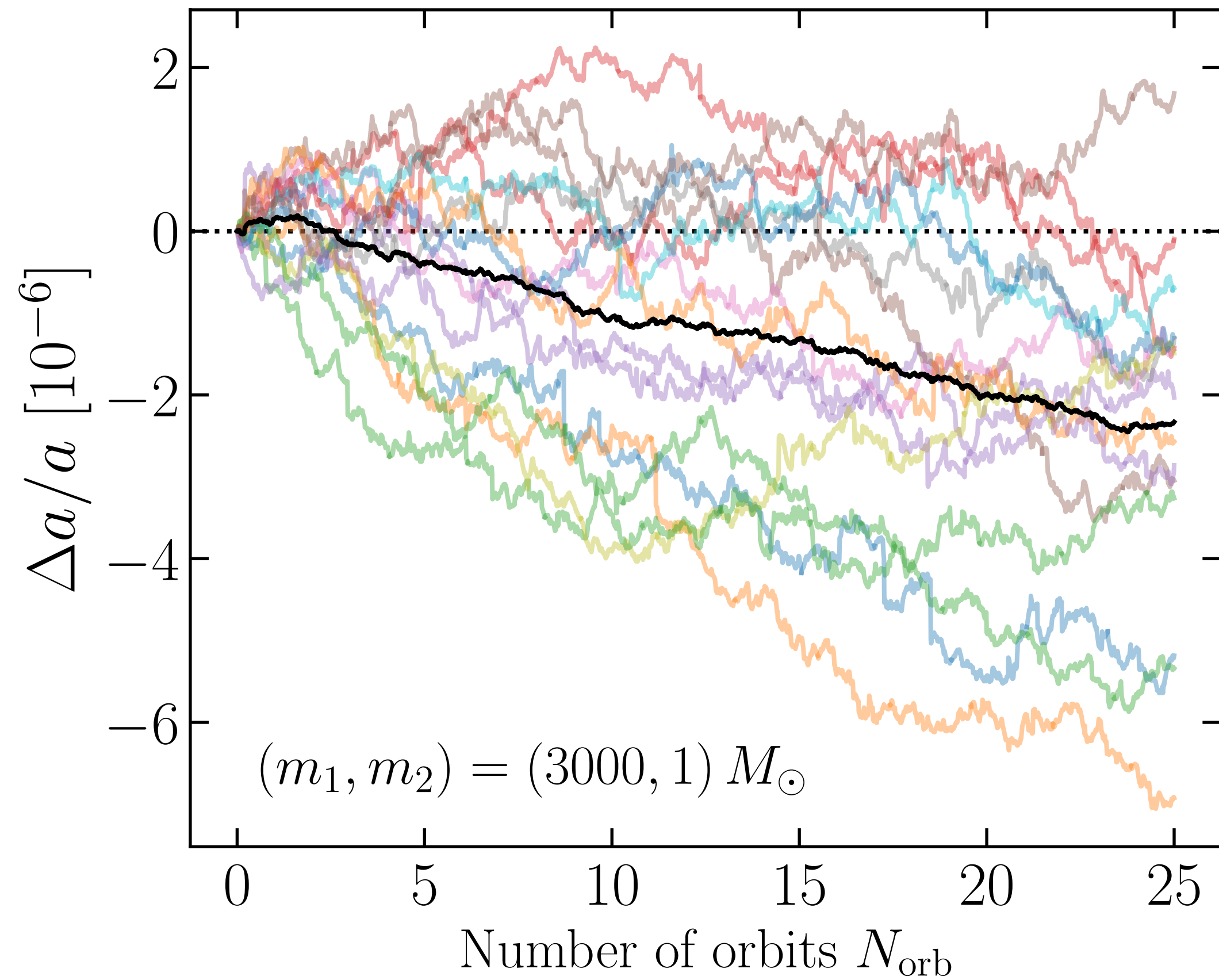
No DM-DM interparticle forces



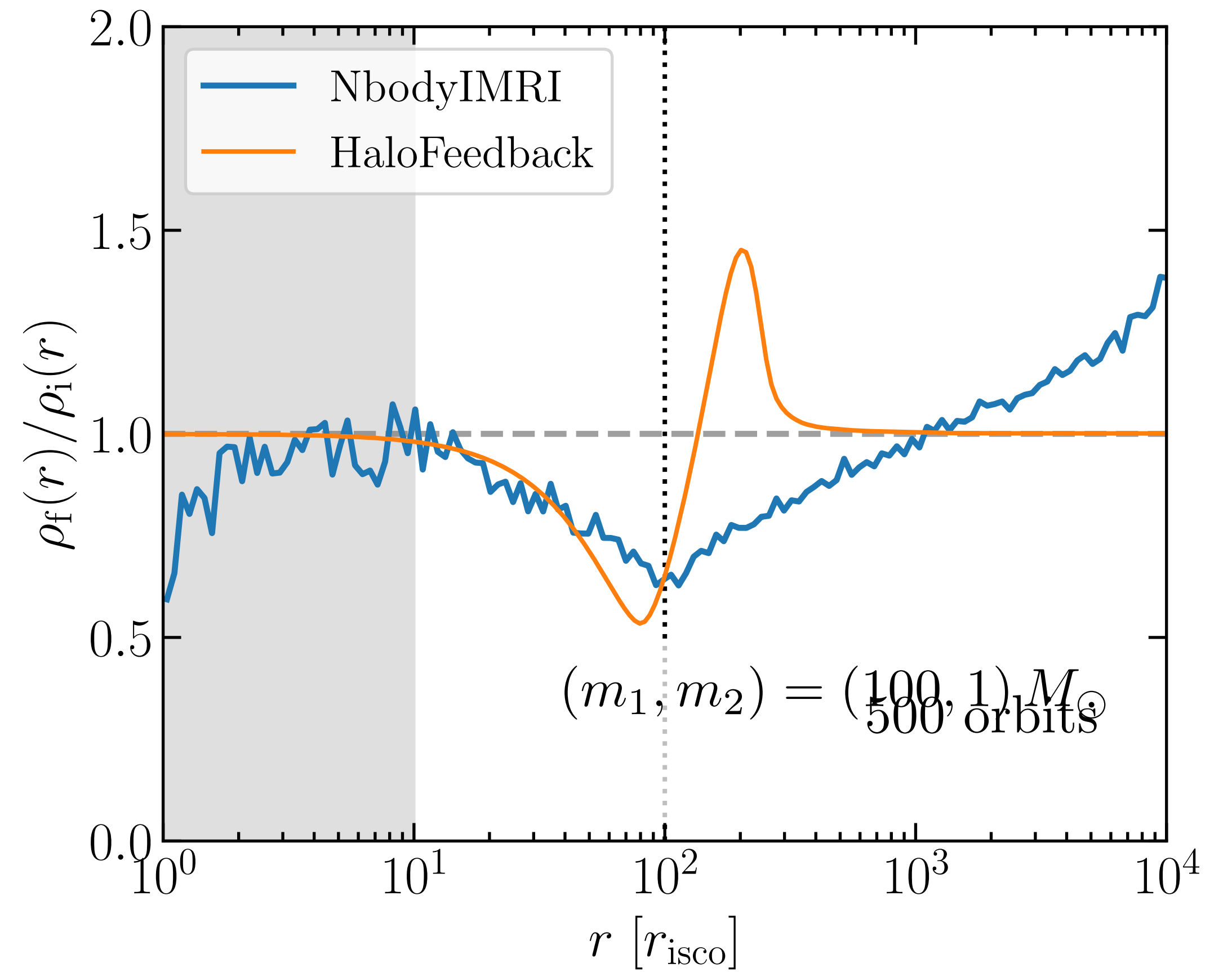
[Code here: github.com/bradkav/NbodyIMRI]



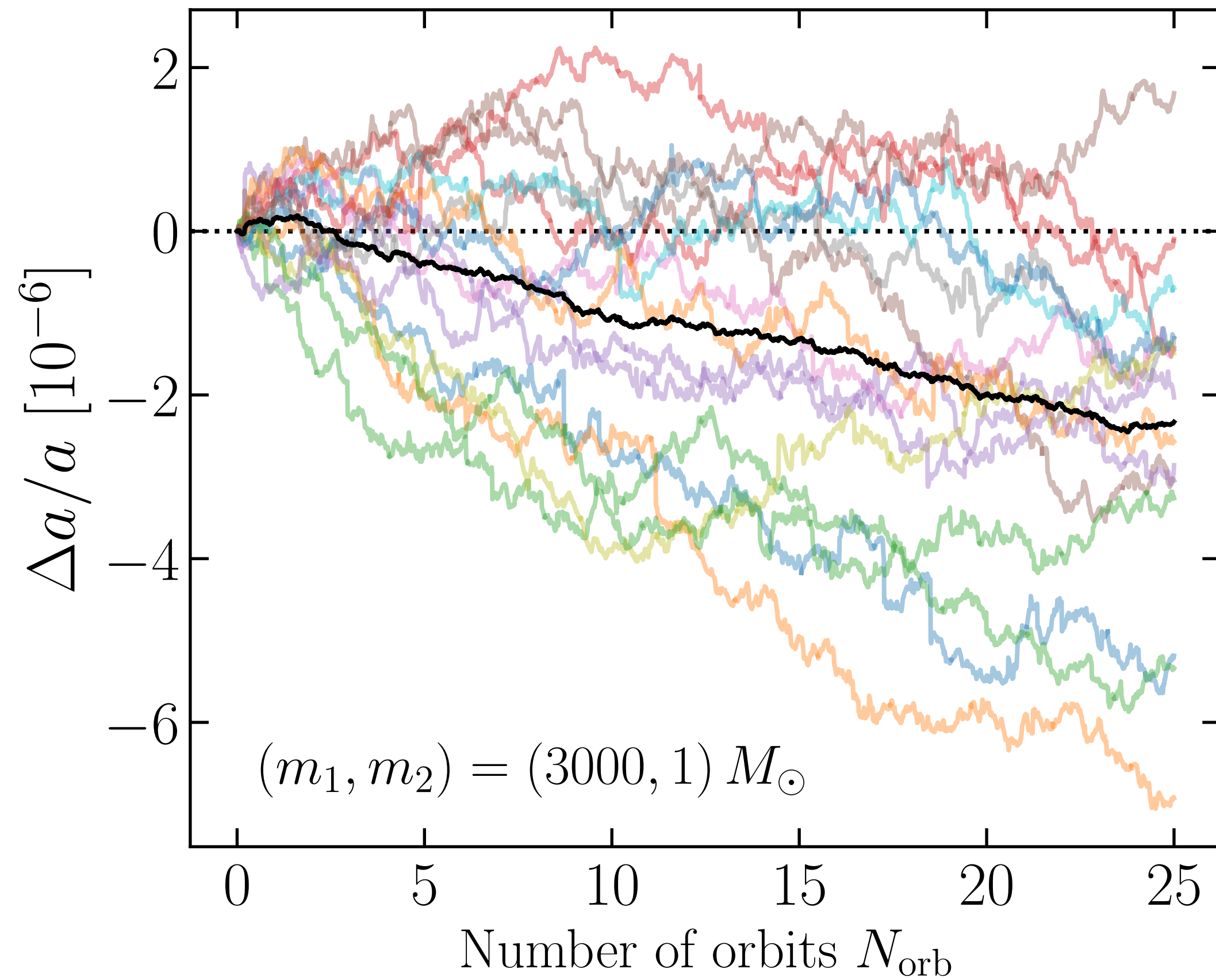
Verify strength of dynamical friction force:



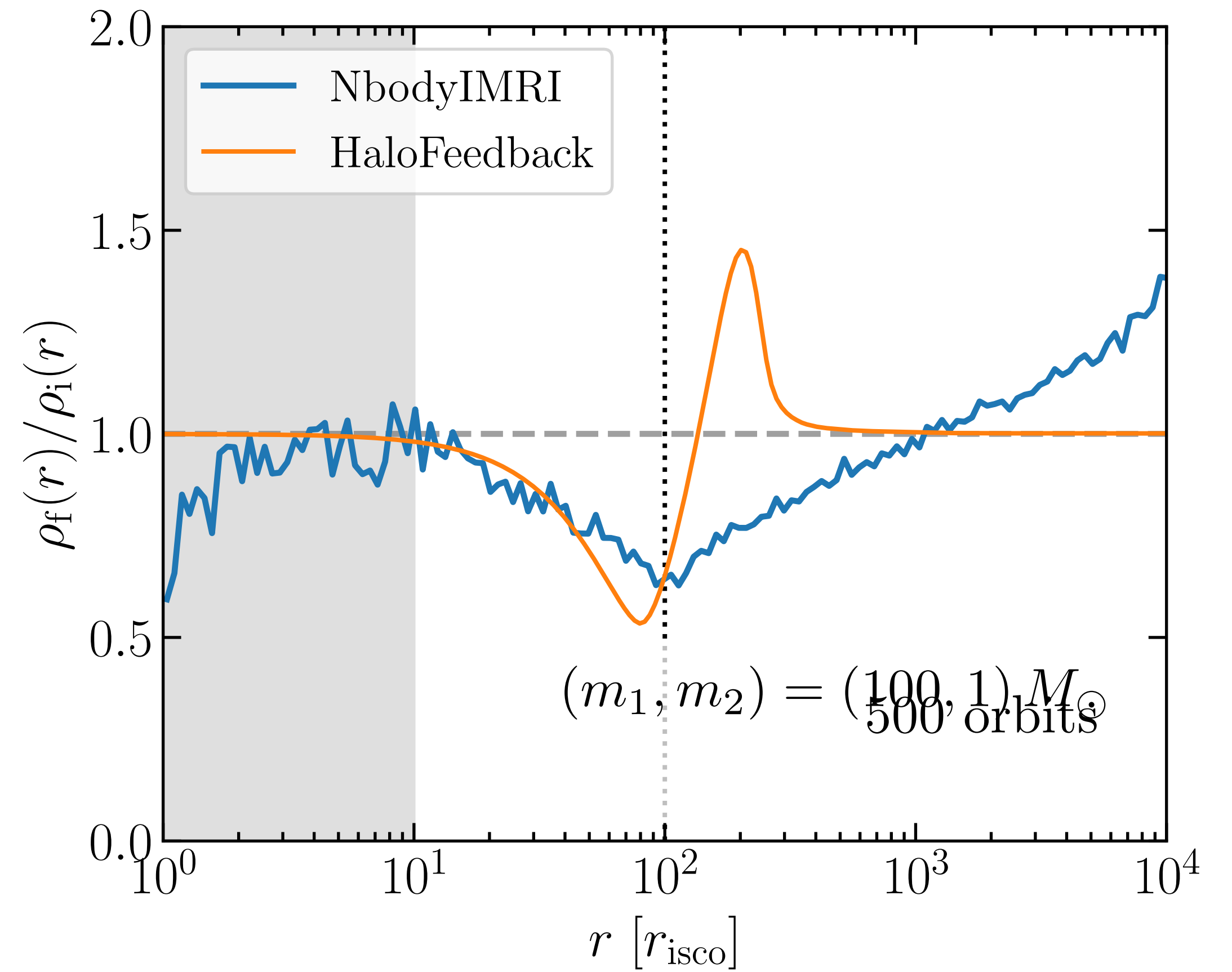
Verify dynamics of the DM halo:



Verify strength of dynamical friction force:



Verify dynamics of the DM halo:



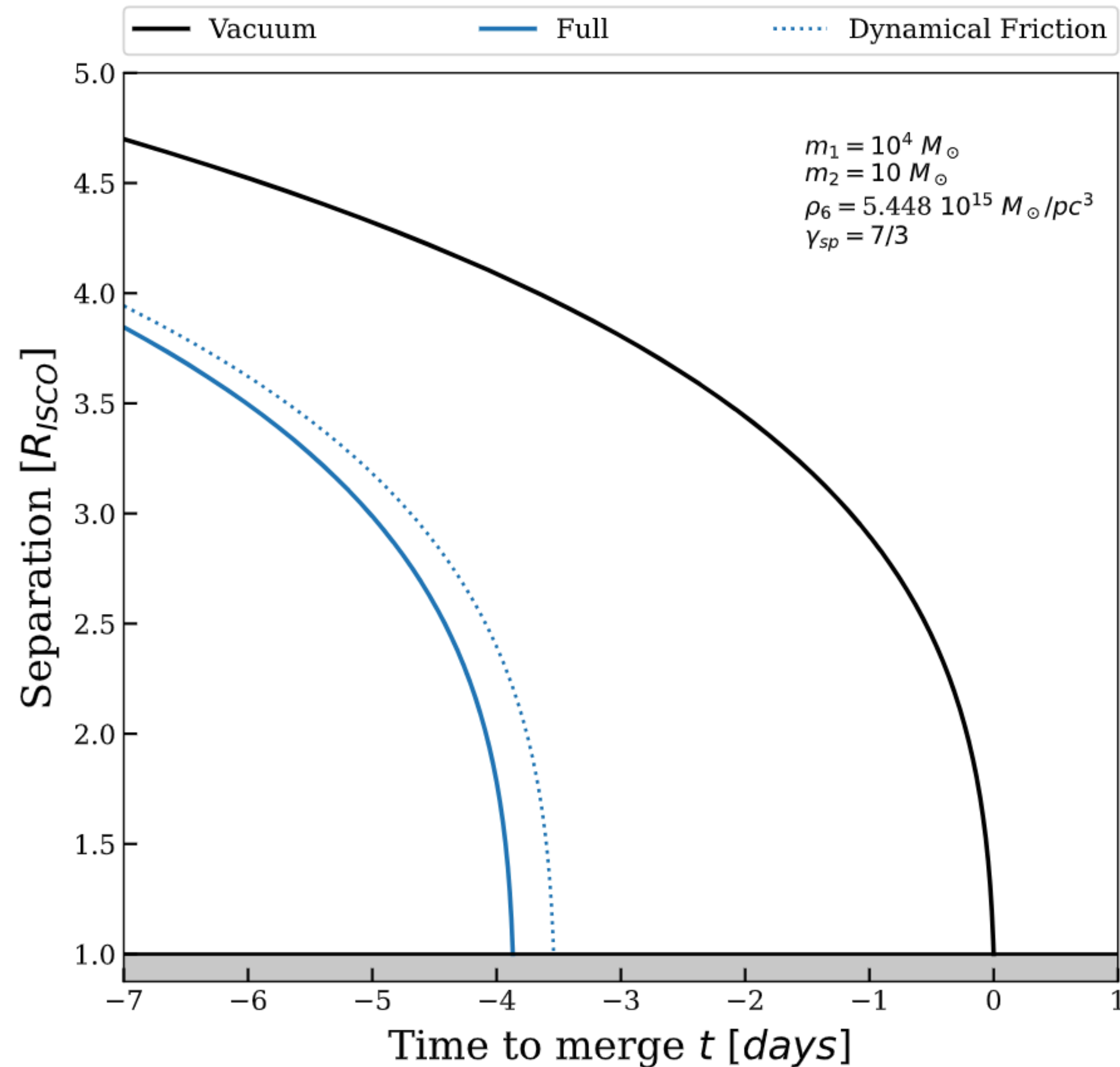
$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$

Accretion of DM by the secondary BH leads to an additional accretion force:

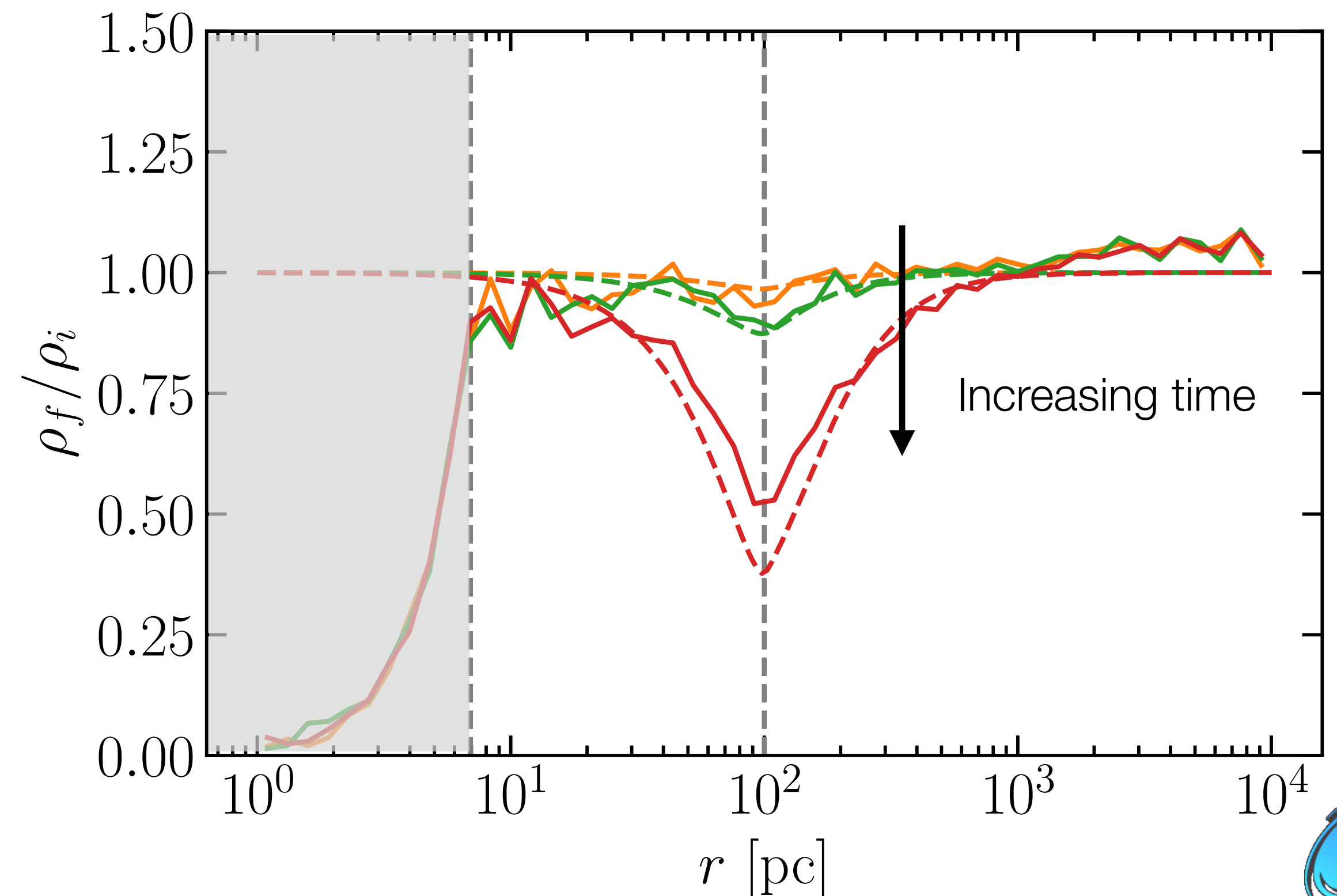
$$F_{\text{acc}} = \int d^3\mathbf{v} \pi b_{\text{acc}}^2(\mathbf{v}_{\text{rel}}) v_{\text{rel}} \rho_{\chi} \mathbf{v}_{\text{rel}} \quad \mathbf{v}_{\text{rel}} = \mathbf{v}_{\text{orb}} - \mathbf{v}$$

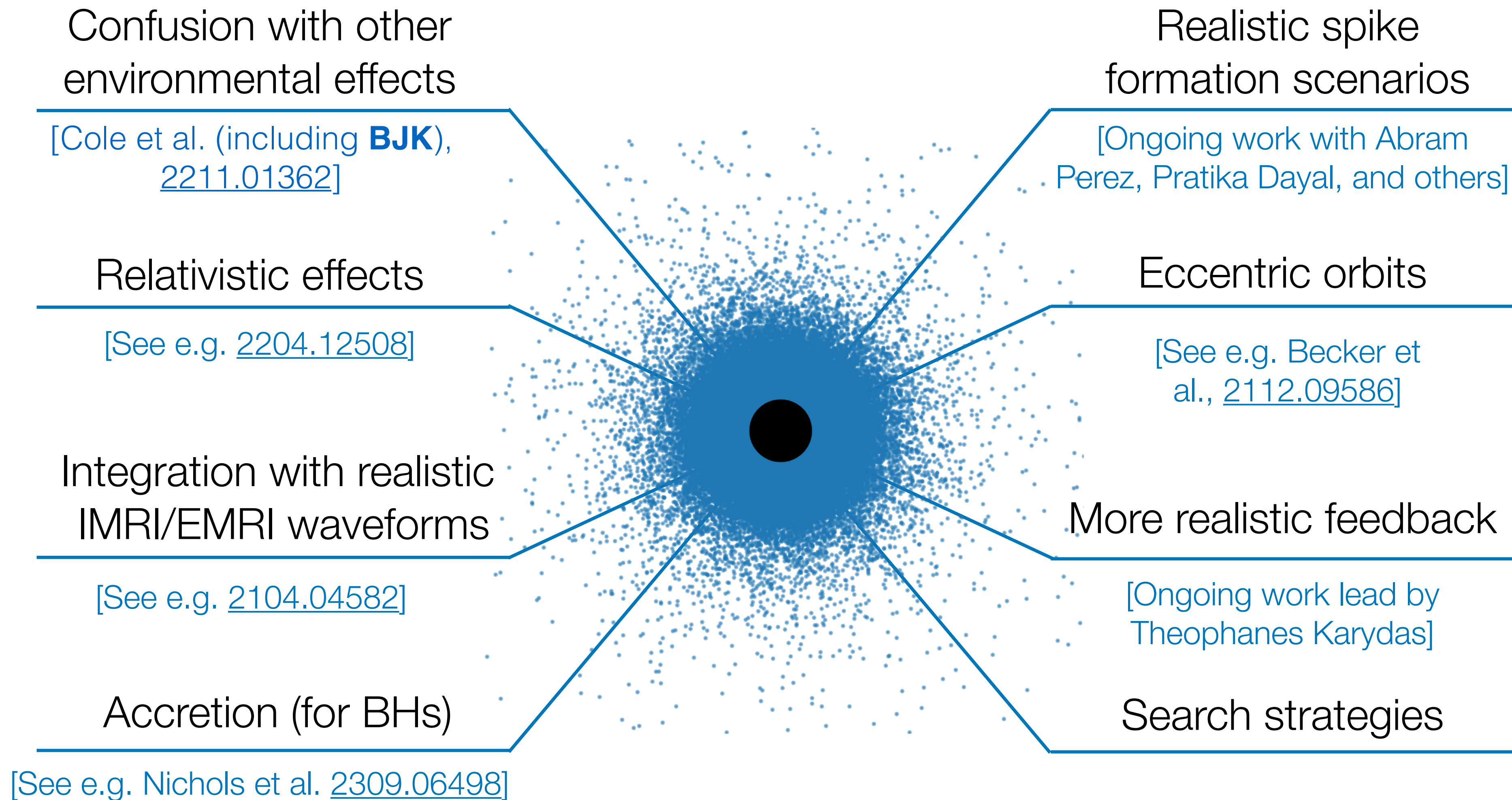
[Work in progress with Theophanes Karydas & Gianfranco Bertone]

[See also Nichols et al. [2309.06498](#)]



Use simulations to validate accretion force and DM spike depletion:





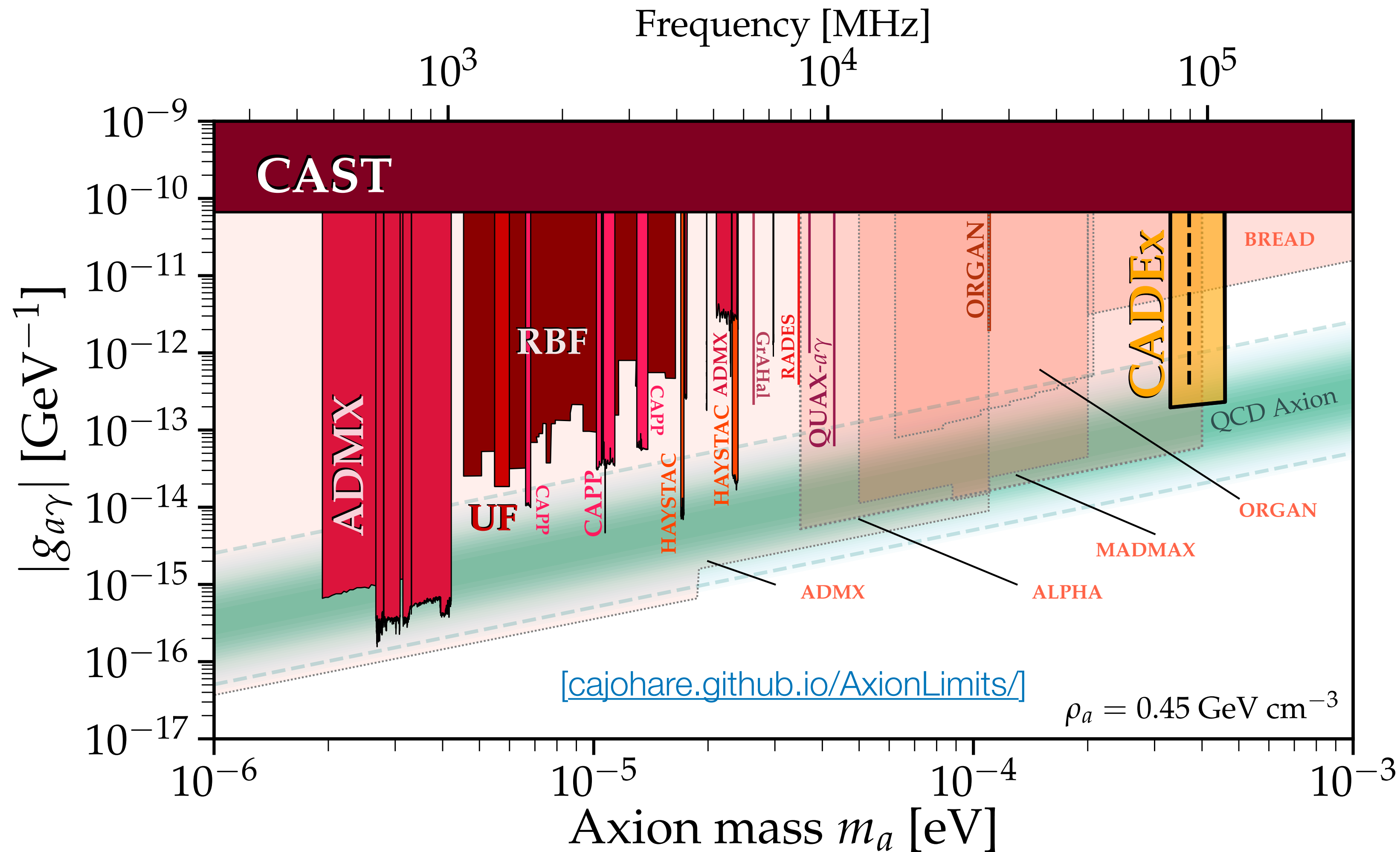
Part 2:
Neutron Stars



Dark Matter could be in the form of light **pseudo-scalar 'axions'**, which may arise as psuedo-Goldstone bosons of a spontaneously broken Peccei-Quinn symmetry $U(1)_{PQ}$.

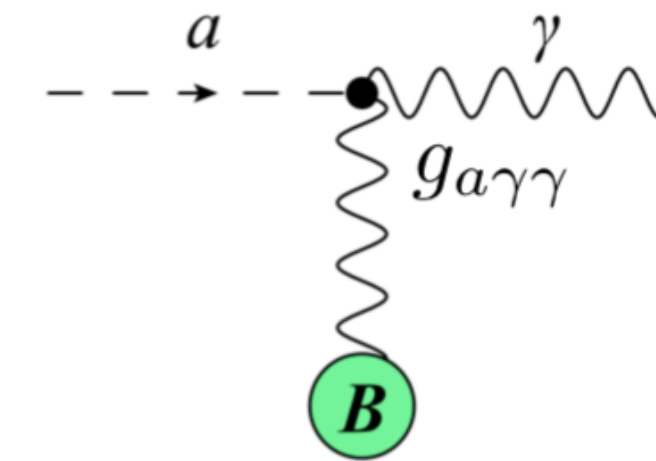
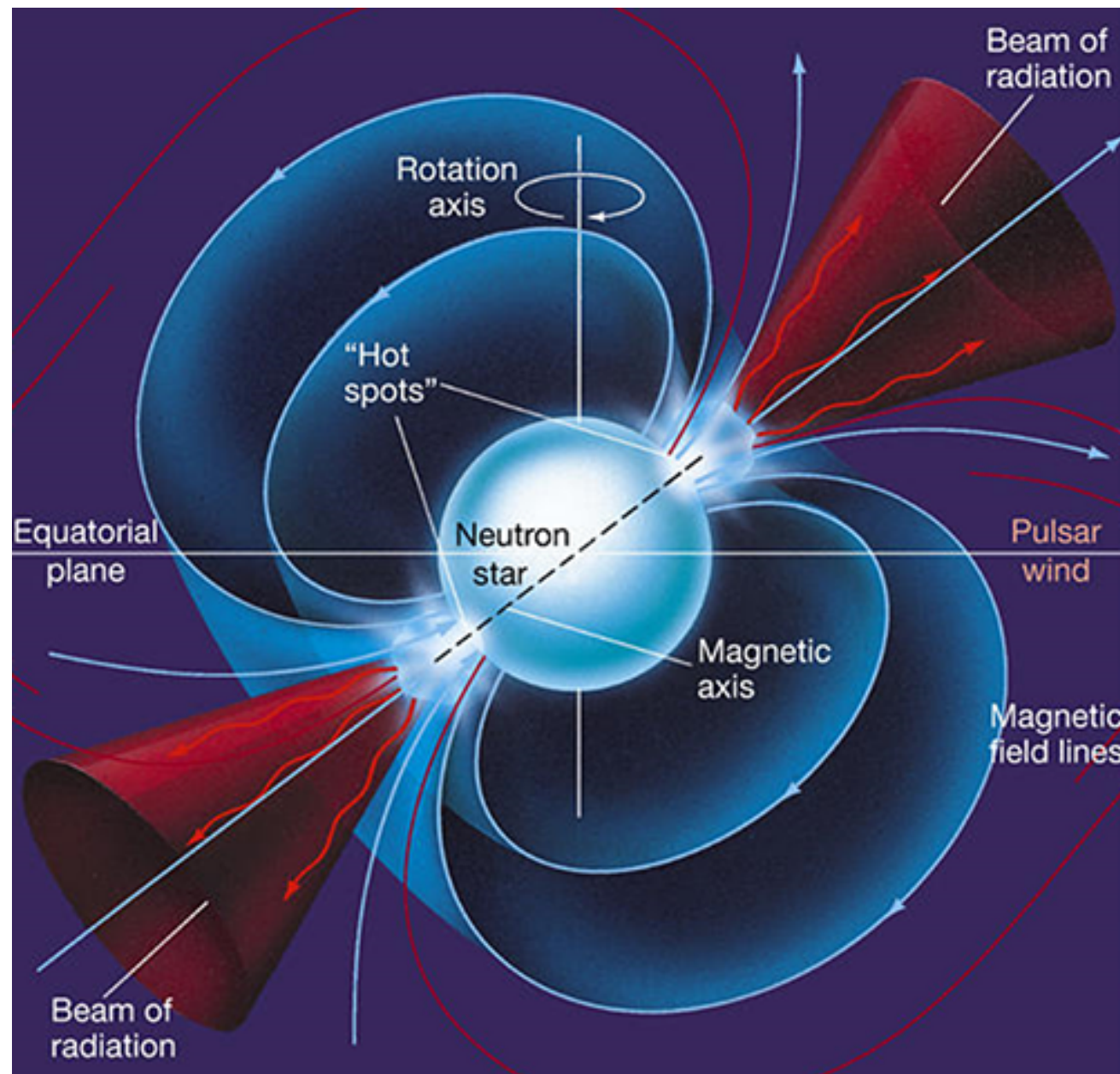
Axions can **convert to photons** (and vice versa) in an external magnetic field:

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ = & -\frac{1}{4} g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B} \end{aligned}$$



Dark Matter could be in the form of light **pseudo-scalar 'axions'**, which may arise as psuedo-Goldstone bosons of a spontaneously broken Peccei-Quinn symmetry $U(1)_{PQ}$.

© 2005 Pearson Prentice Hall, Inc



Neutron Star surrounded by a dense plasma which allows 'resonant' conversion, when **axion mass matches plasma mass**: $\omega_p(B_0, P) = m_a/2\pi$

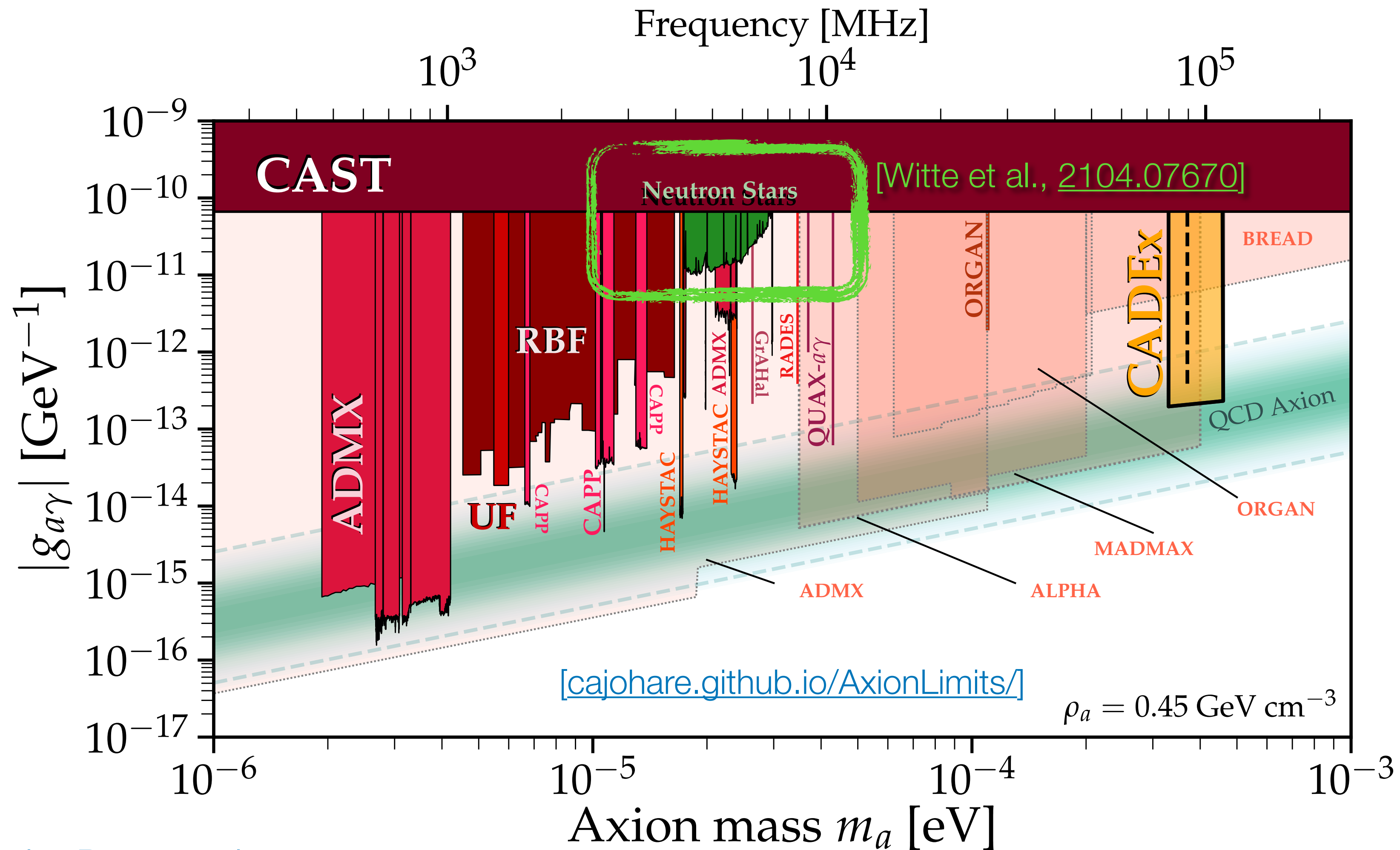
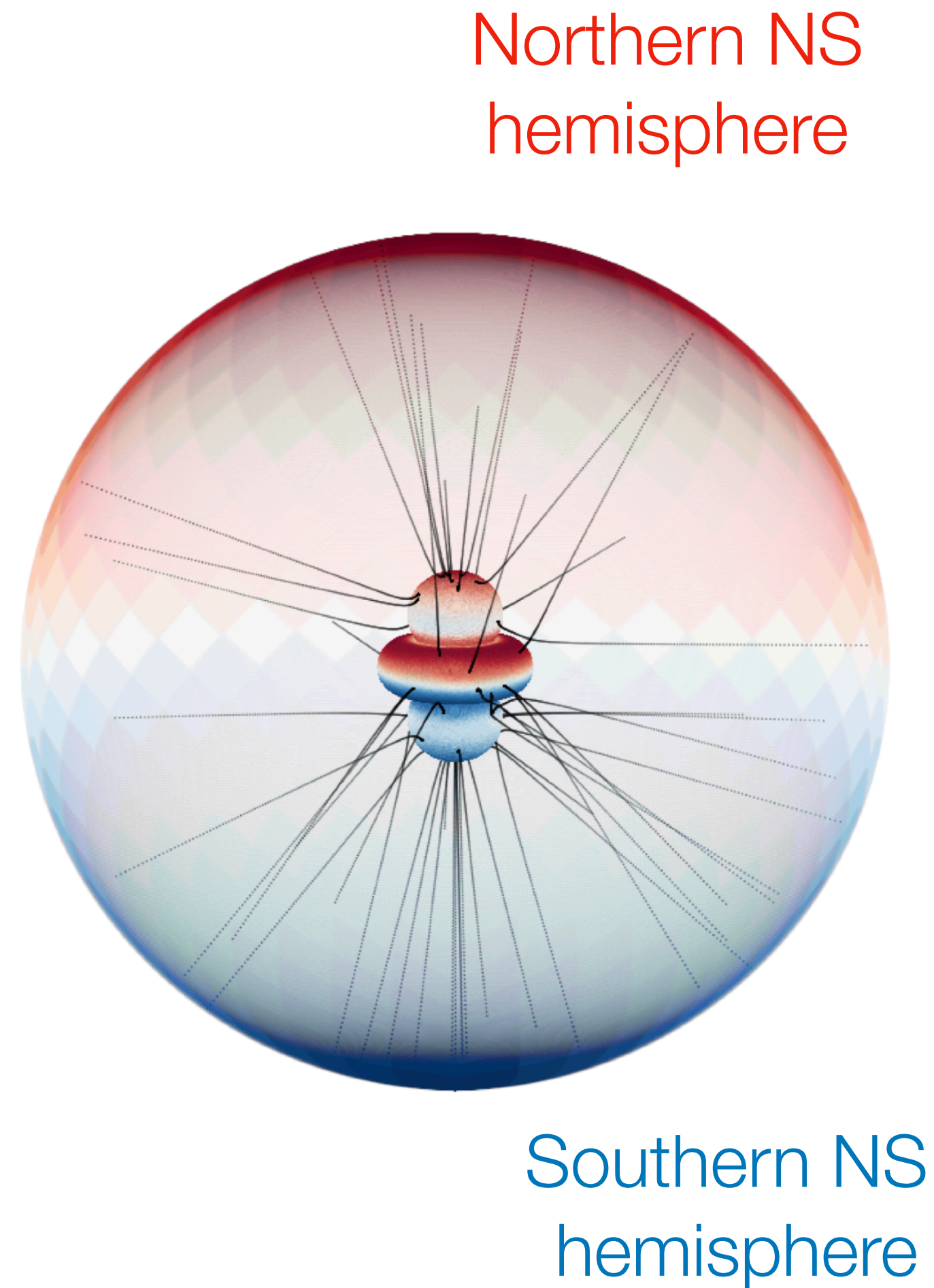
This occurs at the conversion radius r_c , with conversion possible over a distance:

$$L = \sqrt{2\pi r_c v_c / (3m_a)}$$

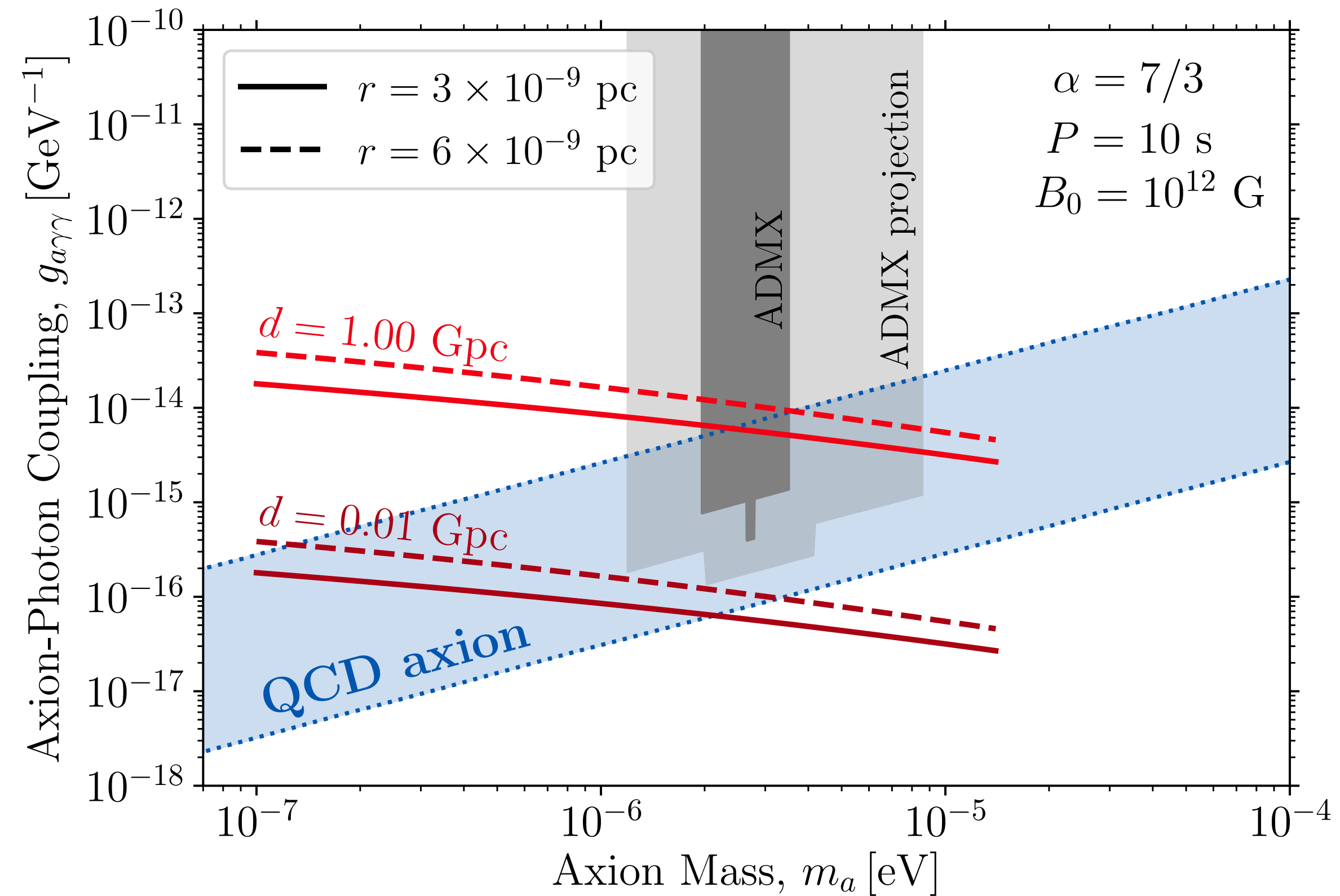
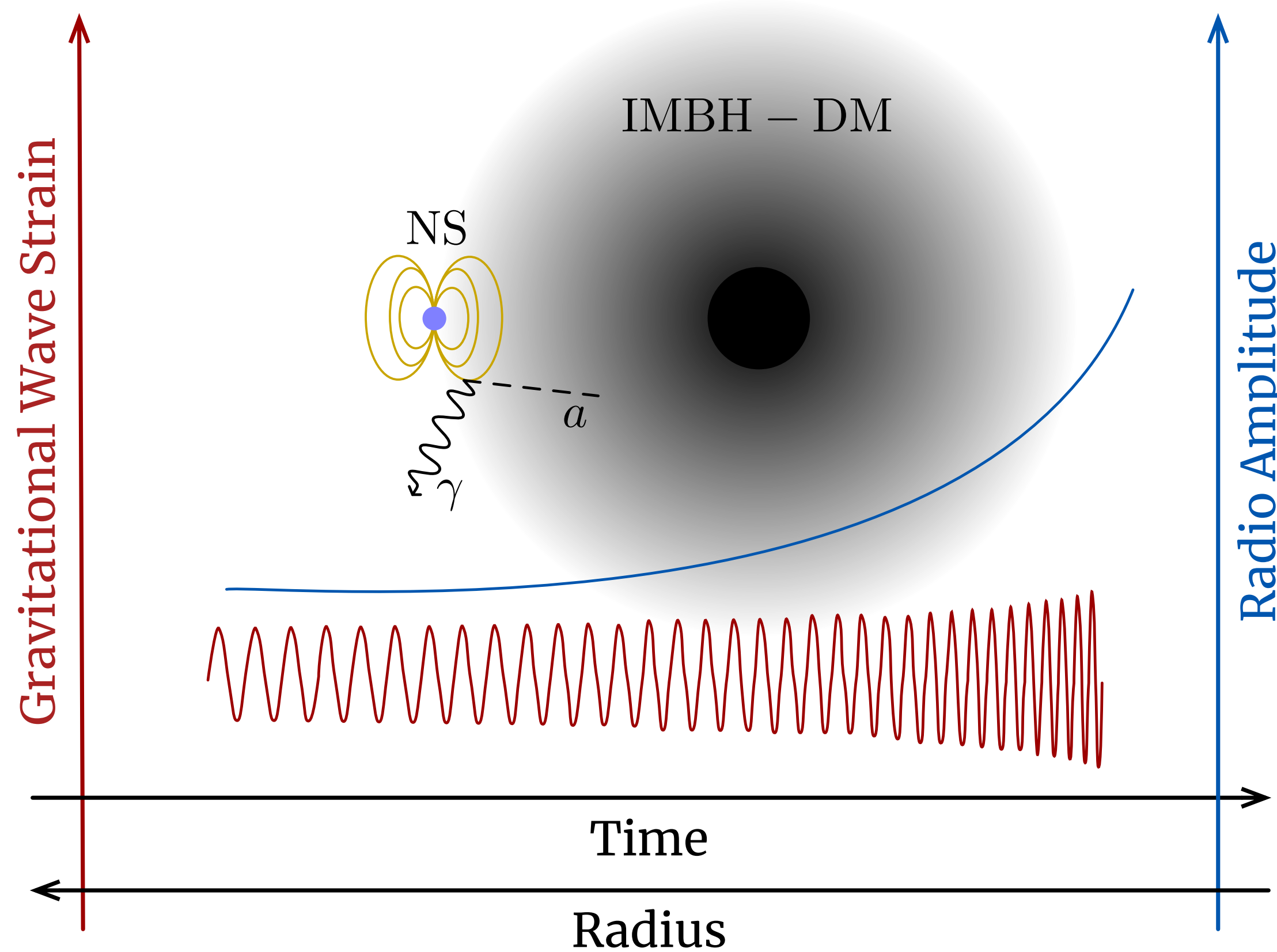
With this, the radiated power is:

$$\frac{d\mathcal{P}(\theta, \theta_m t)}{d\Omega} \approx p_{a\gamma} \rho_{DM}^{r_c} v_c r_c^2 \approx g_{a\gamma}^2 B(r_c)^2 L^2 \rho_{DM}^{r_c} r_c^2$$

Dark Matter could be in the form of light **pseudo-scalar 'axions'**, which may arise as psuedo-Goldstone bosons of a spontaneously broken Peccei-Quinn symmetry $U(1)_{PQ}$.



[For recent modeling developments, see also Battye et al., [1910.11907](#), [2104.08290](#); Leroy et al., [1912.08815](#), Foster et al., [2202.08274](#)]



Future radio observations should be able to probe QCD axion DM in the range $10^{-7} - 10^{-5}$ eV, while LISA would constrain the DM density close to the IMBH!

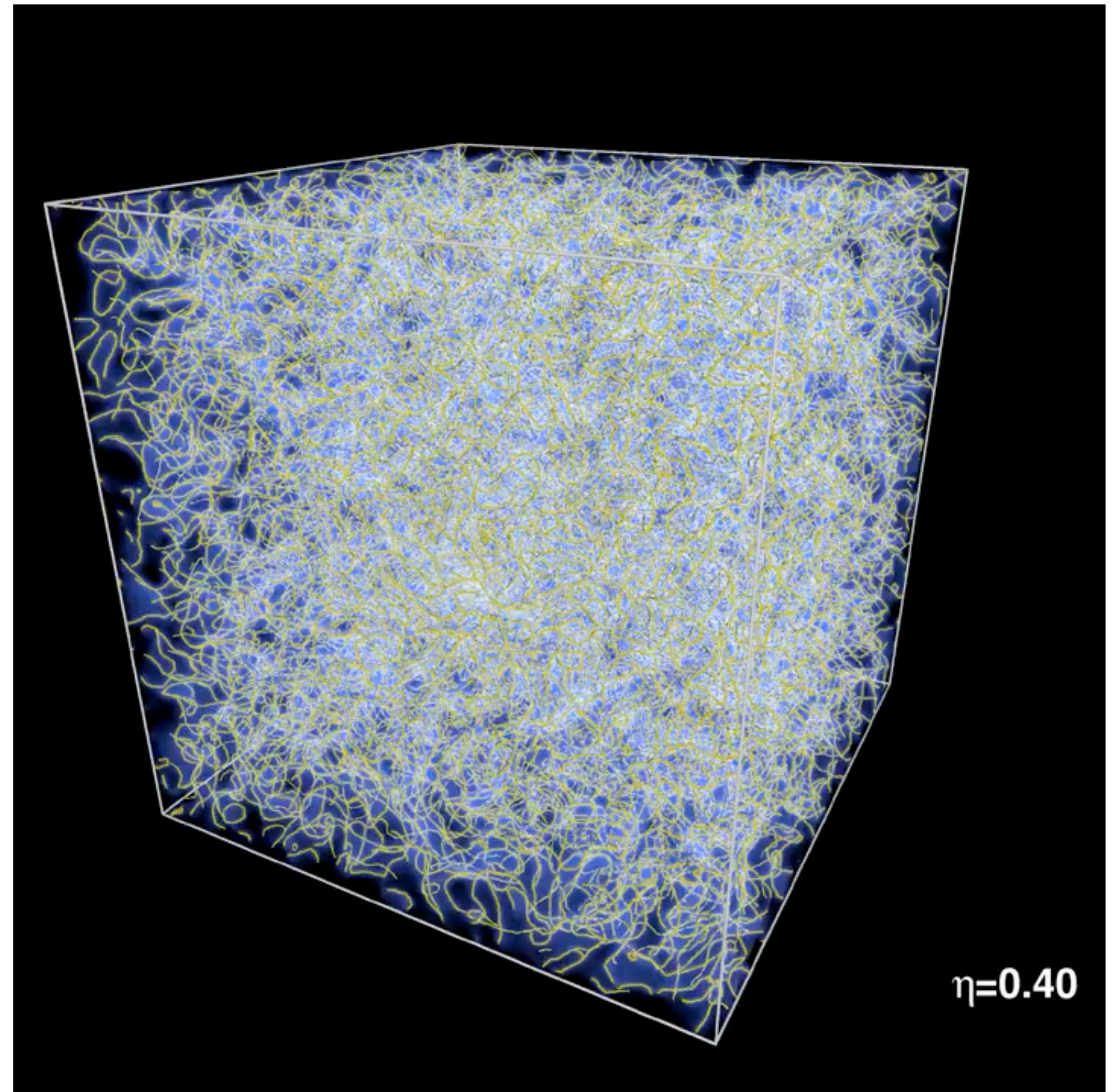
Consider Dark Matter QCD axions,
in which the PQ symmetry is broken
after inflation

Axion field has random initial values in
causally disconnected patches

Need to solve complicated dynamics
of topological defects (axion walls,
strings) in order to determine present-
day DM density.

Simulations are tricky, but point to
masses above $m_a = 20\mu\text{eV}$.

[See e.g. Gorghetto et al., [2007.04990](#)]



[Buschmann et al., [1906.00967](#)]

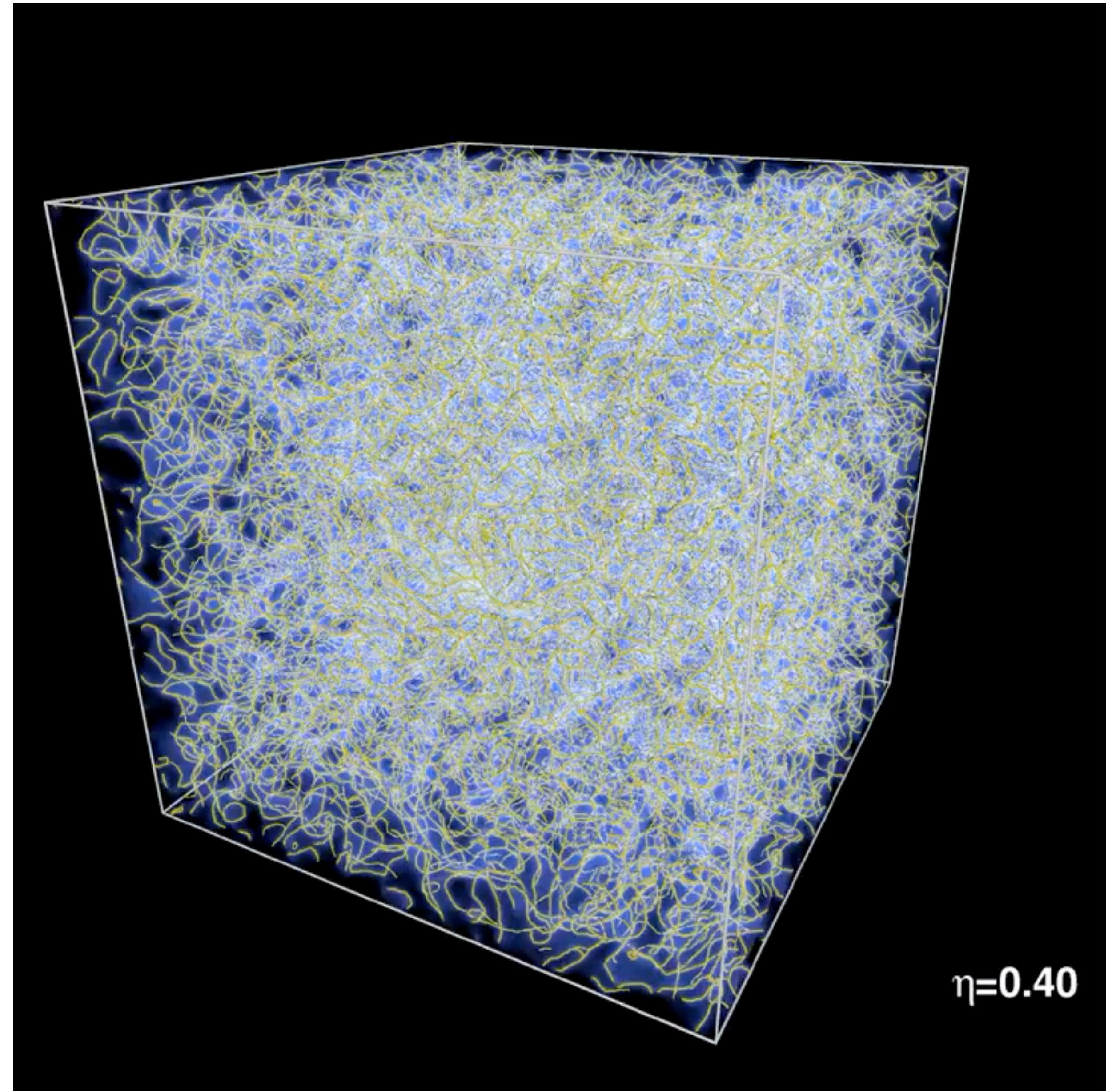
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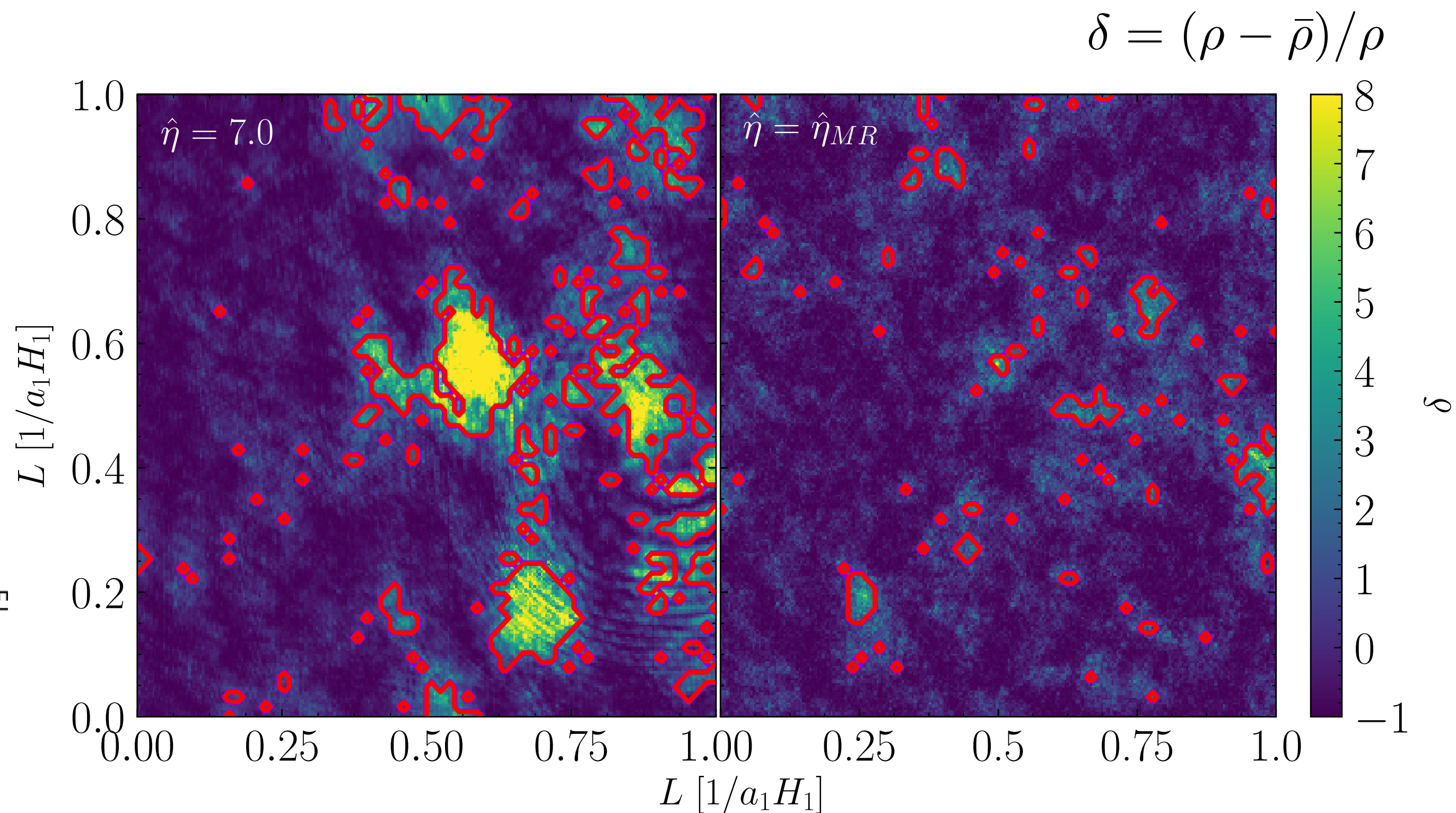
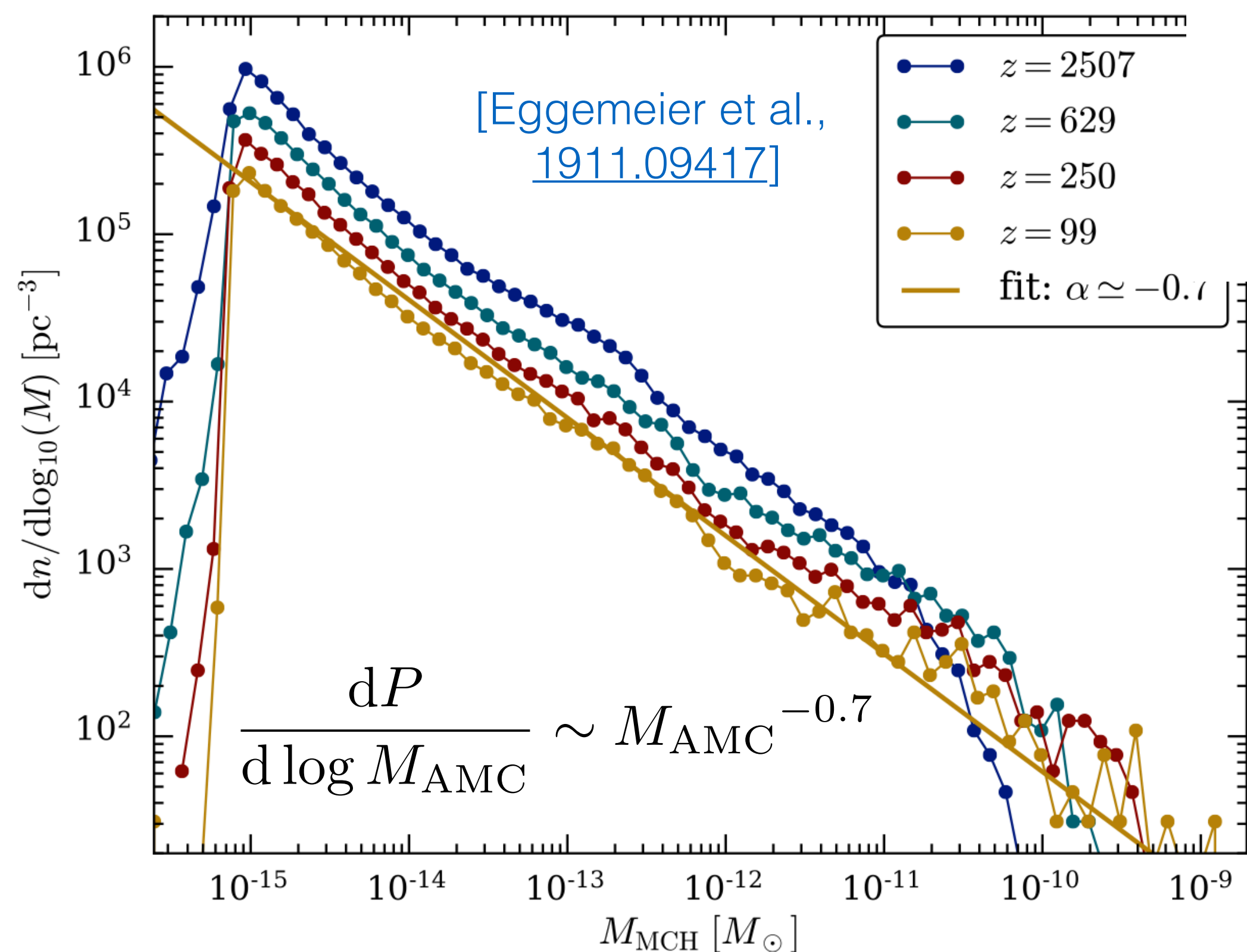
[See e.g. Gorghetto et al., [2007.04990](#)]



[Buschmann et al., [1906.00967](#)]

Overdensities act as 'seeds' for bound **axion miniclusters (AMCs)**

[Kolb & Tkachev, [astro-ph/9403011](#)]



Today, expect minicluster masses in the range
 $M_{\text{AMC}} \sim 10^{-19} - 10^{-5} M_{\odot}$.

But, AMC density profile is uncertain...

[Fairbairn et al., [1707.03310](#)]

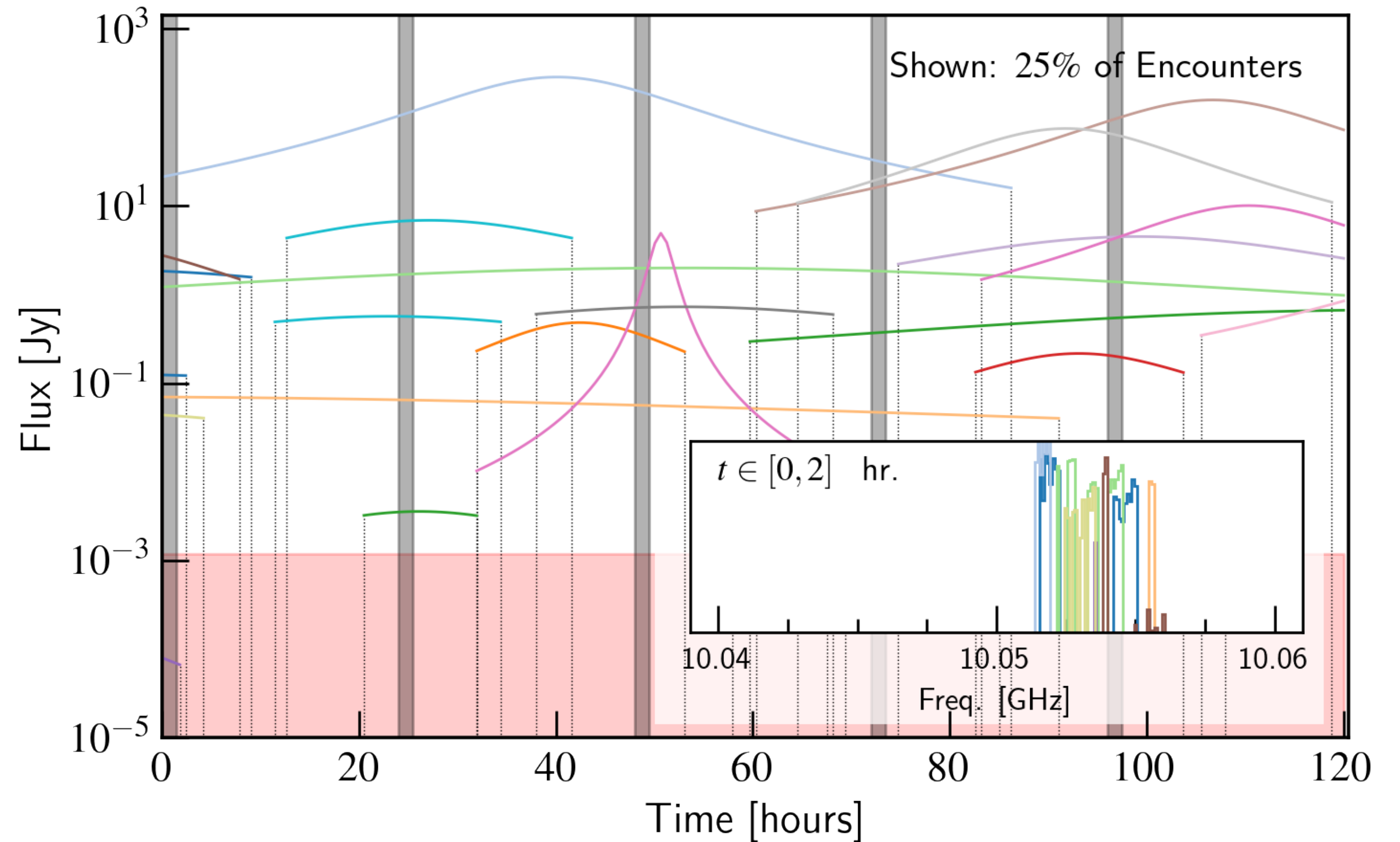
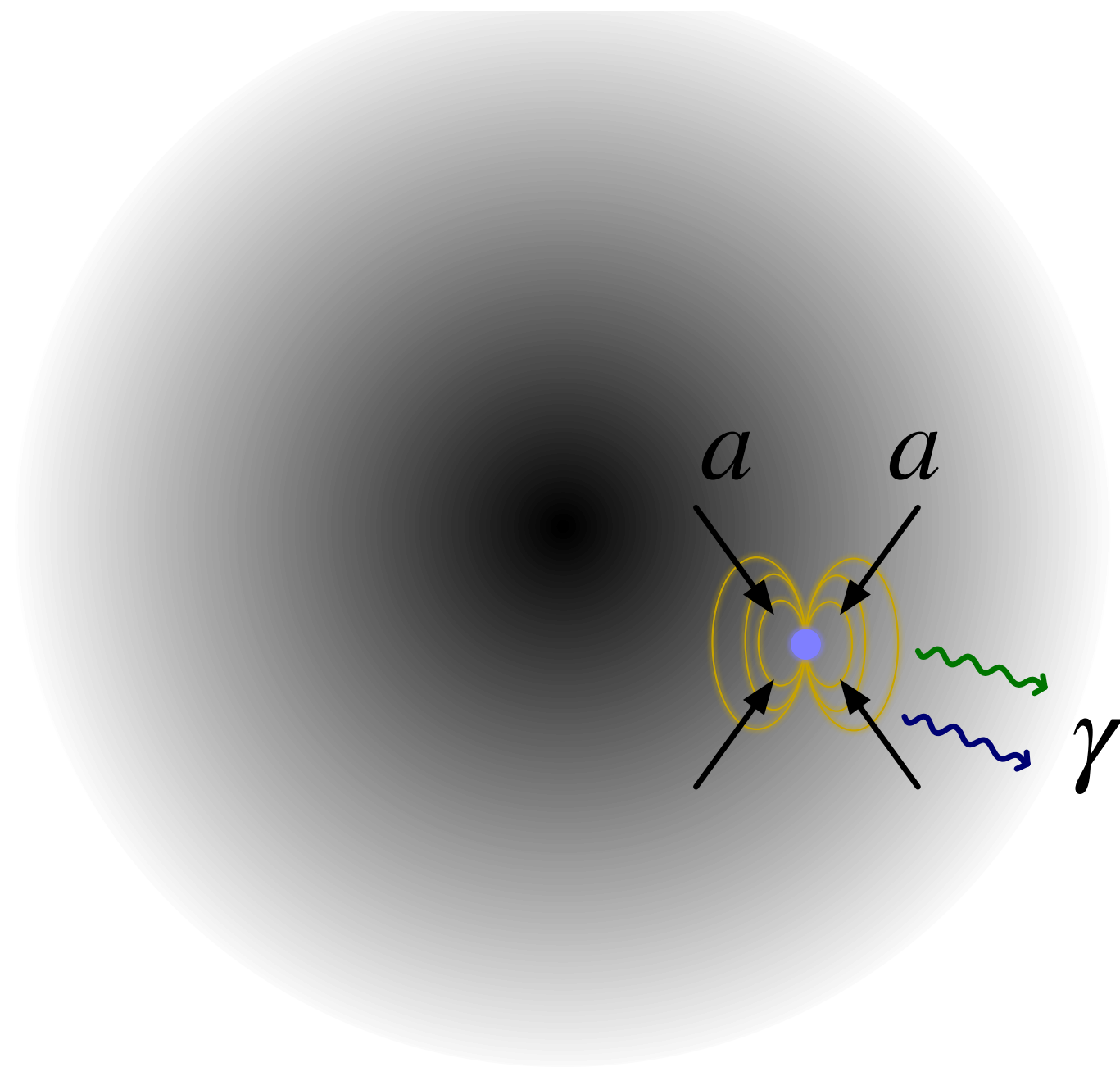
[See also Zurek et al., [astro-ph/0607341](#); Vaquero et al., [1809.09241](#)]

Clumps of axion DM ('**axion miniclusters**' or '**AMCs**') crossing NSs could lead to bright radio transients:

[Hogan & Rees (1988)]

$$M_{\text{AMC}} \sim 10^{-14} M_{\odot}$$

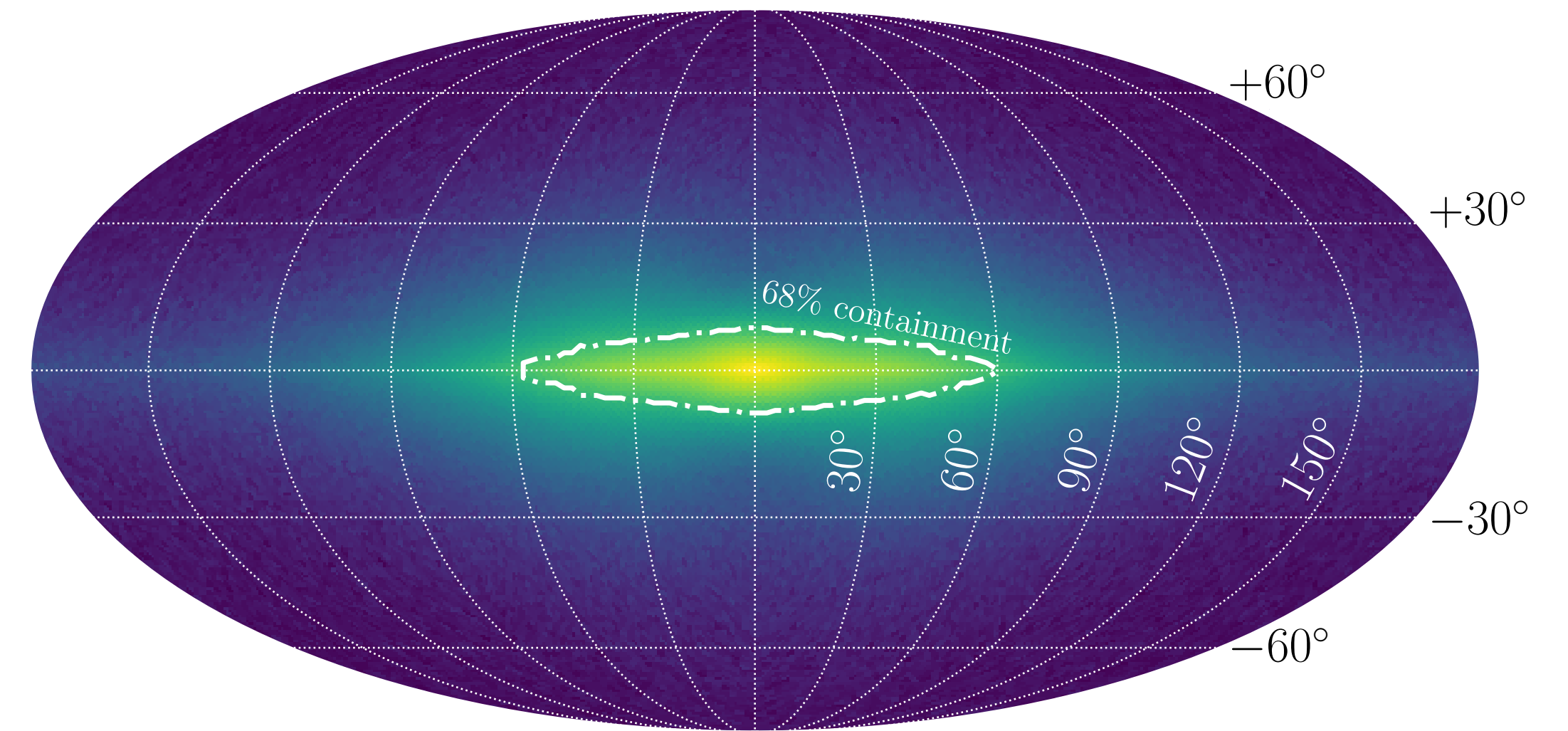
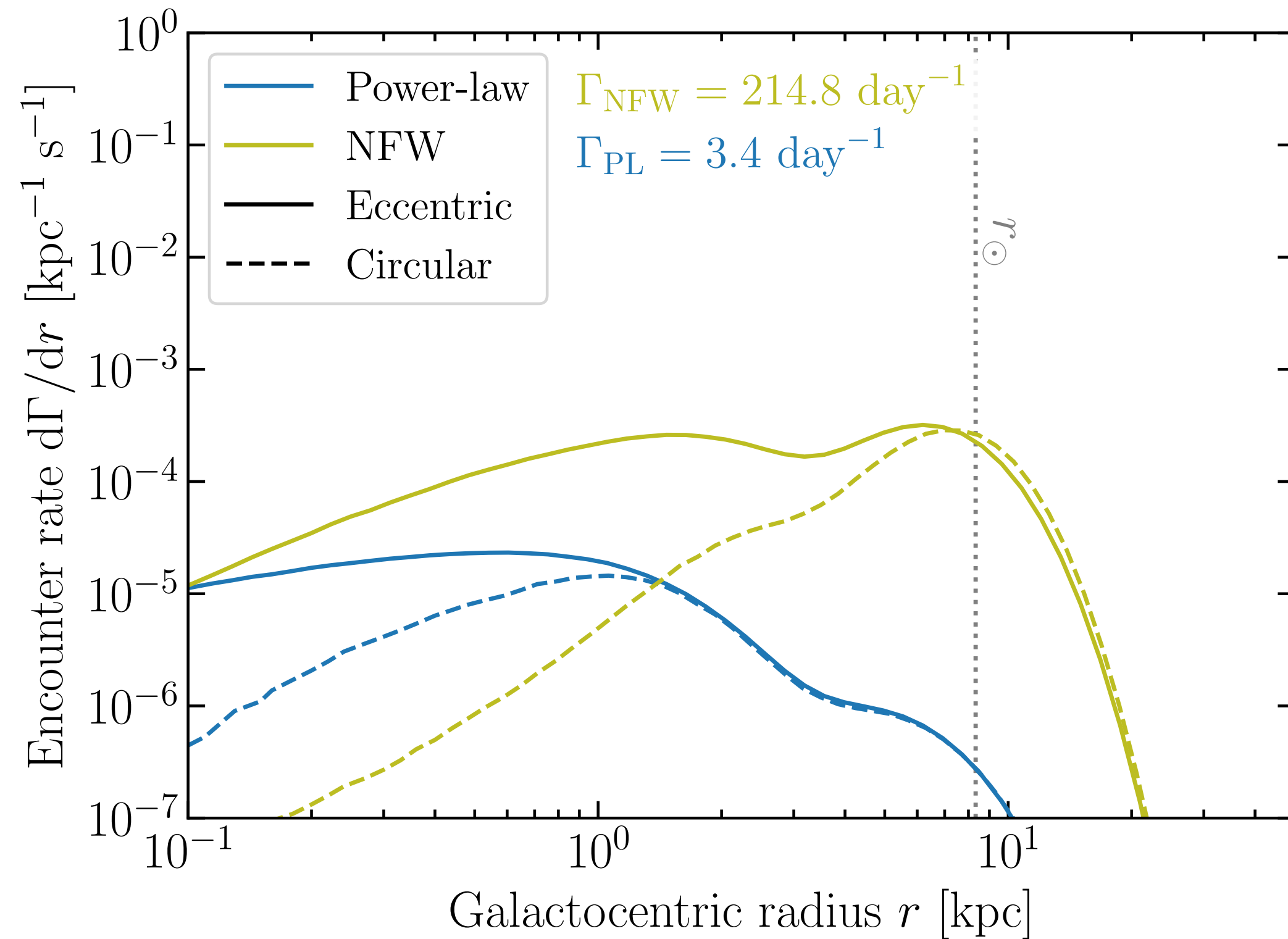
$$R_{\text{AMC}} \sim 10^{-7} \text{ pc} \sim 10^6 \text{ km}$$



[**BJK**, Edwards, Visinelli & Weniger, [2011.05377](#); Edwards, **BJK**, Visinelli & Visinelli, [2011.05378](#); Witte et al., [2212.08079](#)]

[Code: github.com/bradkav/axion-miniclusters]

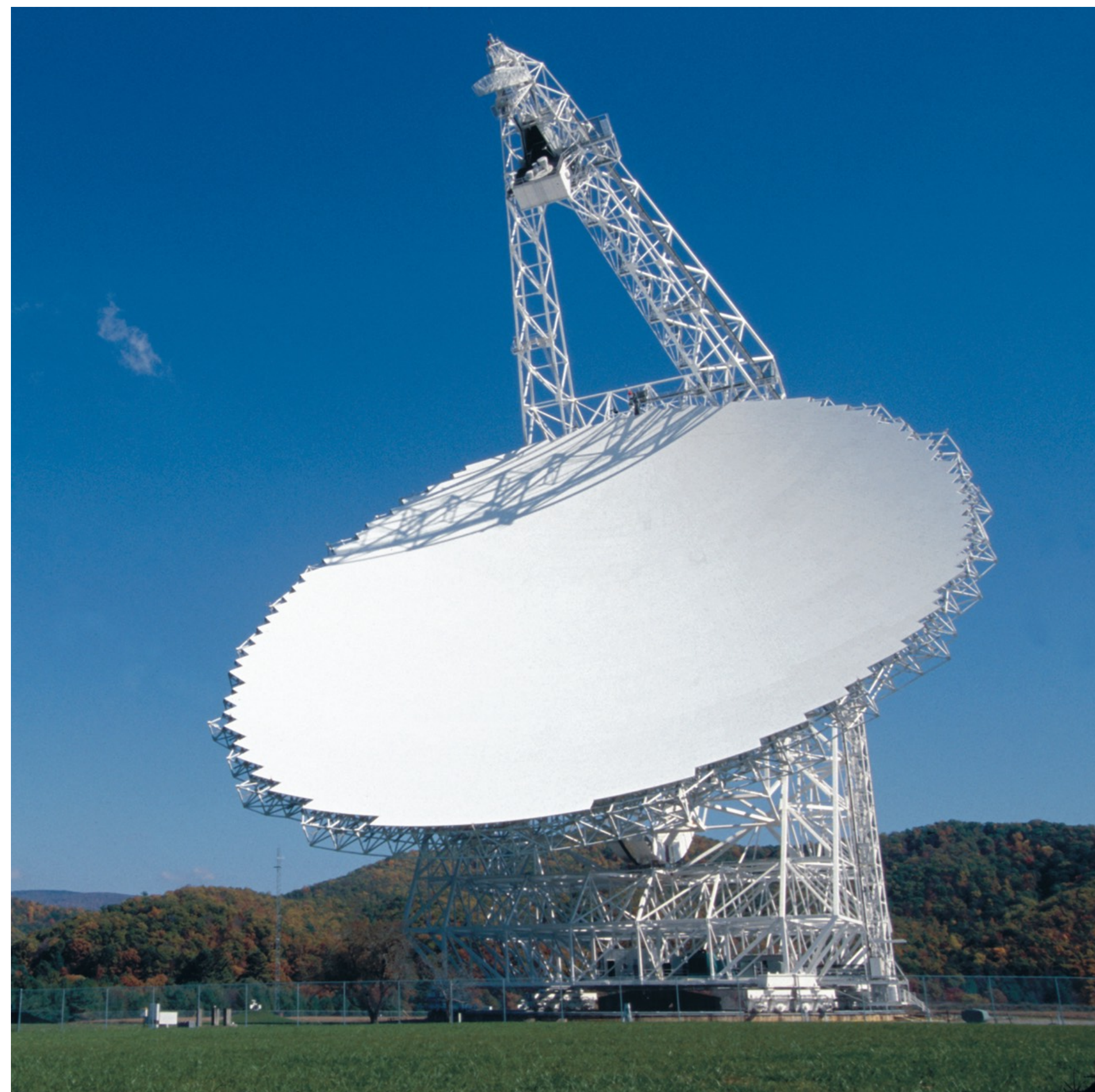
AMC-NS encounter rate obtained by convolving the surviving AMC distribution and the expected distribution of NSs. For the Milky Way:



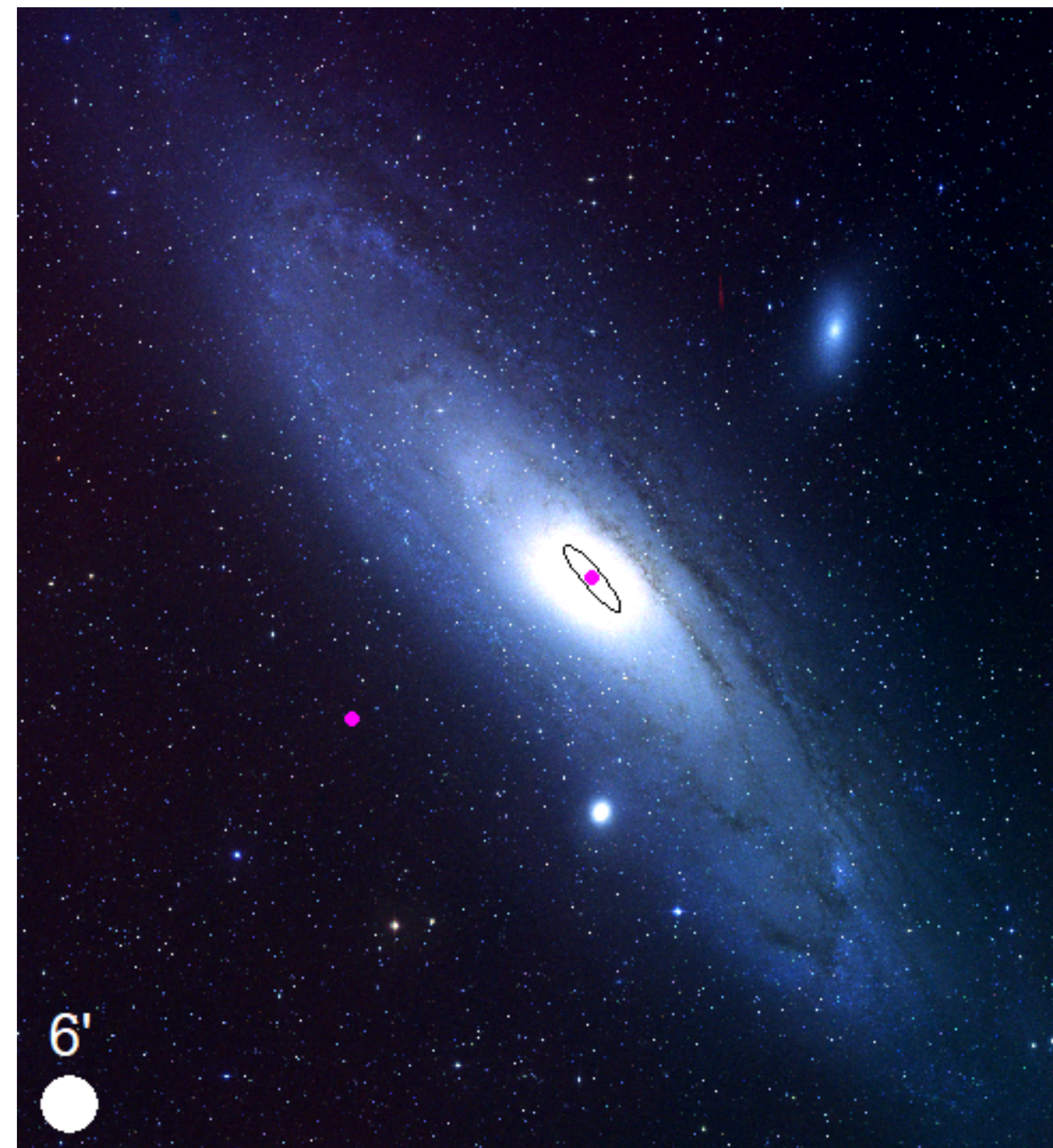
$$\Gamma = \int d^3\mathbf{r} \int dR \frac{dn_{\text{AMC}}(r)}{dR} n_{\text{NS}}(\mathbf{r}) \langle \widetilde{\sigma u} \rangle(r)$$

$$\langle \sigma u \rangle(r) \sim \sigma_u R^2$$

Difficult to catch all of the encounters in a single radio beam. Look at nearby galaxies...



Green Bank Radio Telescope

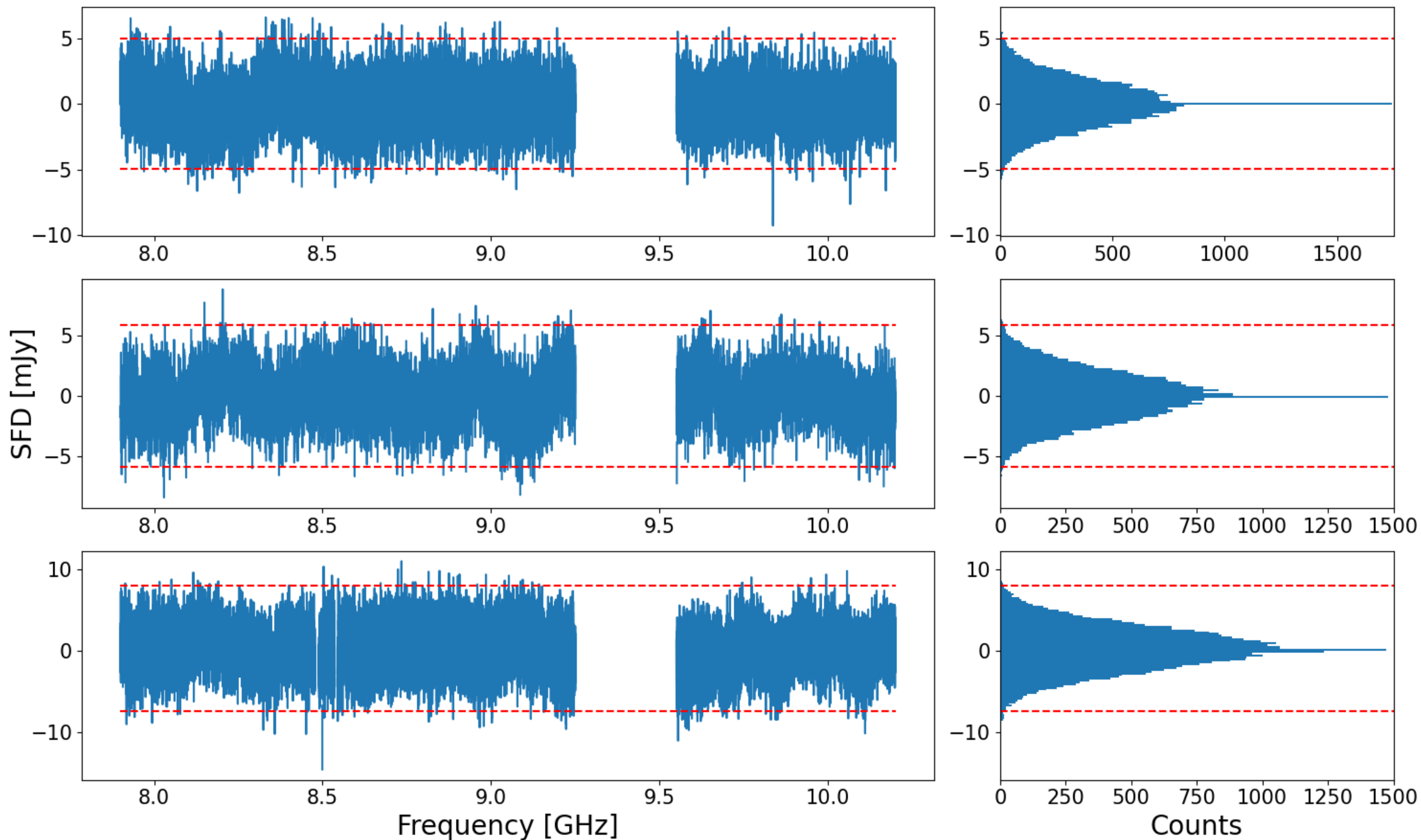


*in close collaboration with University of Virginia Astronomy Instrumentation Group.

GBT observations after reduction, calibration, RFI removal, and signal filtering

PRELIMINARY

Spectral flux density
(Energy per unit area per unit frequency)



$$m_a = 32 \mu\text{eV}$$

$$m_a = 48 \mu\text{eV}$$

Development of signal modeling pipeline

[Witte et al., [2212.08079](#)]

NS populations and magnetic field distributions

[Ongoing work lead by Sam Witte]

Searches for NSs

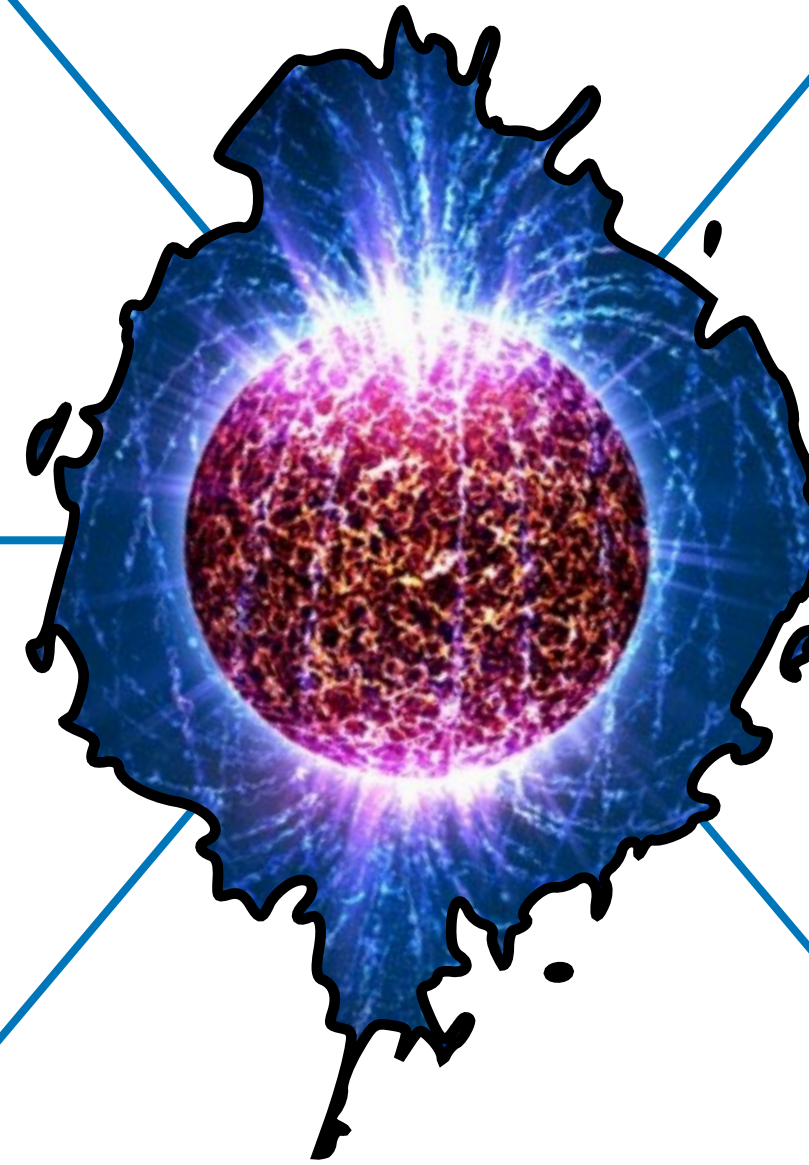
[See e.g. [2205.05048](#)]

AMC distribution and evolution

[See e.g. [2206.04619](#), [2207.11276](#)]

Better modelling of NS magnetospheres

Search strategies?



Dark Matter and Black Holes

Gianfranco Bertone
(GRAPPA, Amsterdam)



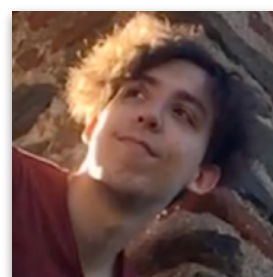
Pratibha Jangra
(IFCA, Santander)



Pippa Cole
(GRAPPA, Amsterdam)



Theophanes Karydas
(GRAPPA, Amsterdam)



Adam Coogan
(Mila, Montreal)



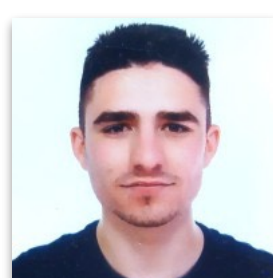
David Nichols
(U. Virginia)



Pratika Dayal
(Groningen University)



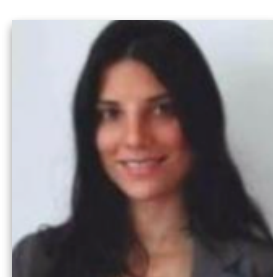
Abram Perez Herrero
(IFCA, Santander)



Jose Maria Diego
(IFCA, Santander)



Francesca Scarcella
(IFT, Madrid)



Daniele Gaggero
(INFN, Pisa)



Gimmy Tomaselli
(GRAPPA, Amsterdam)



Dark Matter and Neutron Stars

Prakamy Agrawal
(U. Virginia)



Scott Ransom
(NRAO)



Joe Bramante
(Queen's University)



Christoph Weniger
(GRAPPA, Amsterdam)



Tom Edwards
(Johns Hopkins)



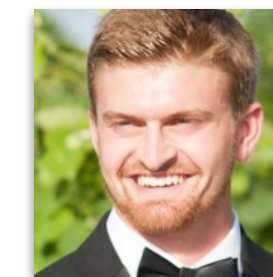
Sam Witte
(GRAPPA, Amsterdam)



Bradley Johnson
(U. Virginia)



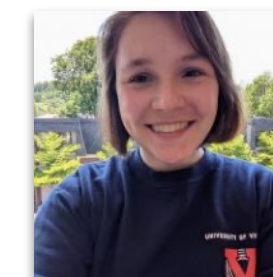
Liam Walters
(U. Virginia)



Doddy Marsh
(KCL, London)



Jordan Shroyer
(U. Virginia)



Nirmal Raj
(TRIUMF)

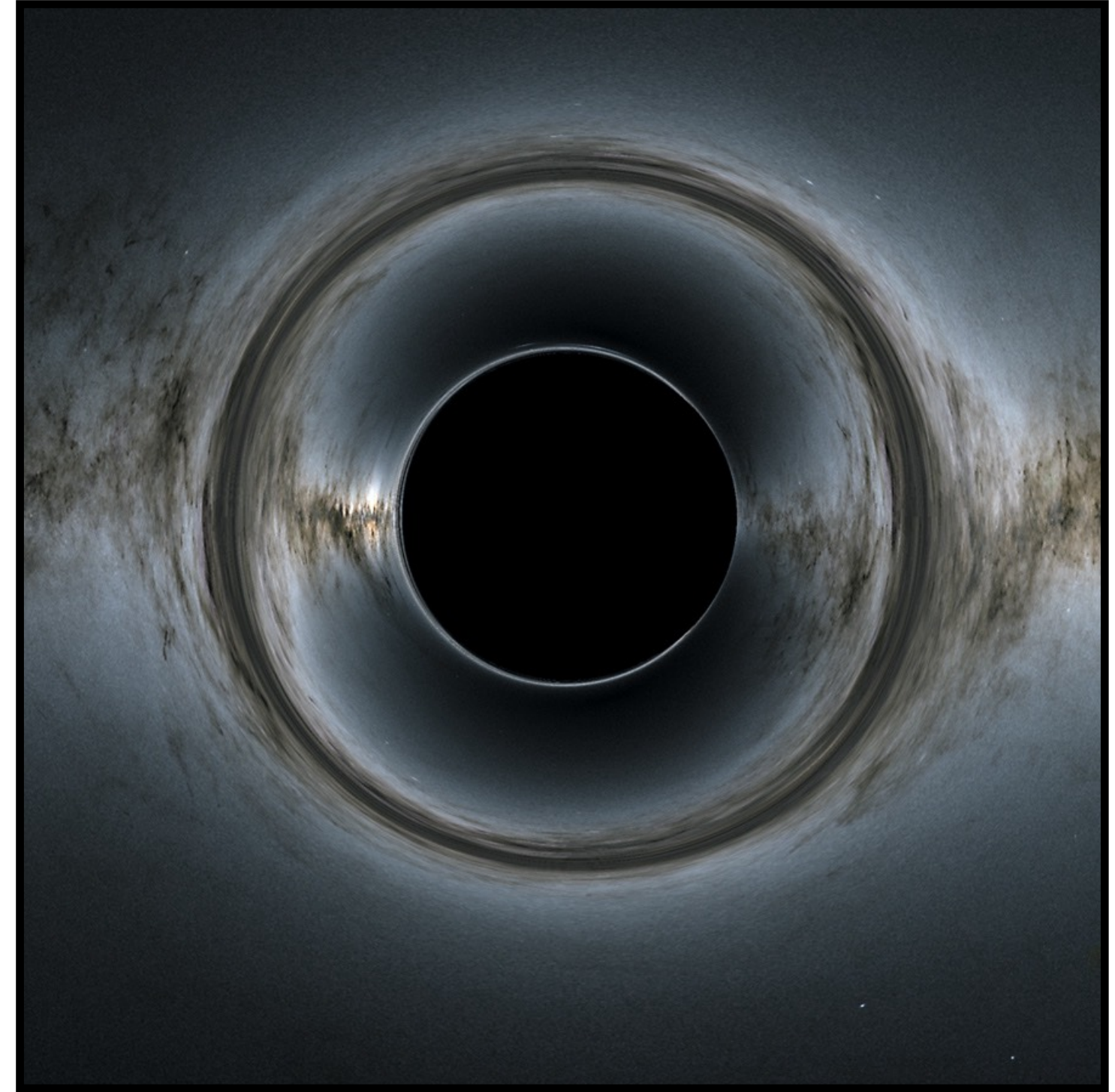


Luca Visinelli
(Shanghai Jiao Tong)



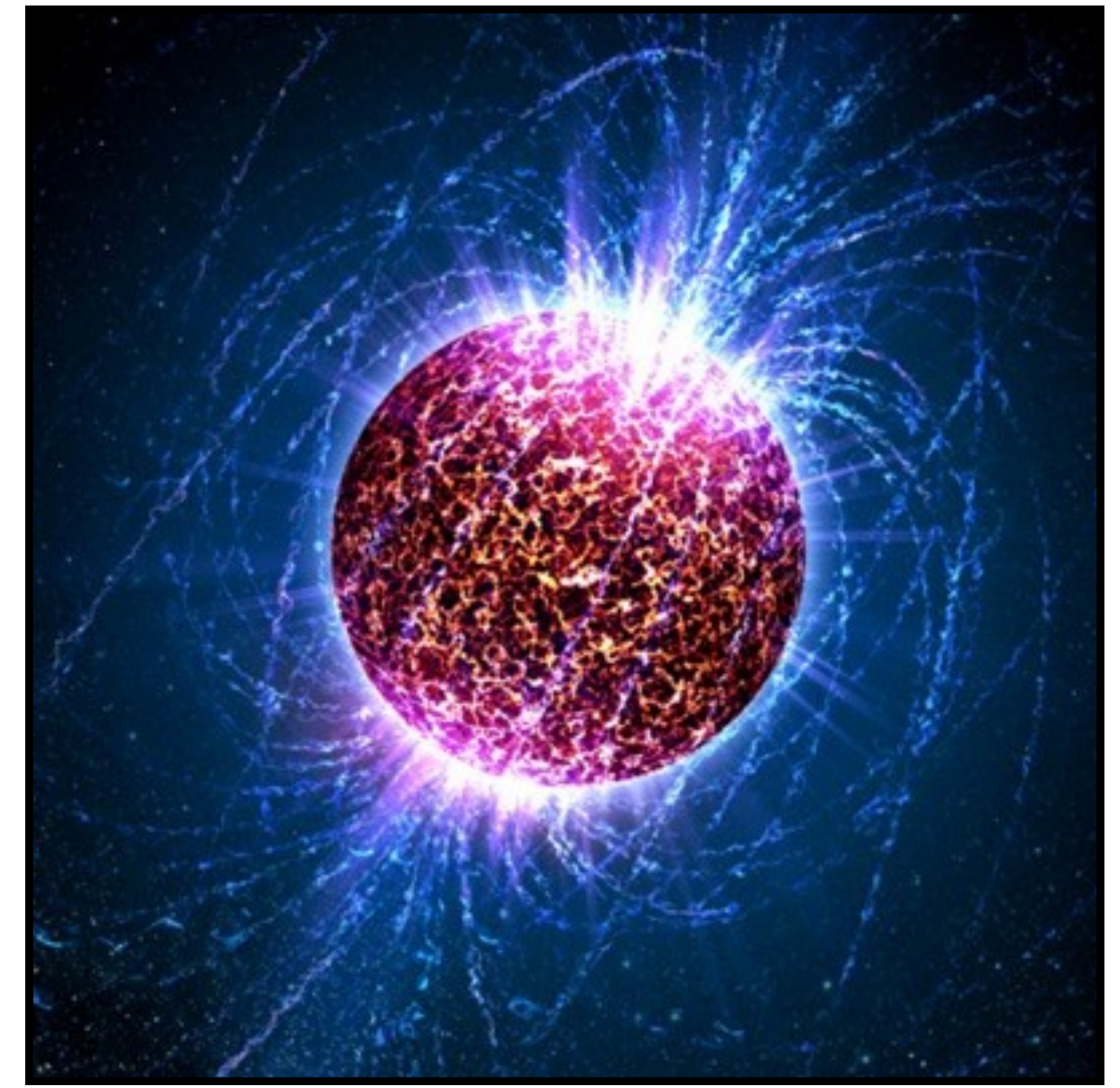
Higher densities, larger magnetic fields, longer timescales...but plenty still to do...

Black Holes



[Credit: NASA's Goddard Space Flight Center; background, ESA/Gaia/DPAC]

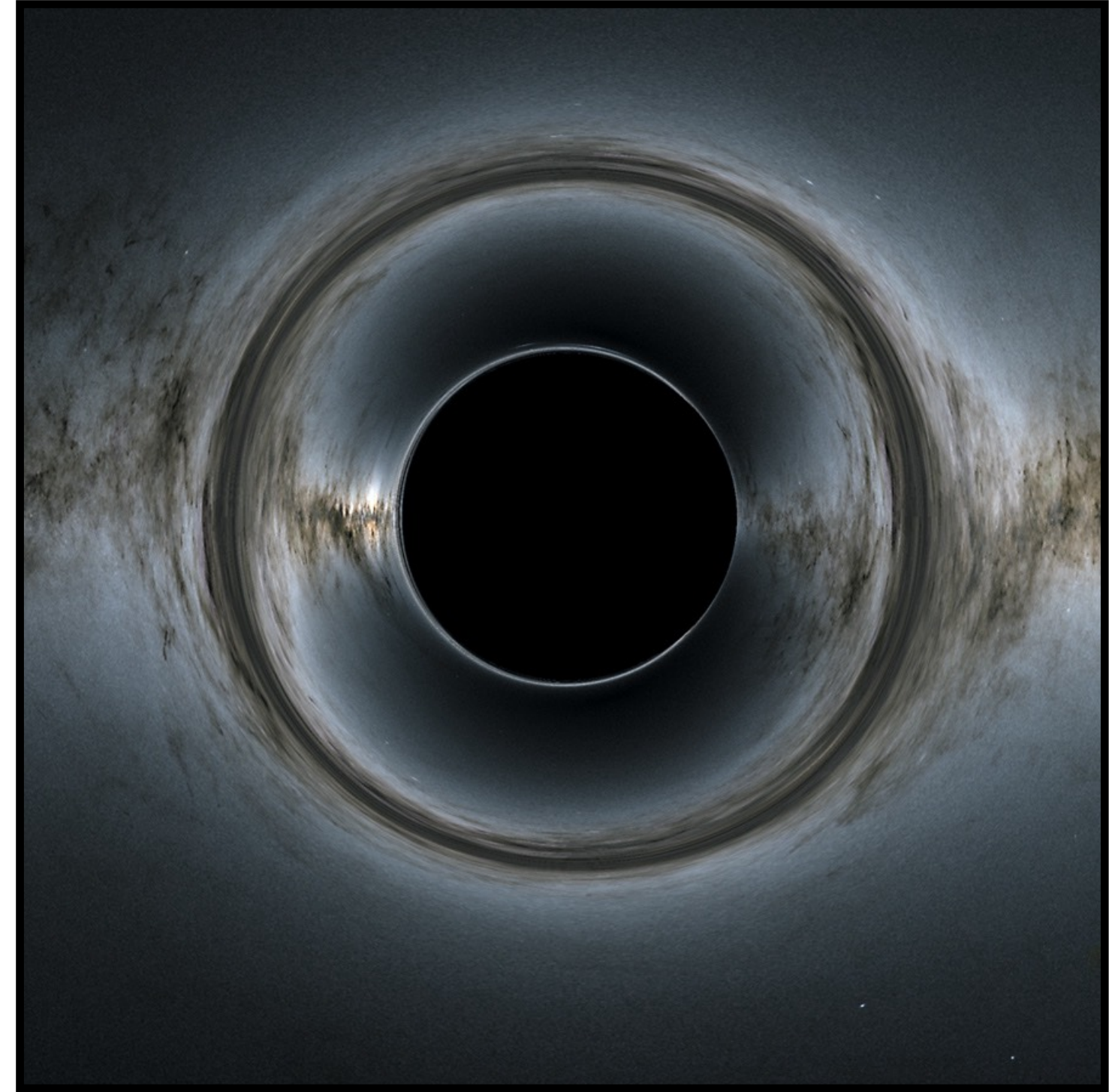
Neutron Stars



[Credit: Casey Reed (Penn State University), Wikimedia Commons]

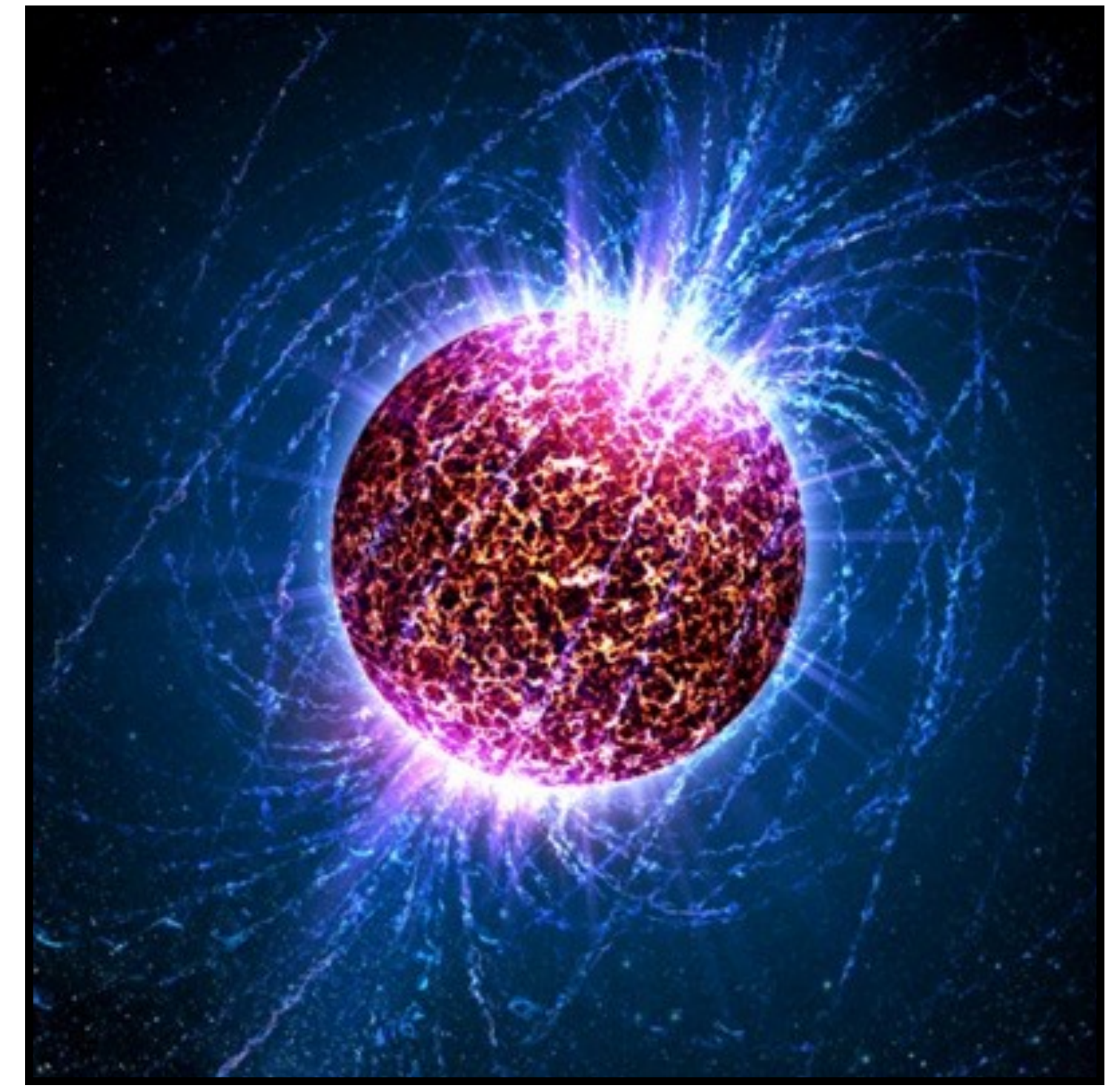
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Neutron Stars



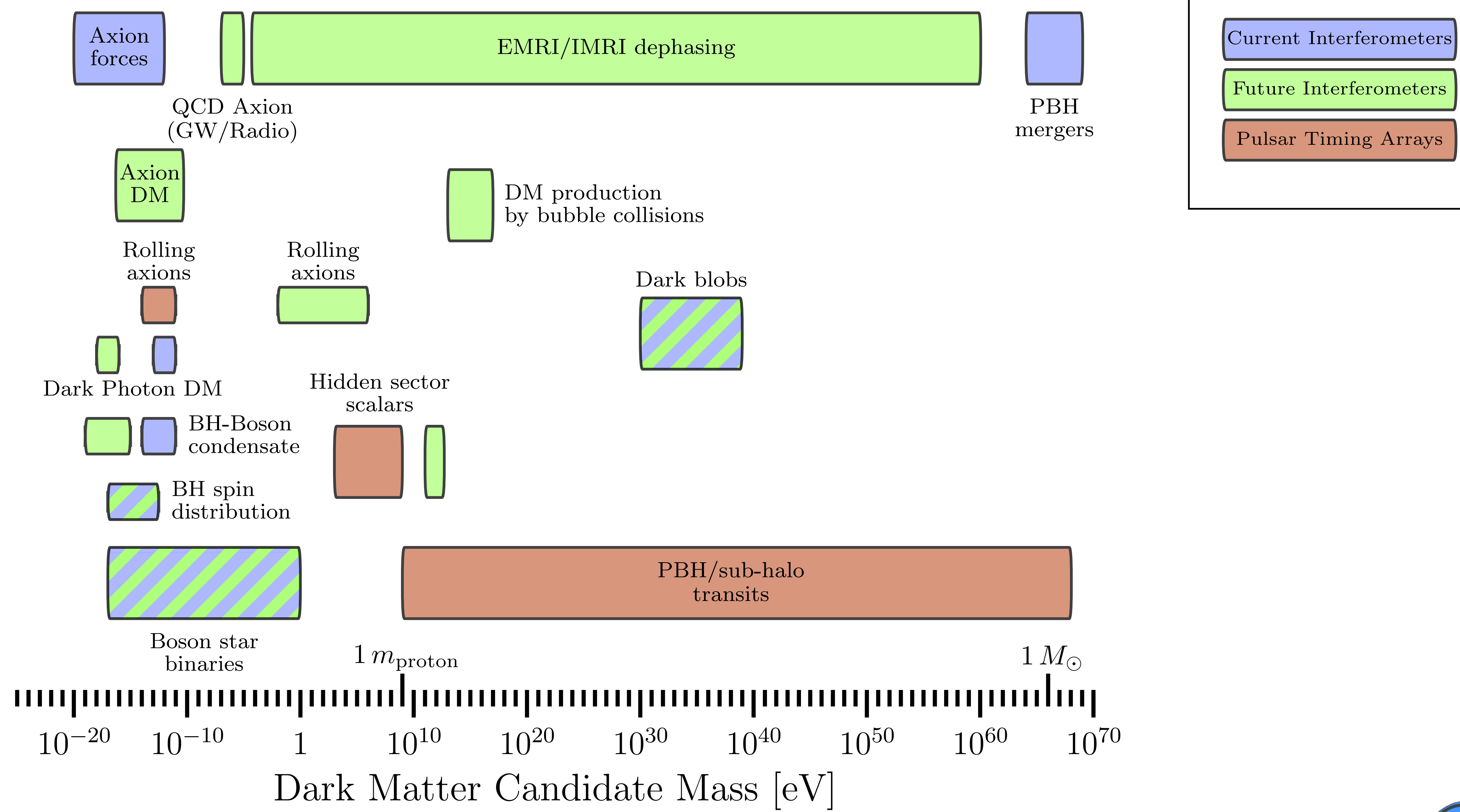
[Credit: Casey Reed (Penn State University), Wikimedia Commons]

Thank you!

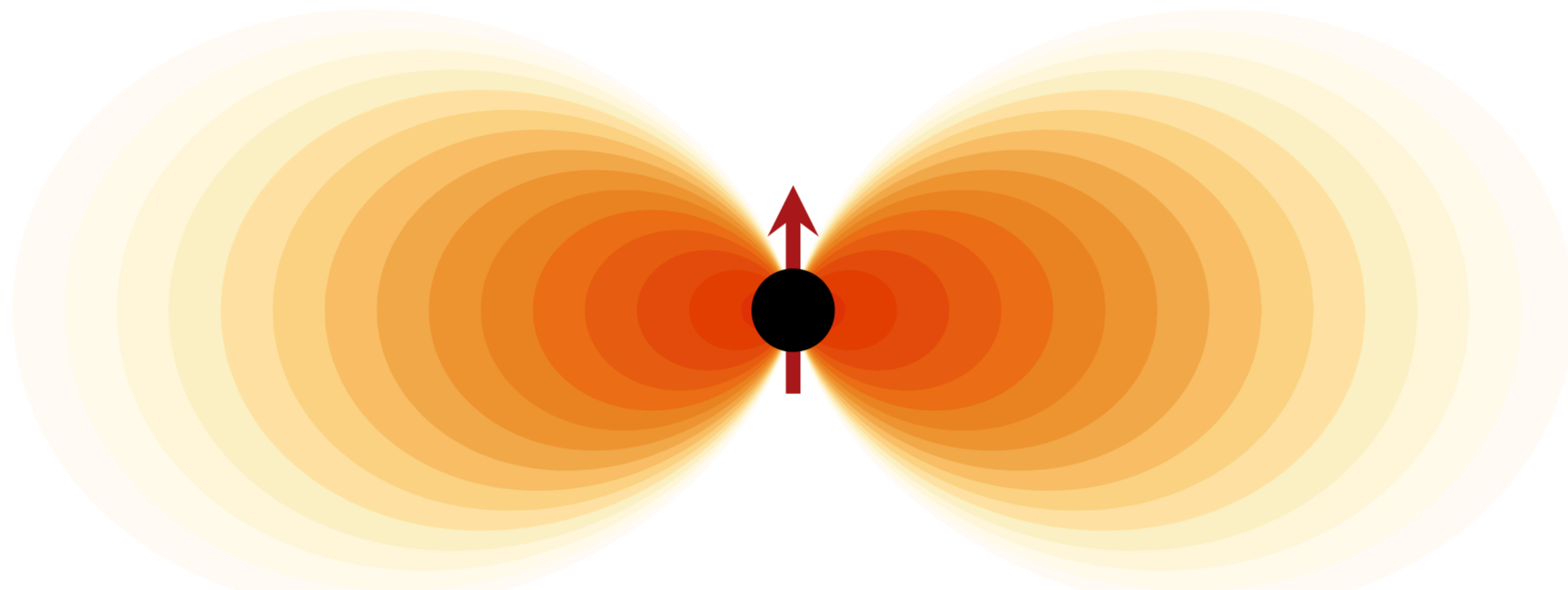
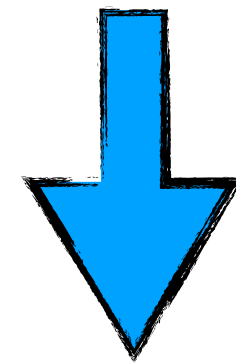
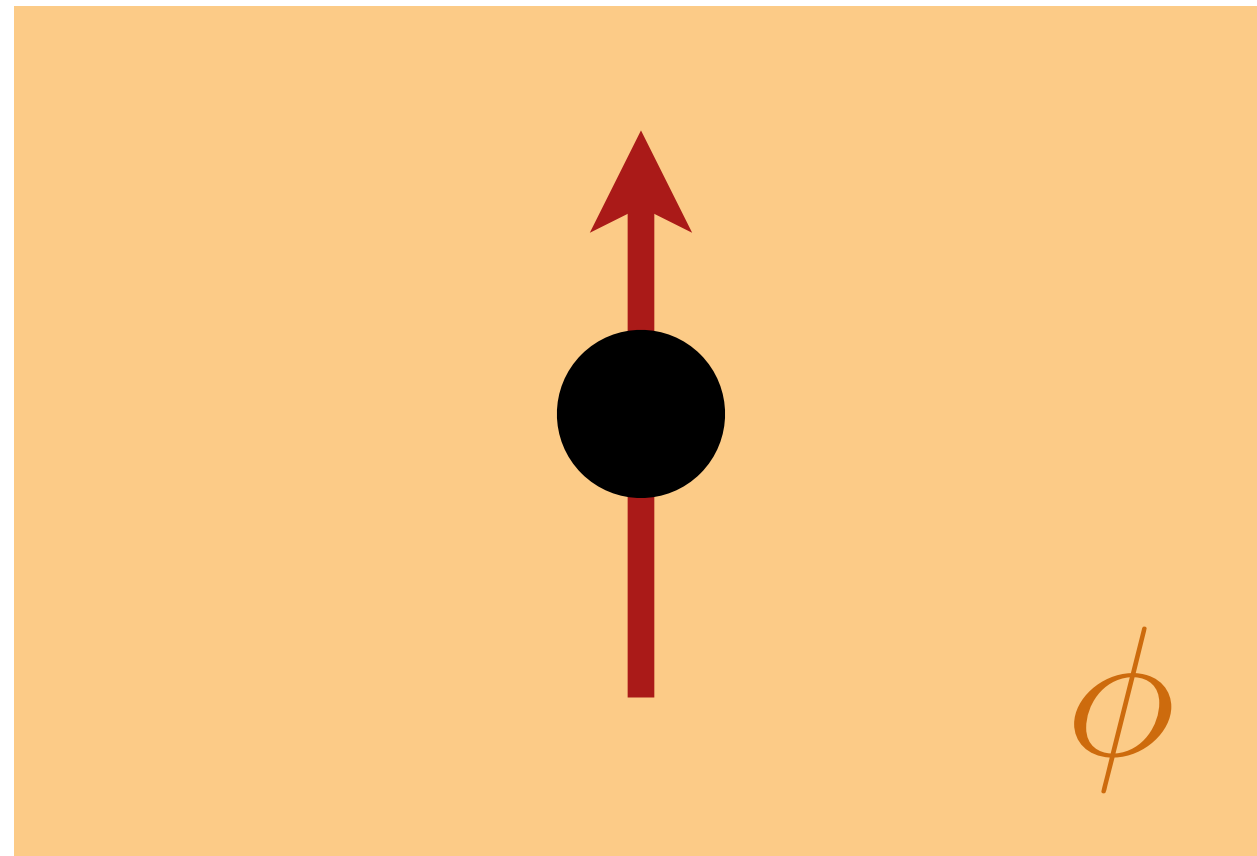


Backup Slides





[Bertone, Croon, et al (including **BJK**), [1907.10610](https://arxiv.org/abs/1907.10610)]



Compton wavelength of a light scalar field:

$$\lambda_c \simeq 2 \text{ km} \left(\frac{10^{-10} \text{ eV}}{\mu} \right)$$

Super-radiance (and growth of a ‘**gravitational atom**’) when:

$$r_g \sim GM_{\text{BH}}/c^2 < \lambda_c$$

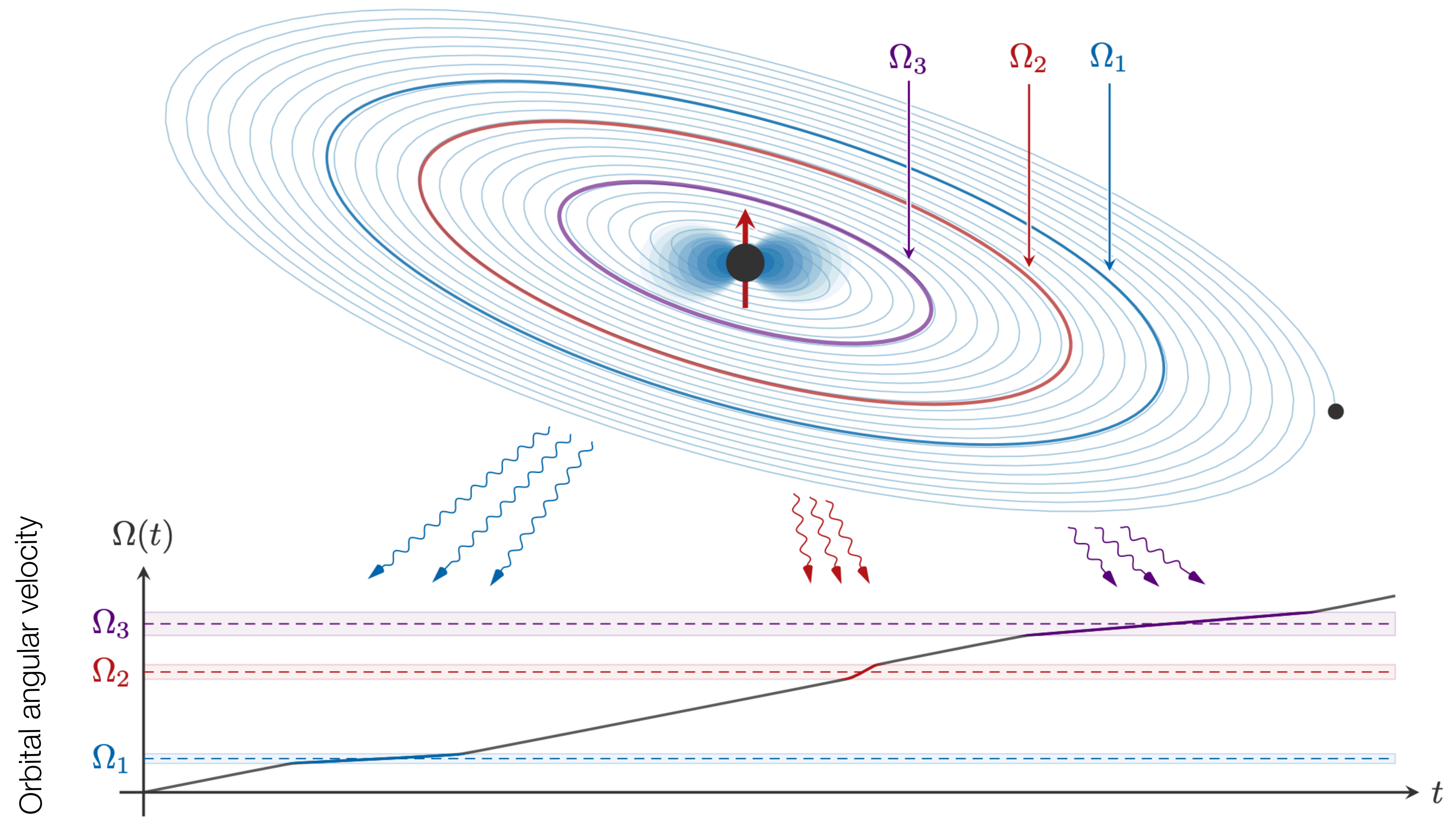
$$M_{\text{BH}} \in [1, 10^{10}] M_{\odot}$$

$$\rightarrow m_{\phi} \in [10^{-20}, 10^{-10}] \text{ eV}$$

[Chia, [2012.09167](#)]

[E.g. Baumann et al., [1804.03208](#), [1908.10370](#), [1912.04932](#), [2112.14777](#)]

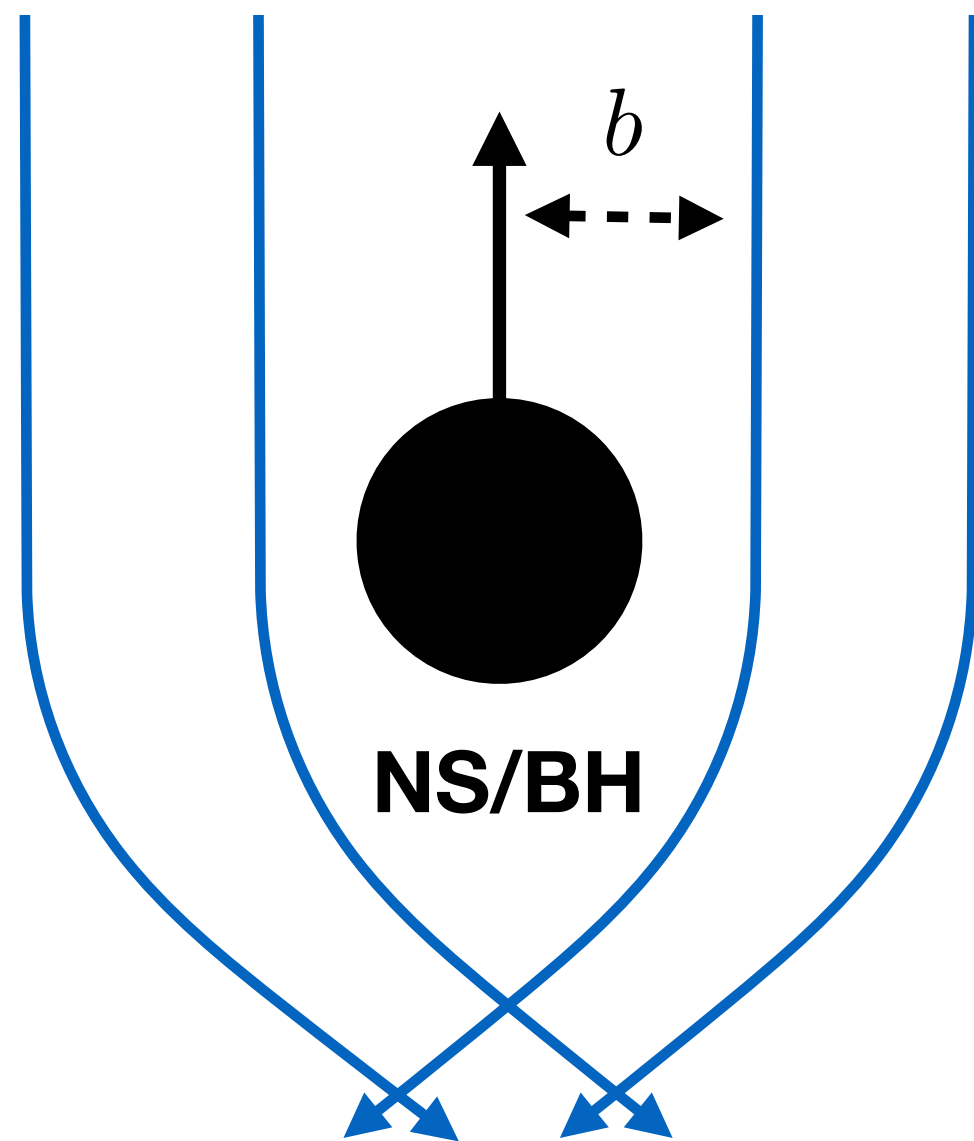
Gravitational Atoms



[Baumann et al., [1804.03208](#), [1908.10370](#), [1912.04932](#), [2012.09167](#), [2112.14777](#)]

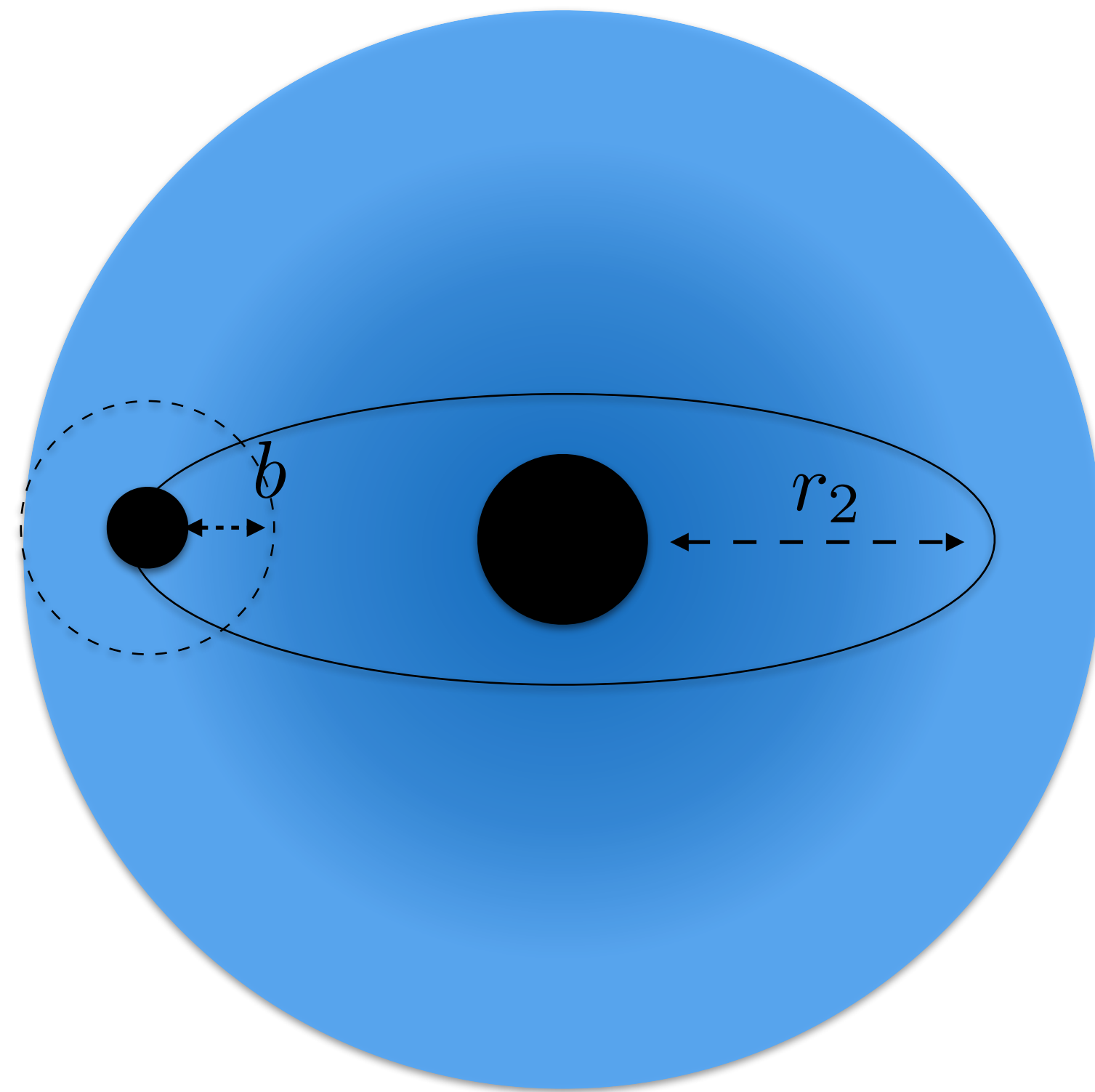
Two body scattering problem relates energy exchange to impact parameter:

$$\Delta\mathcal{E}(b) = -2v_0^2 \left[1 + \frac{b^2 v_0^4}{G^2 m_2^2} \right]^{-1}$$

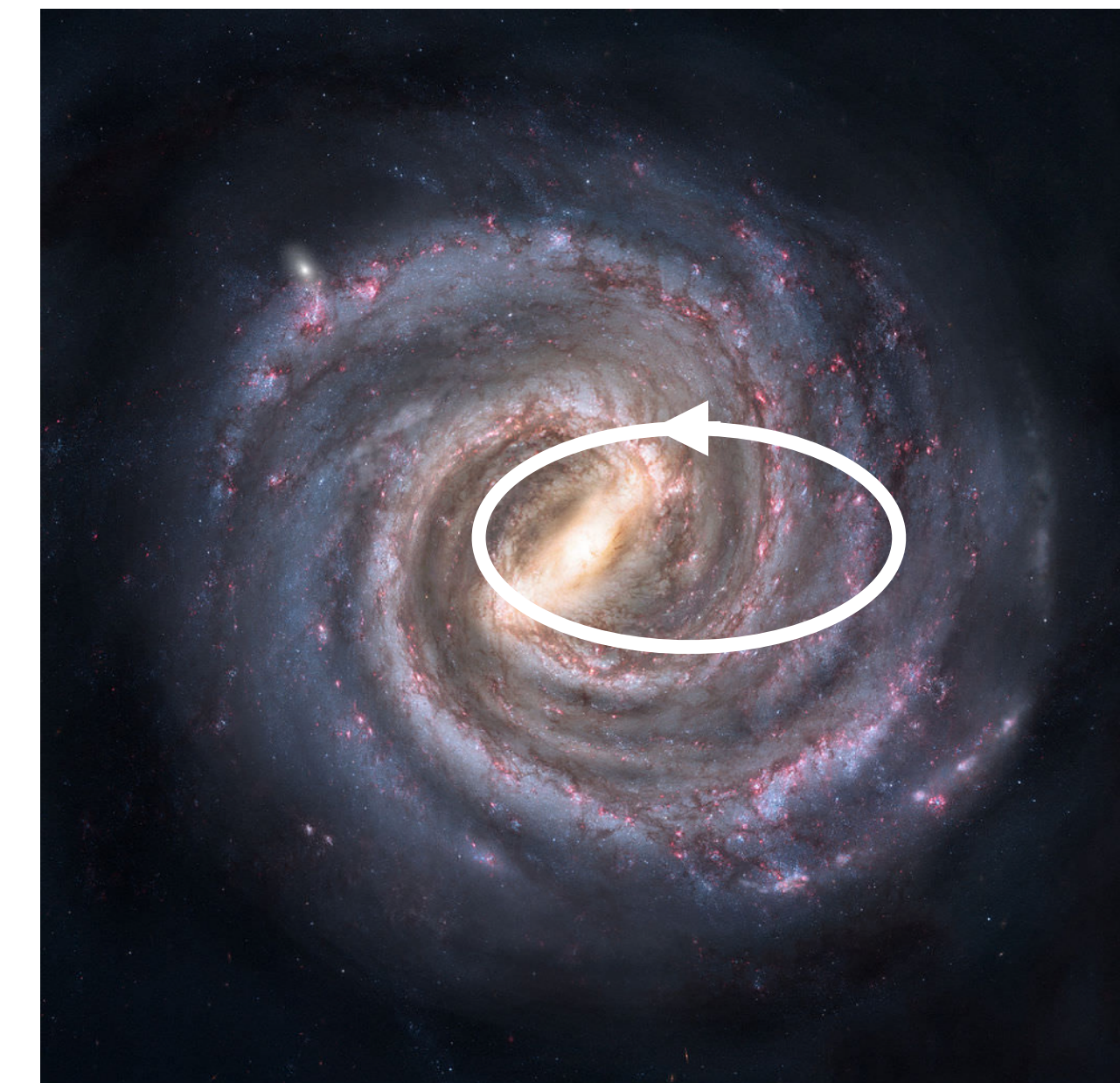
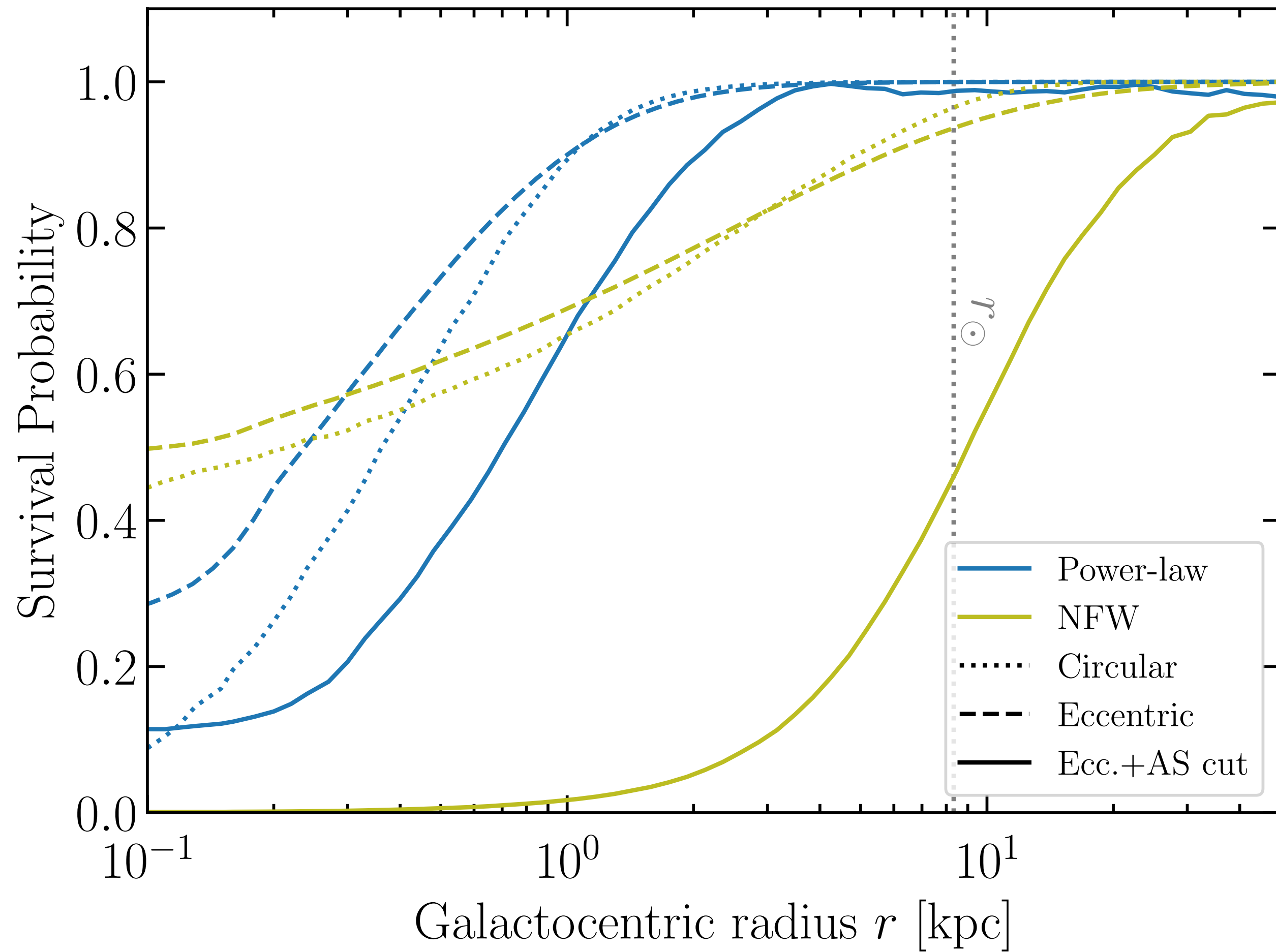


Scattering probability becomes a *geometric* problem:

$$P_{\mathcal{E}}(\Delta\mathcal{E}) \propto P(b|\mathcal{E})$$



Code available online: github.com/bradkav/HaloFeedback
(See also <https://github.com/DMGW-Goethe/imripy>)



Survival probability at Solar circle:
 $\mathcal{O}(40\%)$ for NFW profiles
 $\mathcal{O}(99\%)$ for PL profiles

But remember that even 'surviving' AMCs may be drastically altered.

[**BJK**, Edwards, Visinelli & Weniger, [2011.05377](#); Edwards, **BJK**, Visinelli & Visinelli, [2011.05378](#)]

[See also previous work, e.g. Tinyakov et al., [1512.02884](#); Dokuchaev et al., [1710.09586](#); and more recent work e.g. Dandoy et al., [2206.04619](#), Shen et al., [2207.11276](#)]