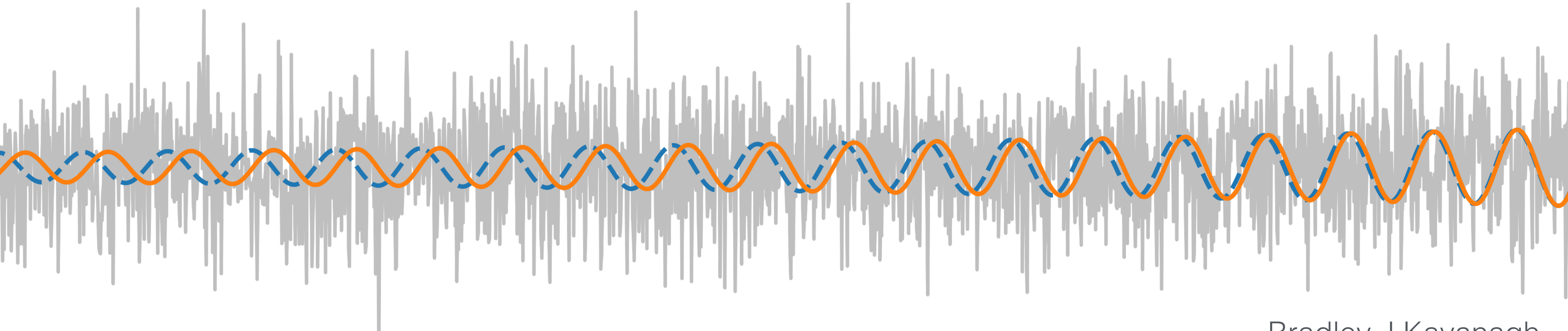


Gravitational Wave Signatures of Dark Matter around Black Holes



 kavanagh@ifca.unican.es

 @BradleyKavanagh

Bradley J Kavanagh
Instituto de Física de Cantabria
(CSIC-Universidad de Cantabria)

5th July 2023 - GRAPPA@10

GRAPPA Introduction

Postdoc at GRAPPA:
September 2017 - February 2020

GRAPPA Introduction Document

The idea of this document is to gather all the useful information for when a new member of GRAPPA joins the group. Hopefully it will help with a smooth transition to working at a new institution, possibly in a new country too. Lots of useful information can also be found on the University A-Z - <https://medewerker.uva.nl/en/science/az/a-z.html>.

First Days

One of the first things to do is to get yourself on the GRAPPA mailing list (GRAPPA-

Coffee

There is a hierarchy of coffee at UvA. In order of increasing goodness:

- Free coffee/hot chocolate/etc. is available from the big black coffee machines in the kitchen areas scattered around the building. Just scan your staff card on the contactless reader.
- Free coffee capsules are also available from the IoP secretaries' office. These can be used in the silver coffee machines in the theory lounge (C4.278) and the coffee lounge (at the opposite end of the building from GRAPPA).
- Note that there are also nice tea bags and occasionally sweet treats in the IoP secretaries' office so visit often.
- :: • Lovely take-away coffee can be bought downstairs at the coffee bar in the main hall.
- Hipster coffee is available from Coffee Virus, in the start-up village a few minutes walk from Science Park 904. Since February 2019 CoffeeVirus runs two restaurants in Science Park, the other being in building Science Park 408/410, just behind Universum. They also sell take-away sandwiches and salads.

In collaboration with
Tom Edwards

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GRAPPA@5

Celebrating 5 years of the GRAPPA Centre of Excellence in Astroparticle Physics

GRAPPA: Research Highlights

1. Discovery of the Higgs boson (2012)
Phys. Lett. B 716 (2011) 1-29 (and GRAPPA members: D. Burgi, P. de Jong, S. Sennrich)
2. XENON100 dark matter direct detection results (2012)
Phys. Rev. Lett. 107 (2011) 191301 (and S. Sennrich)
3. Dissecting X-ray-Emitting Gas Around the Center of Our Galaxy (2013)
Nature 501 (2013) 541 (and S. Sennrich)
4. Characterisation of the Fermi GeV Excess (2015)
Phys. Rev. Lett. 115 (2015) 081301 (and S. Sennrich, C. Williams, C. Williams)
5. Evidence for dark matter in the inner Milky Way (2015)
Nature Phys. 11 (2015) 546-549 (and S. Sennrich)
6. Origin of PeV Neutrinos discovered by IceCube (2015)
Phys. Rev. Lett. 115 (2015) 081301 (and S. Sennrich, P. de Jong, S. Sennrich, J. Sennrich)
7. Discovery of Pevaron at the Galactic Centre (2014)
Nature 511 (2014) 521 (and S. Sennrich)
8. GRAPPA-wide paper - search for Primordial Black Hole (2017)
Phys. Rev. Lett. 118 (2017) 061301 (and S. Sennrich, S. Sennrich, S. Sennrich, S. Sennrich, S. Sennrich)

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GRAPPA: Research Highlights


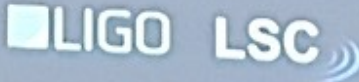

1. **Discovery of the Higgs boson (2012)**
Phys. Lett. B716 (2012) 1-29 (incl. GRAPPA members D. Bergs, P. de Jong, S. Bontveit)
2. **XENON100 dark matter direct detection results (2012)**
Phys. Rev. Lett. 109 (2012) 181301 (incl. P. Duvvuri)
3. **Dissecting X-ray-Emitting Gas Around the Center of Our Galaxy (2013)**
Science (2013) 341, 104 (incl. S. Portinari)
4. **Characterisation of the Fermi GeV Excess (2015)**
JCAP 1302 (2015) 058 (incl. F. Calzavara, C. Weniger); Phys. Rev. Lett. 114 (2014) 051102 (incl. R. Bernabè, S. Cristoforetti, C. Weniger)
5. **Evidence for dark matter in the inner Milky Way (2015)**
Nature Phys. 11 (2015) 245-248 (incl. G. Bertone)
6. **Origin of PeV Neutrinos discovered by IceCube (2015)**
Phys. Rev. Lett. 115 (2015) 071101 (incl. S. Ando); Phys. Rev. Lett. 115 (2015) 221101 (incl. S. Ando, I. Tamborra, F. Zandorini)
7. **Discovery of Pevatron at the Galactic Centre (2016)**
Nature 531 (2016) 476 (HESS Collaboration and D. Bergs)
8. **GRAPPA-wide paper - search for Primordial Black Hole (2017)**
Phys. Rev. Lett. 118 (2017) no. 24, 241101 (incl. D. Goggin, G. Bertone, F. Calzavara, R. M.T. Connors, M. Lovell, S. Portinari and E. Steig)

GRAPPA: Research Highlights

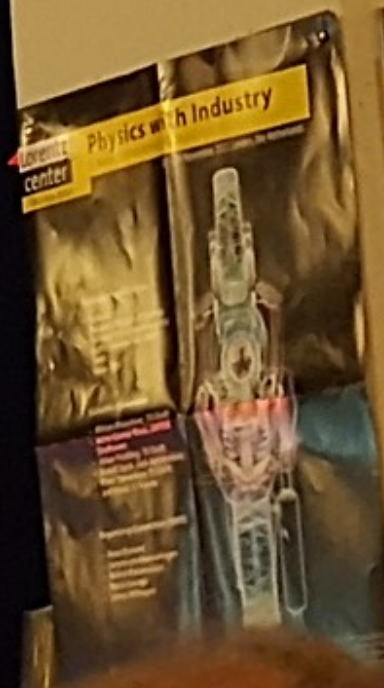
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National Science Foundation
LIGO-VIRGO COLLABORATION
UPDATE ON THE SEARCH FOR GRAVITATIONAL WAVES

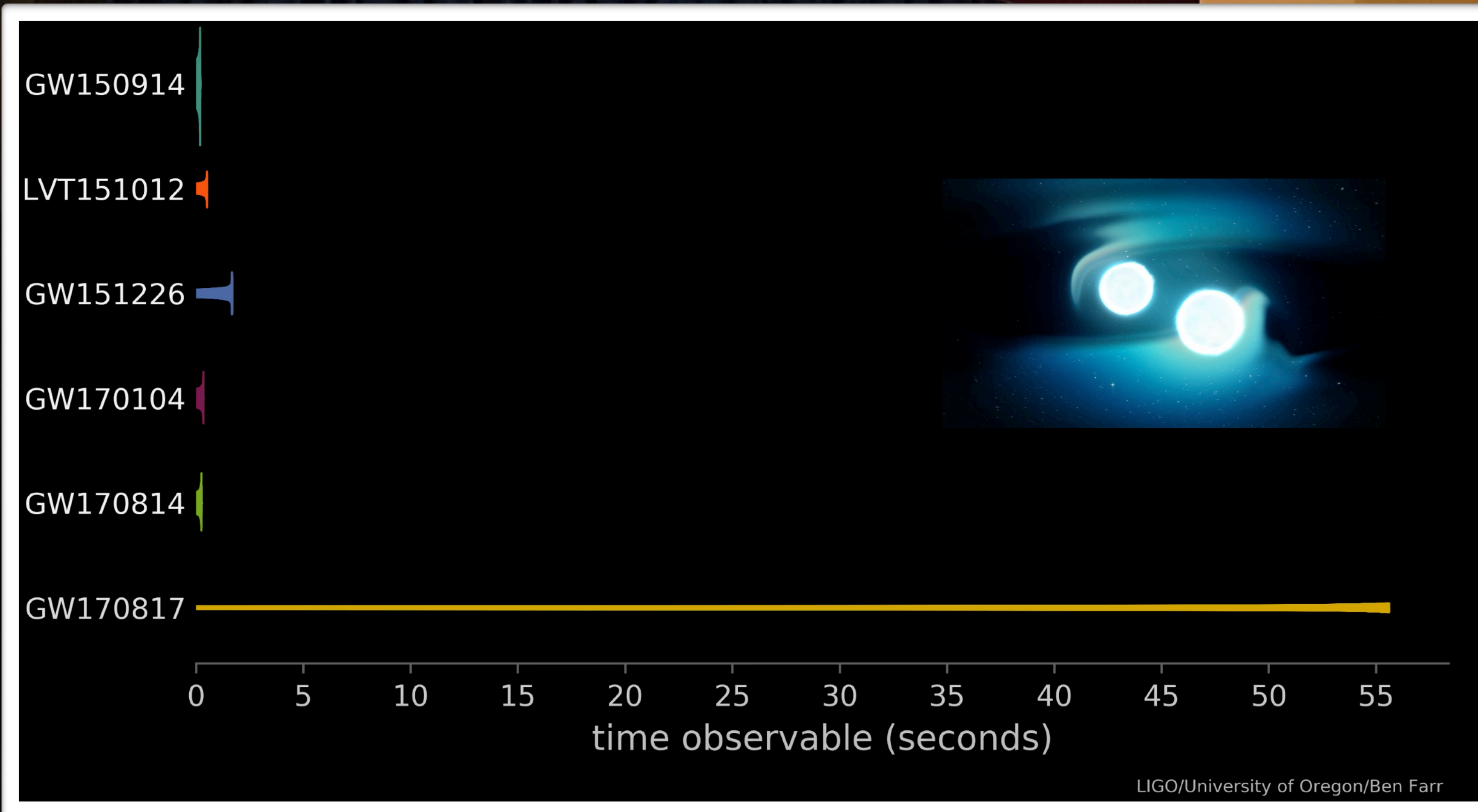
Physics with Industry
center



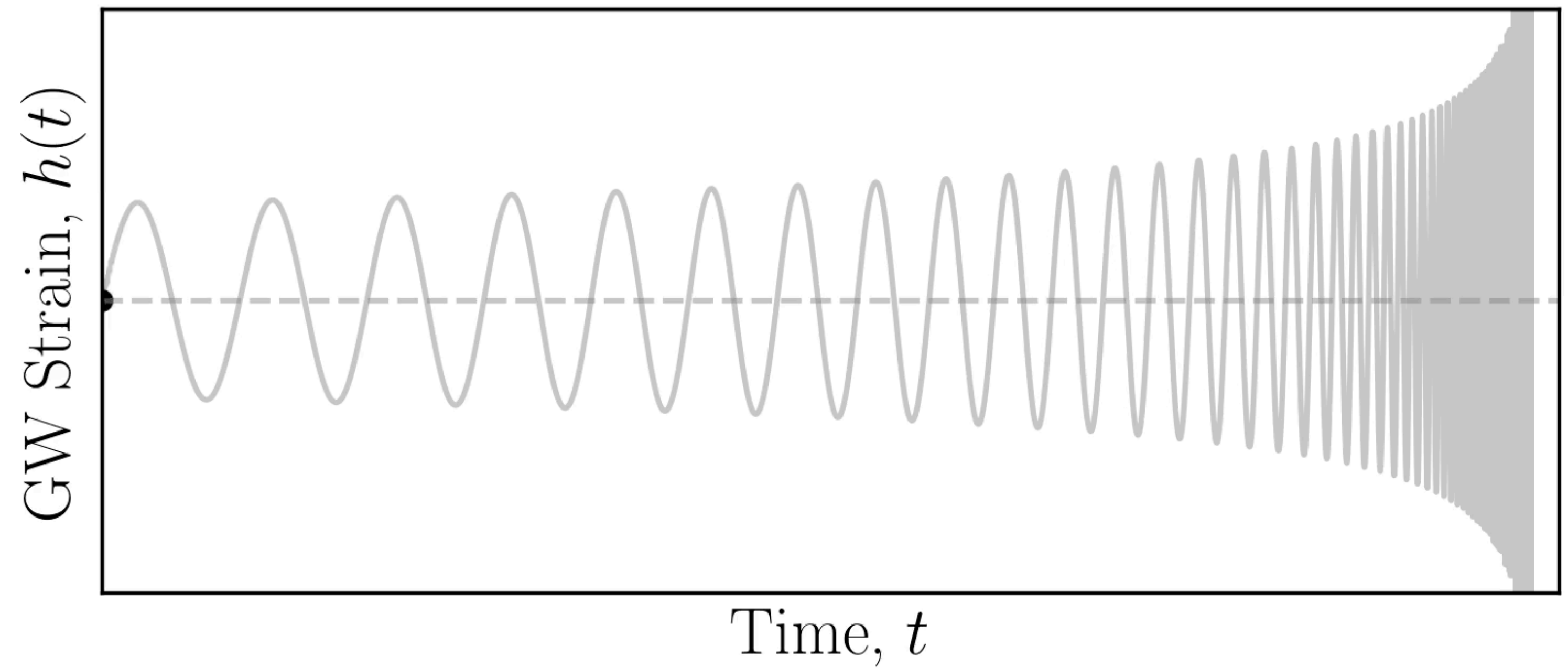
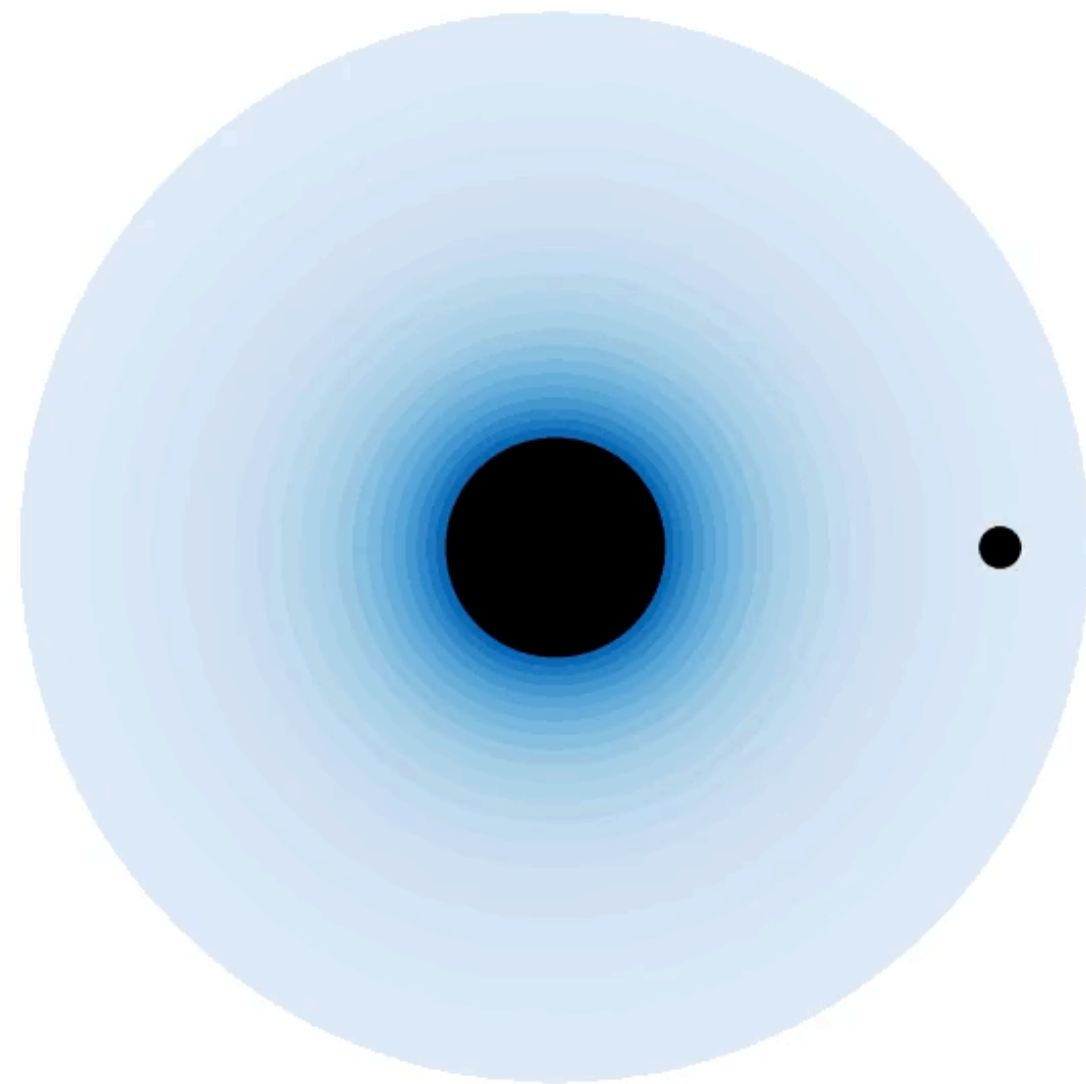
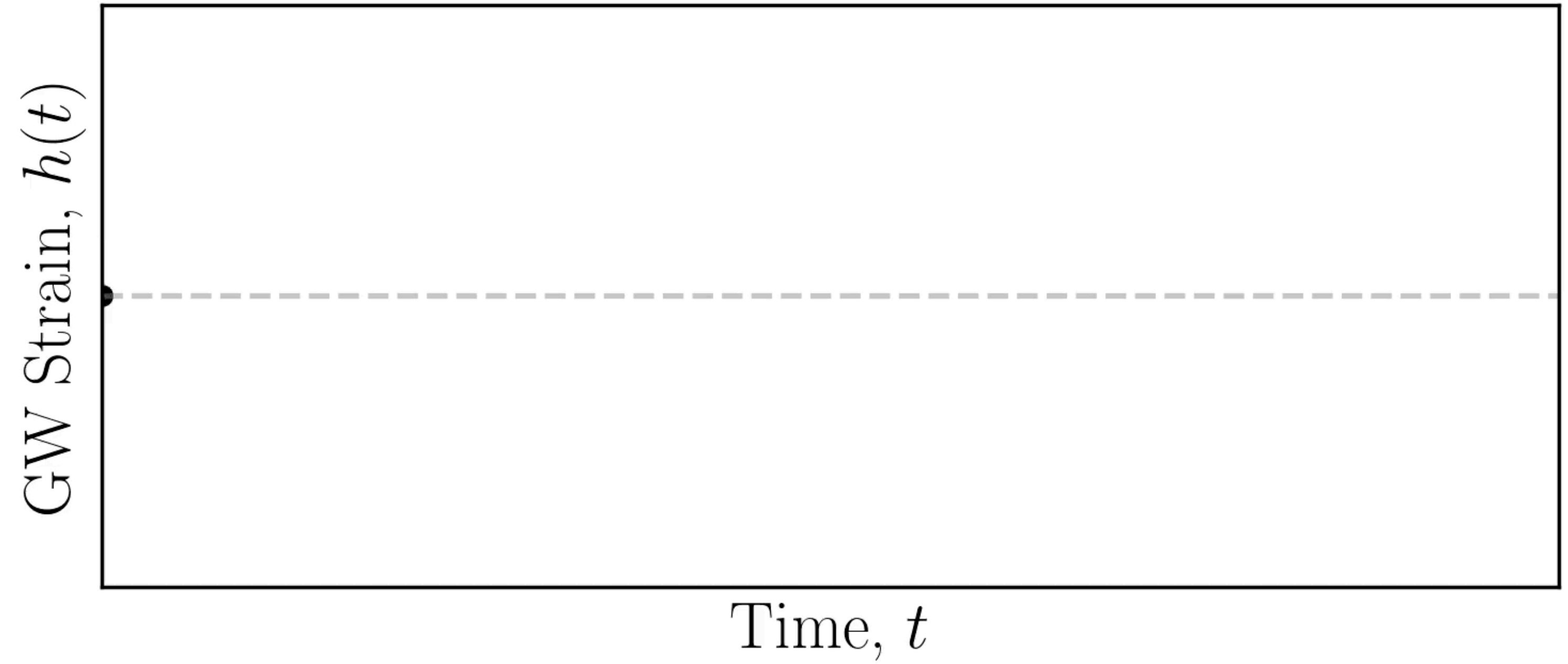
BREAKING NEWS:
PHYSICS
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READING ABSTRACTS
& SEPTEMBER 2017
PROGRAM REGISTRATION
18 NOVEMBER 2017 23-26
JANUARY 2018

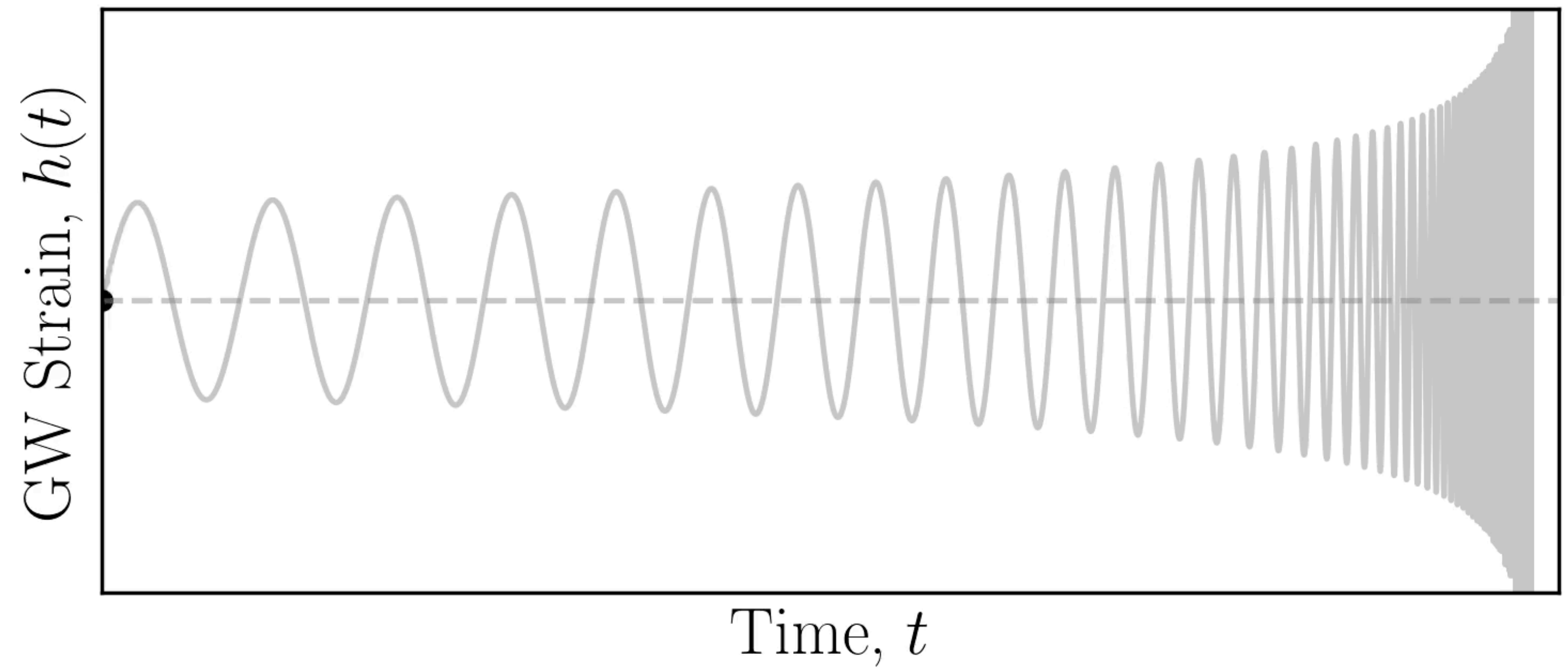
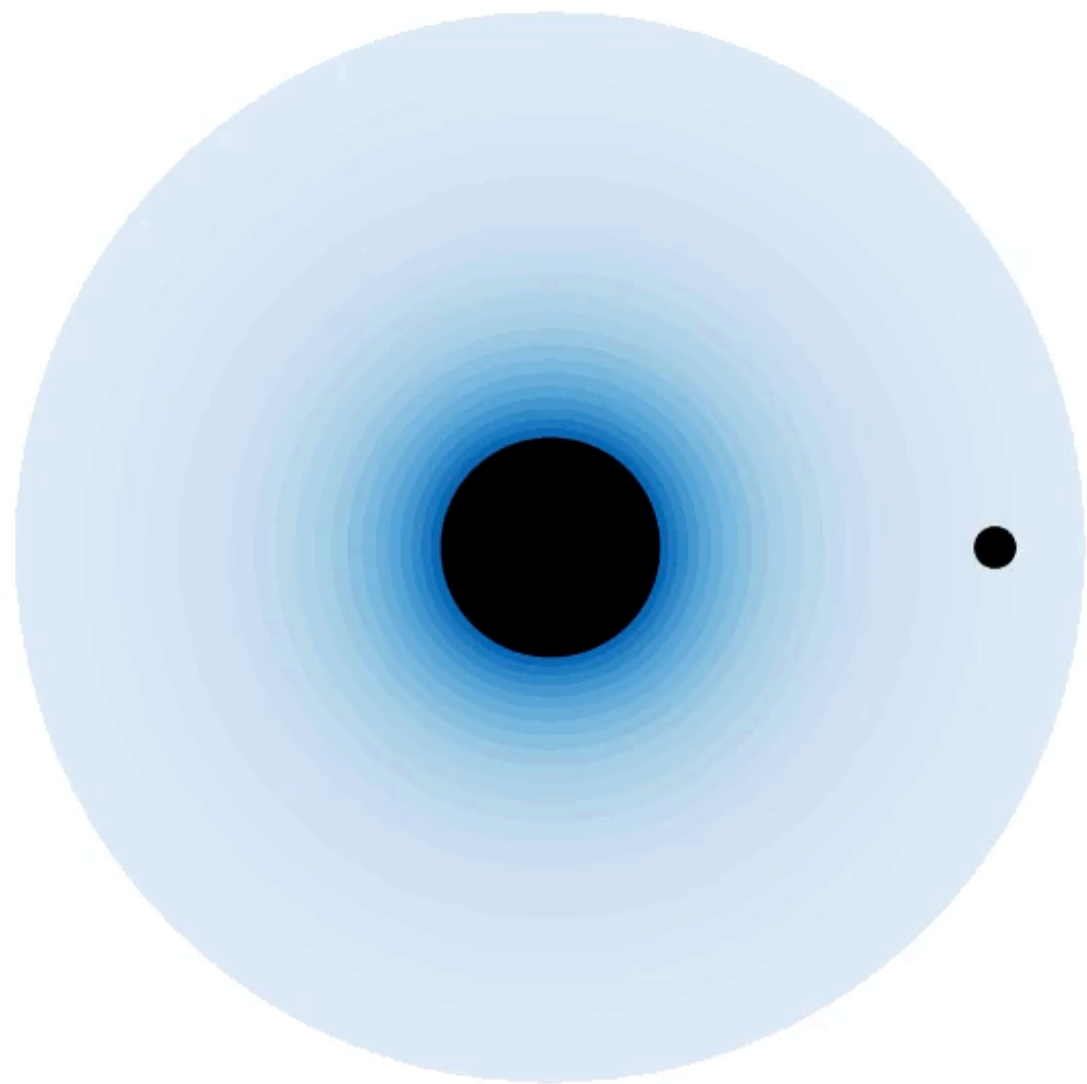
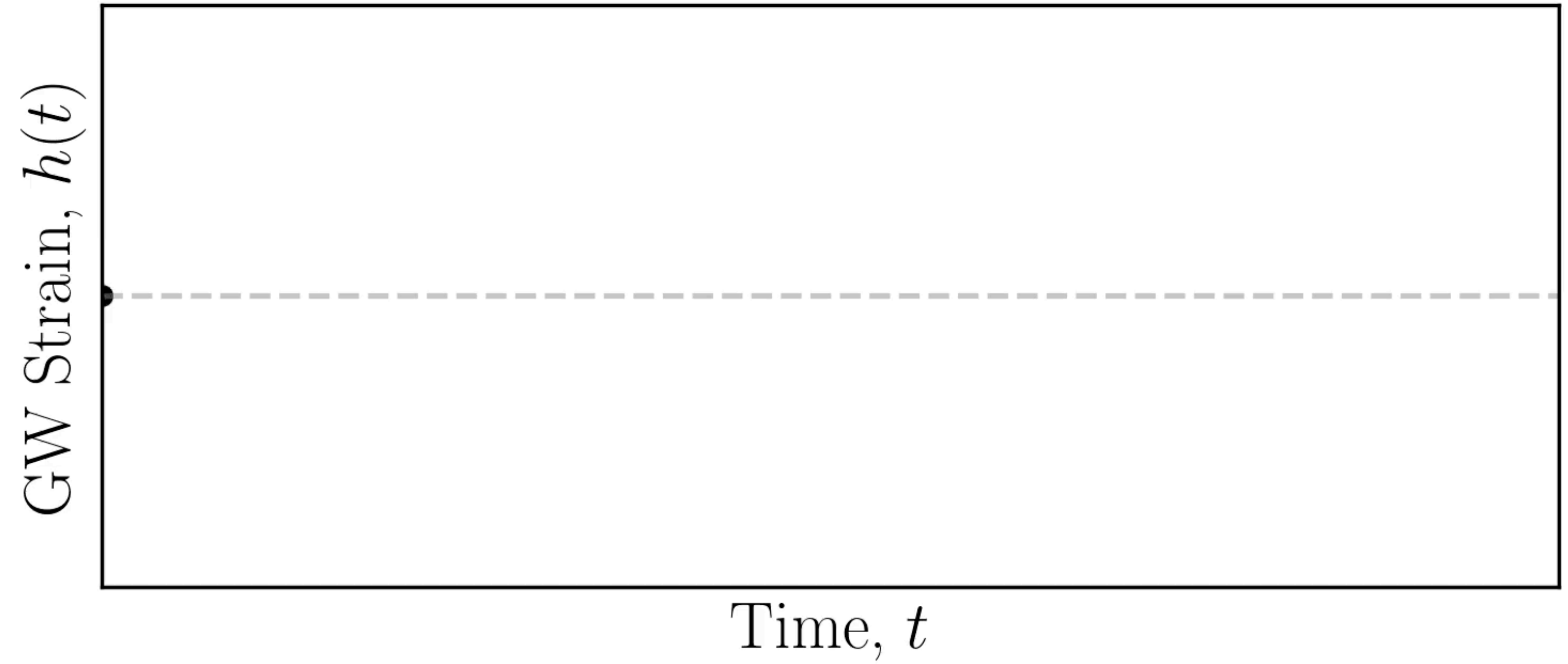
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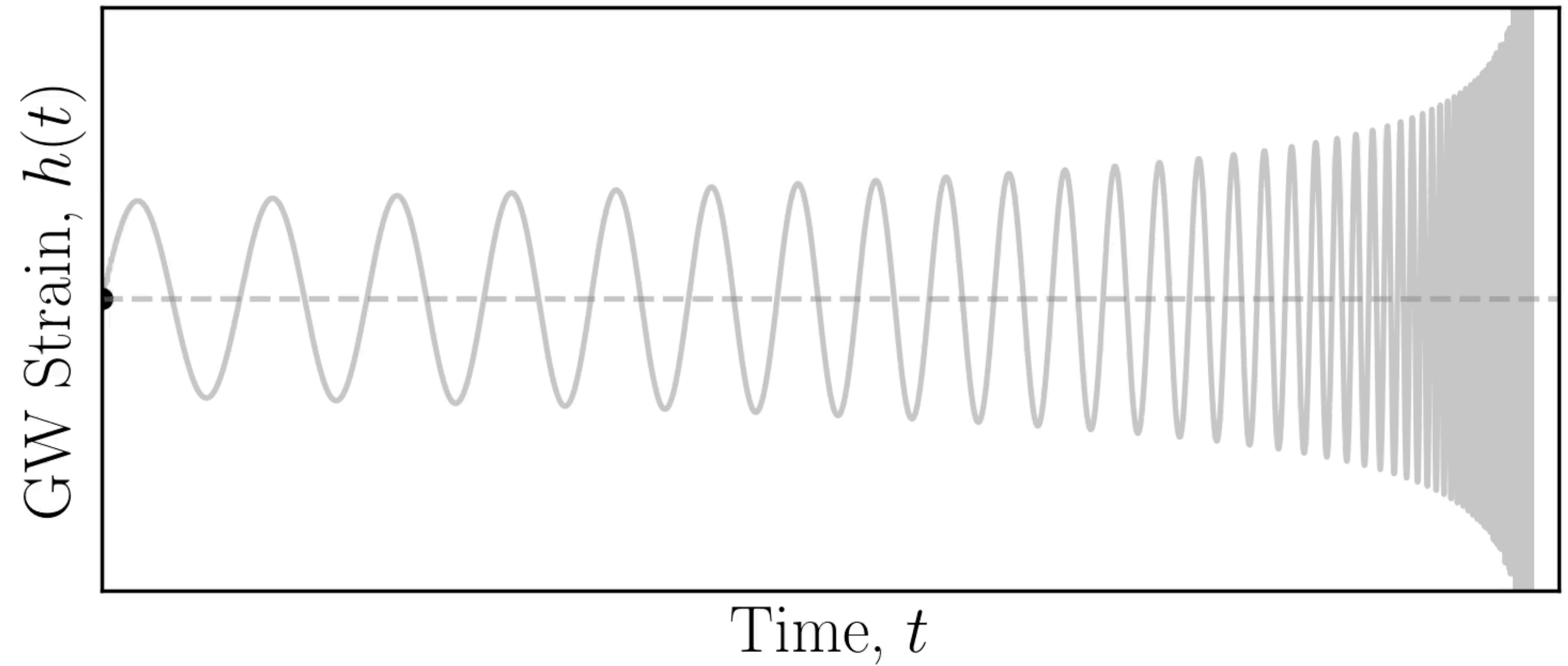
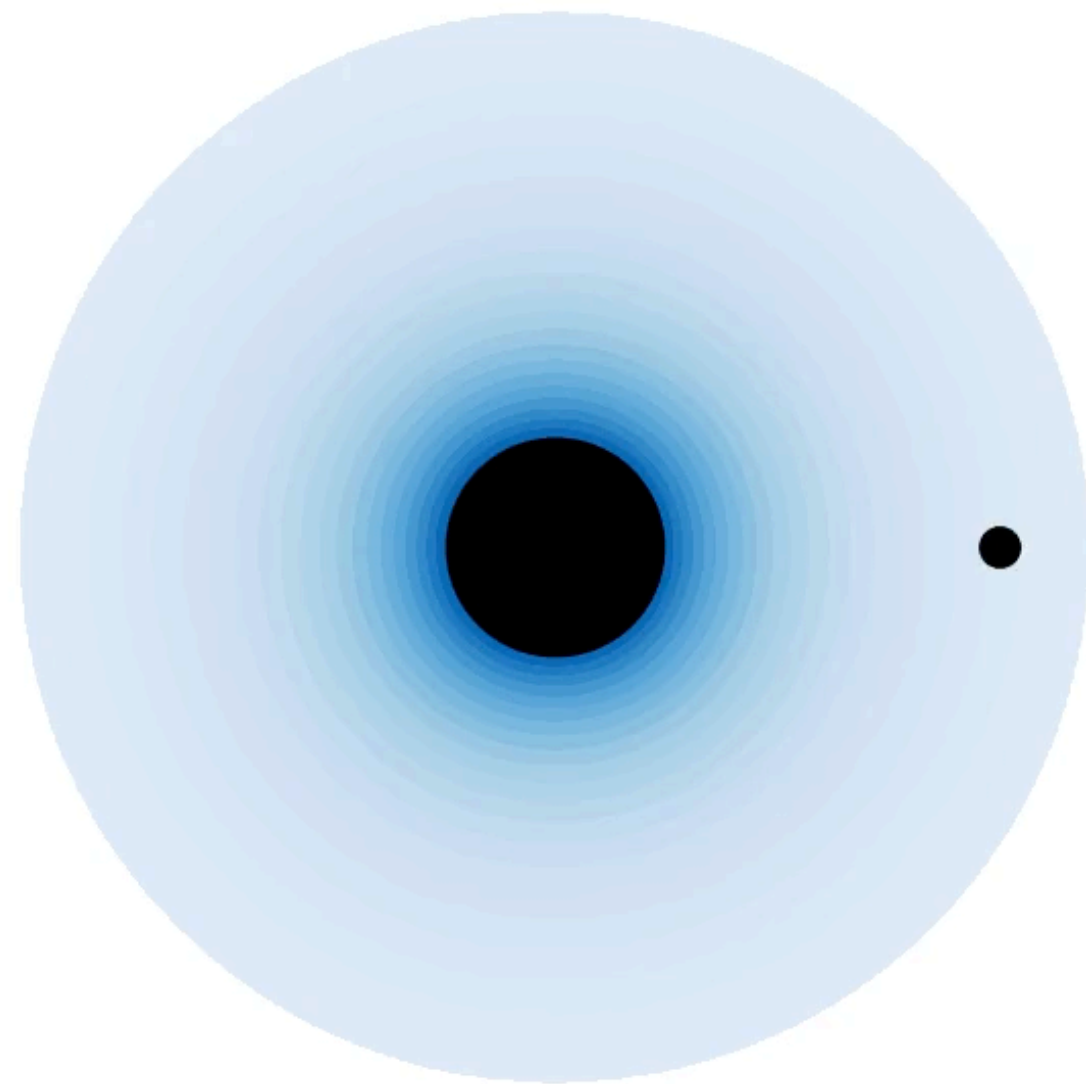
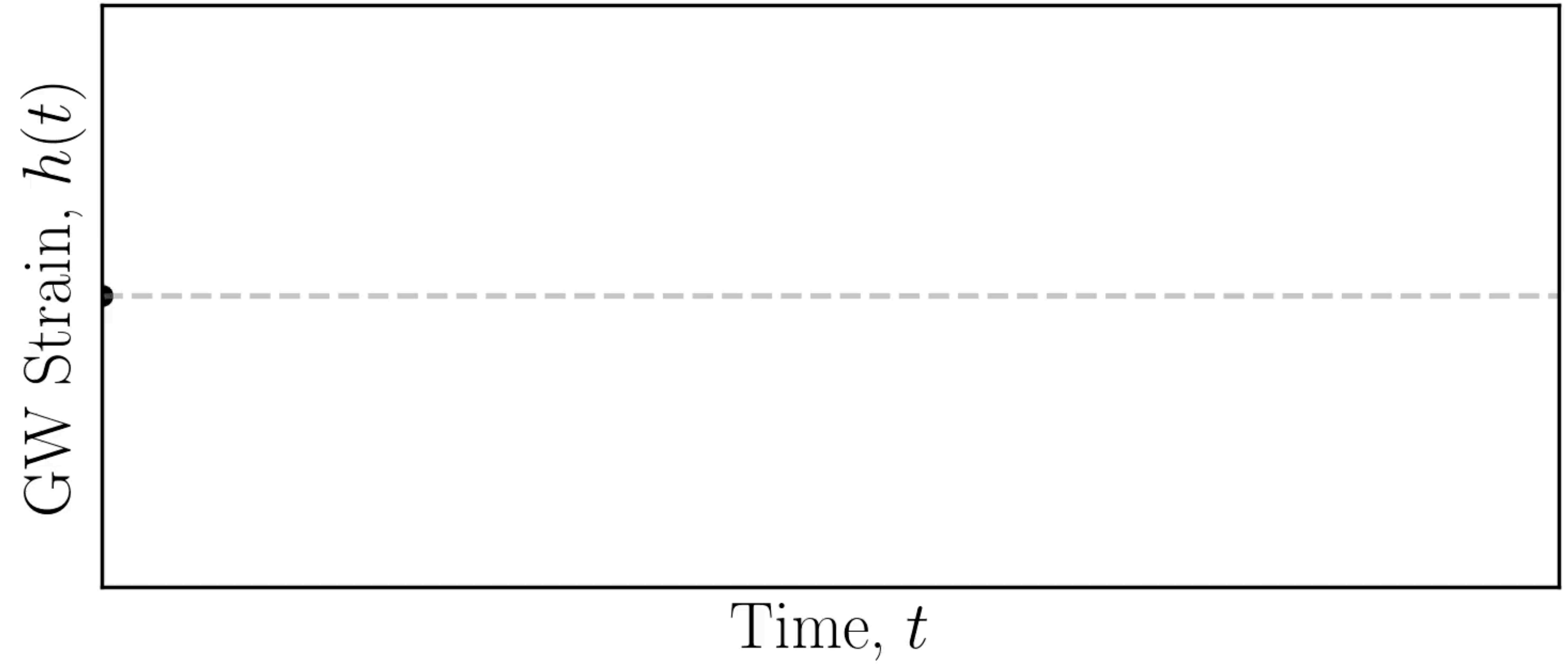
“De-phasing” of GWs



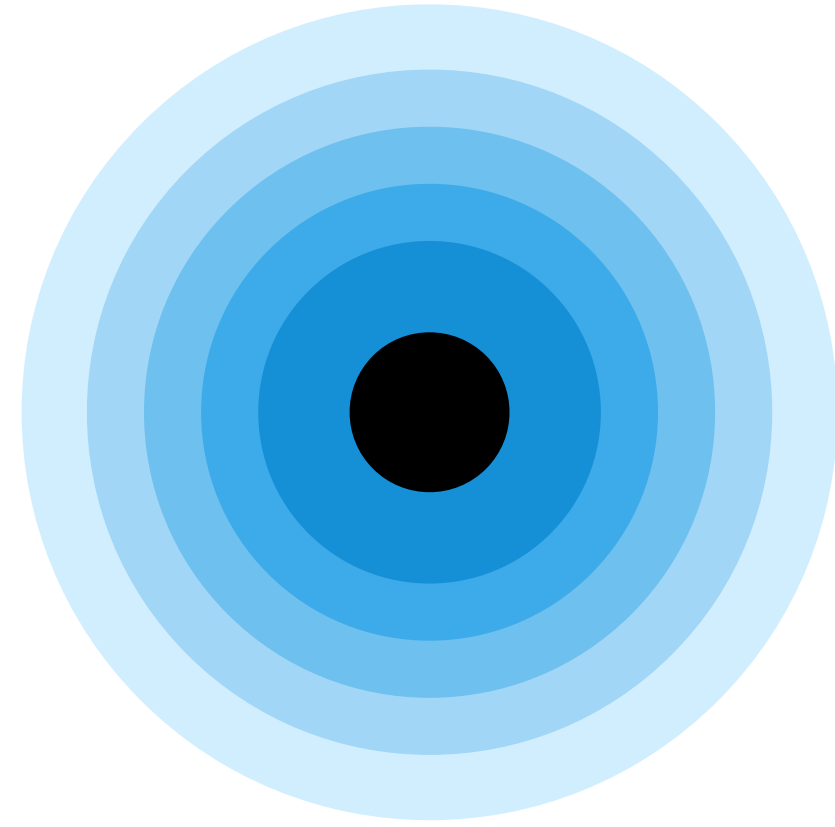
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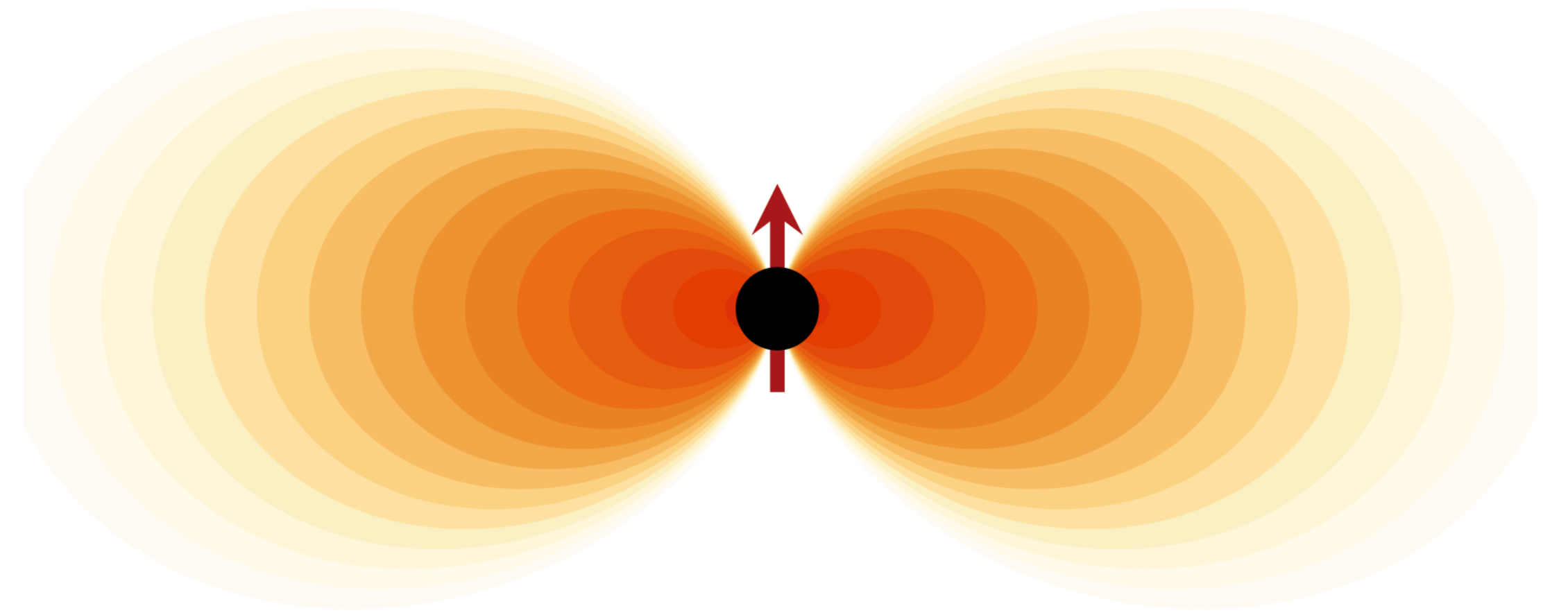
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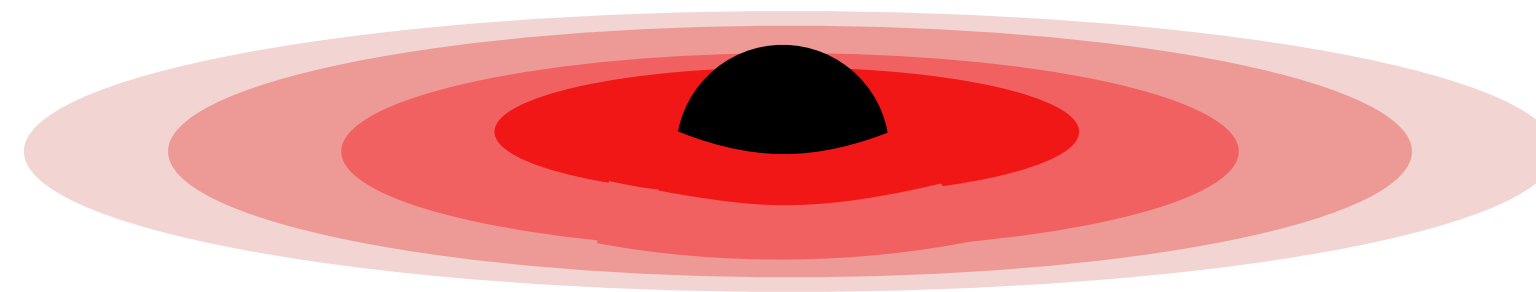
Black Hole Environments



Particle Dark Matter
'**Spikes**' or '**Dresses**'



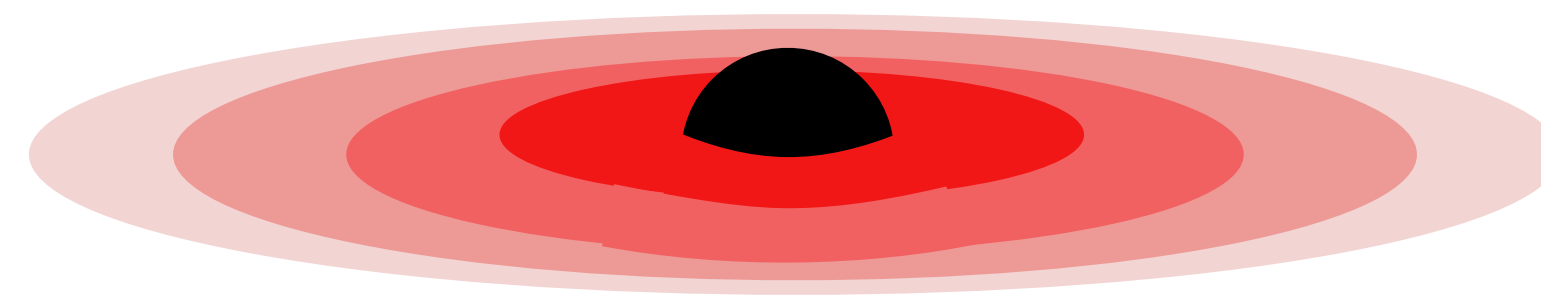
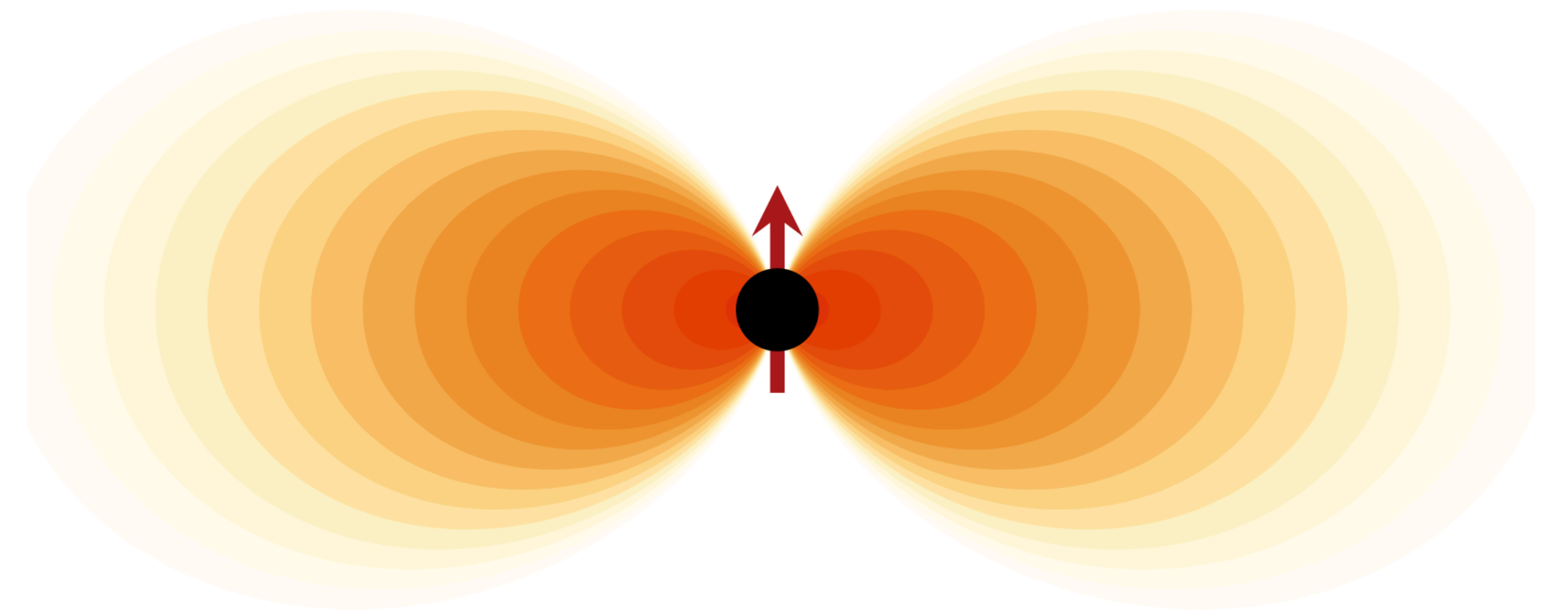
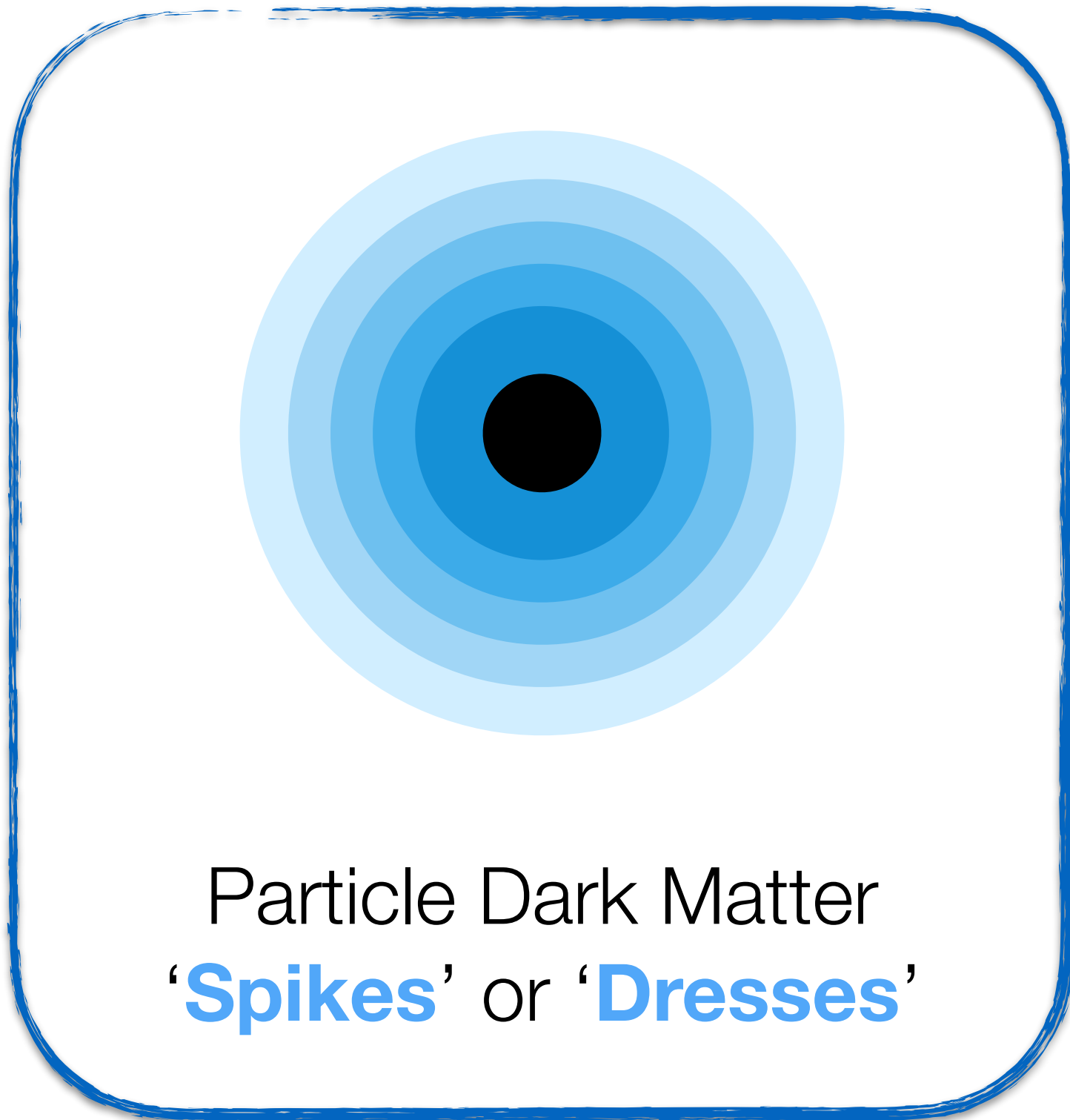
'**Gravitational Atoms**' of
Ultralight Bosons



Baryonic
Accretion Disks

Black Holes (BHs) are extreme environments allowing us to probe high densities, long timescales...

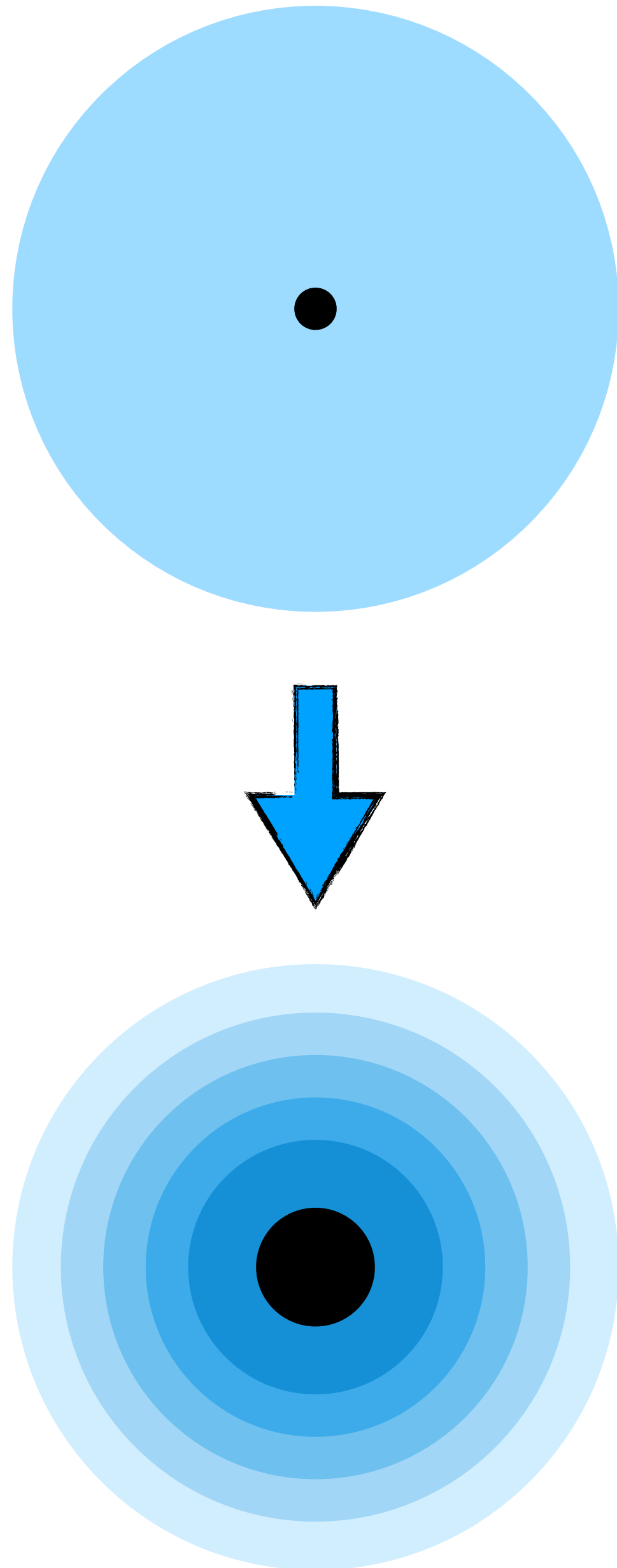
Black Hole Environments



Baryonic
Accretion Disks

Black Holes (BHs) are extreme environments allowing us to probe high densities, long timescales...

Dark Matter Spikes (1)



‘Spikes’ or ‘dresses’ of cold, particle-like DM may form around BHs:

From the slow (‘adiabatic’) growth of a BH at the centre of a DM halo

“Astrophysical scenario”

[[astro-ph/9906391](#), [astro-ph/0509565](#), [1305.2619](#), ...]

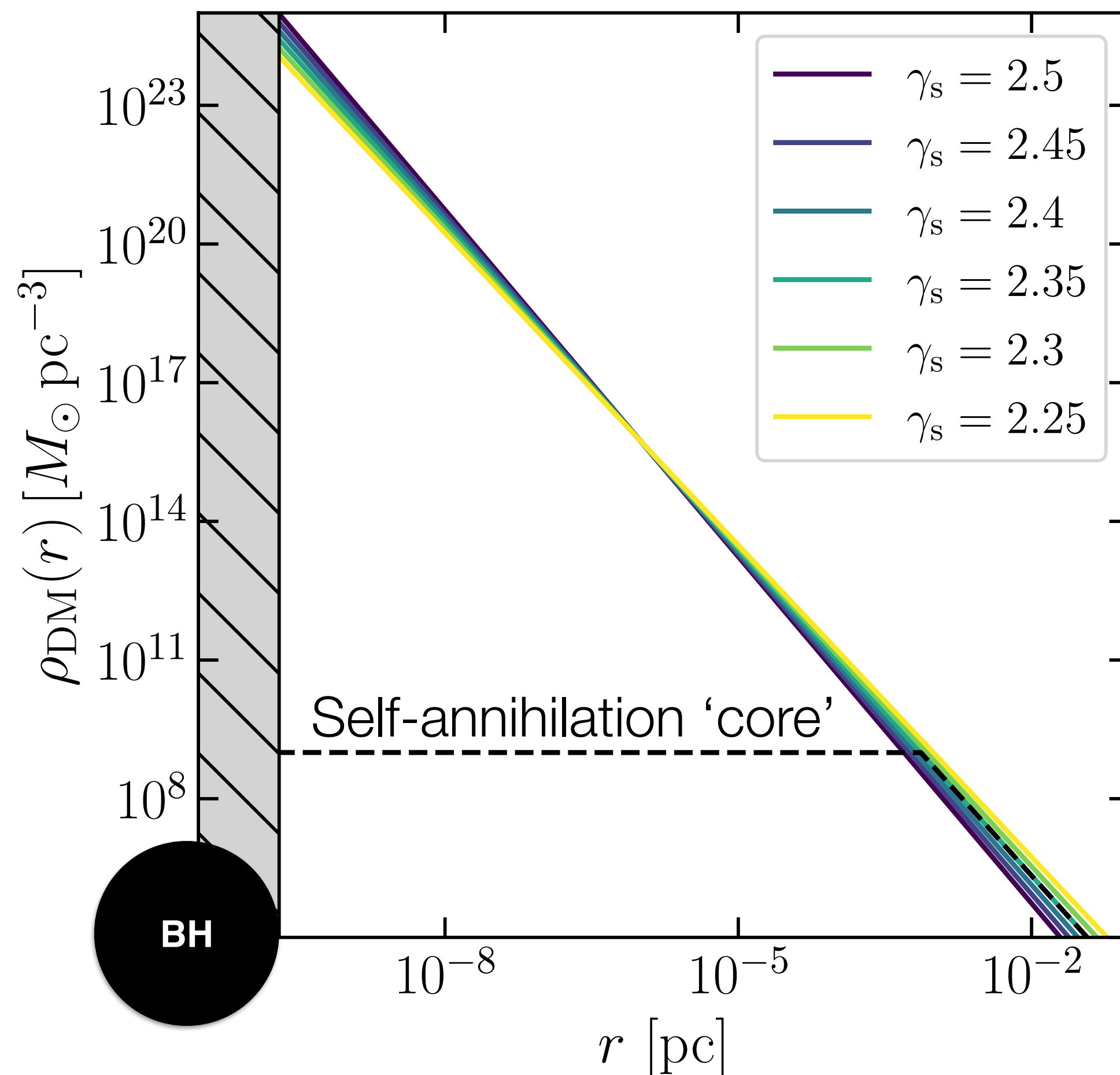
Around BHs which form from large density fluctuations in the early Universe (i.e. Primordial Black Holes)

“PBH scenario”

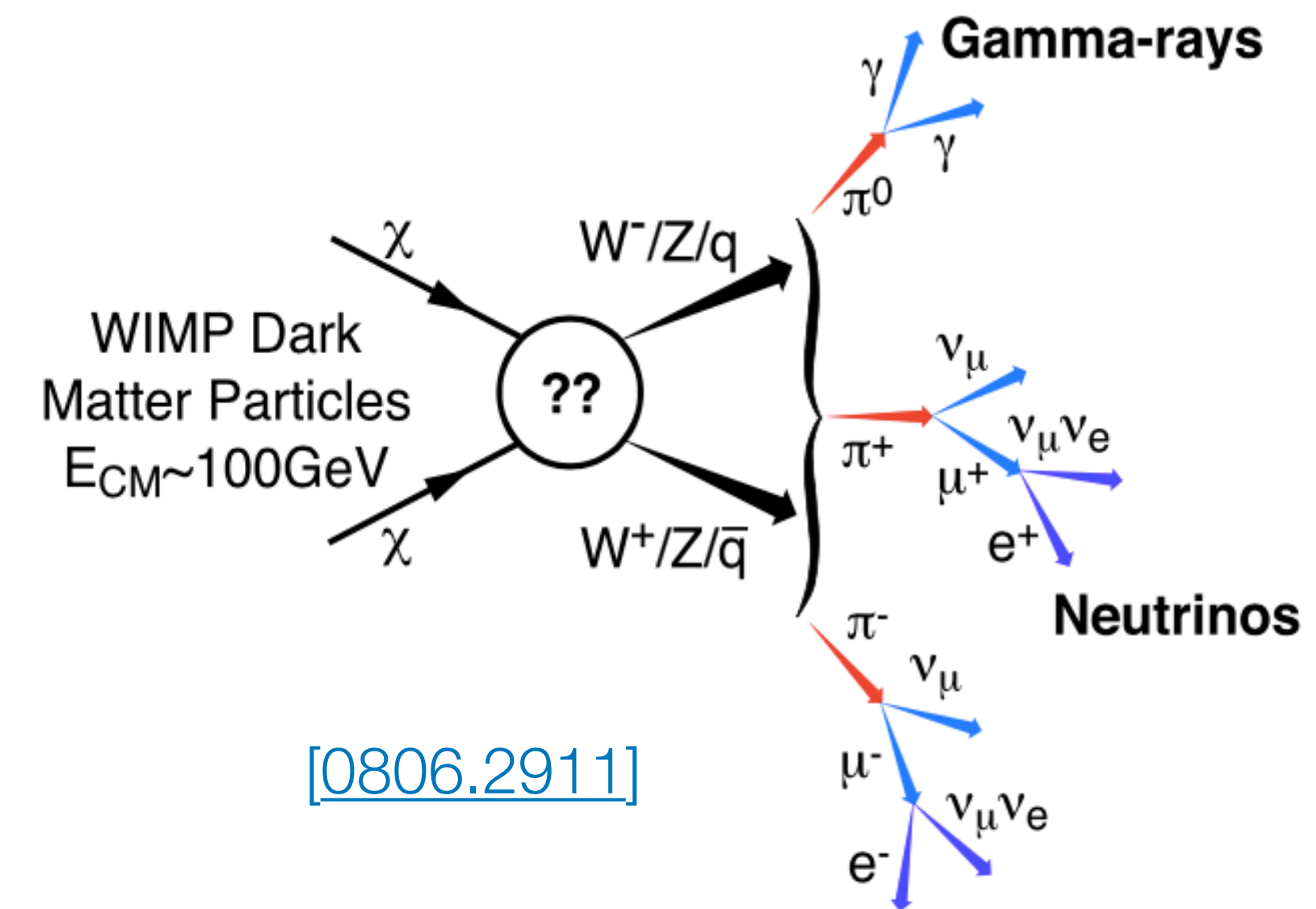
[[Bertschinger \(1985\)](#), [astro-ph/0608642](#), [1901.08528](#), ...]

Dark Matter Spikes (2)

$$\rho_{\text{DM}} = \rho_6 \left(\frac{10^{-6} \text{ pc}}{r} \right)^{\gamma_{\text{sp}}}$$



$$\rho_{\text{DM, local}} \sim 10^{-2} M_{\odot} / \text{pc}^3$$



DM self-annihilation can suppress the spike density, but can still lead to large (diffuse and point source) fluxes of gamma-rays and neutrinos

[E.g. Bertone, Coogan, Gaggero, **BJK** & Weniger, [1905.01238](#)]

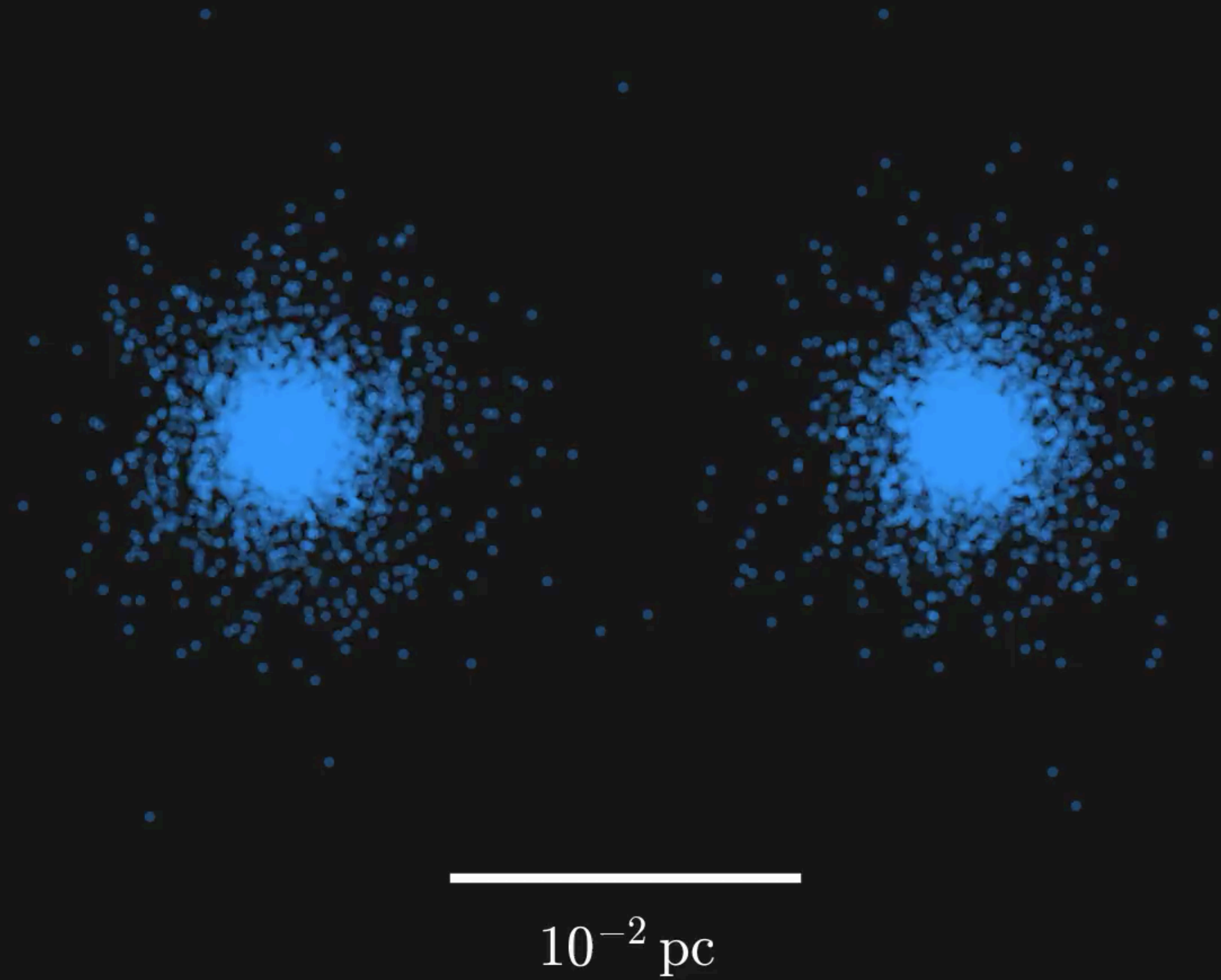
Think instead about **non-annihilating DM**

Equal-mass binaries

[[BJK](#), Gaggero & Bertone, [1805.09034](#)]

[[Movies here](#)]

$$\begin{aligned} M_{\text{PBH}} &= 30 M_{\odot}; a_i = 0.01 \text{ pc}; e_i = 0.995 \\ T &= 0.00 \text{ kyr} \end{aligned}$$

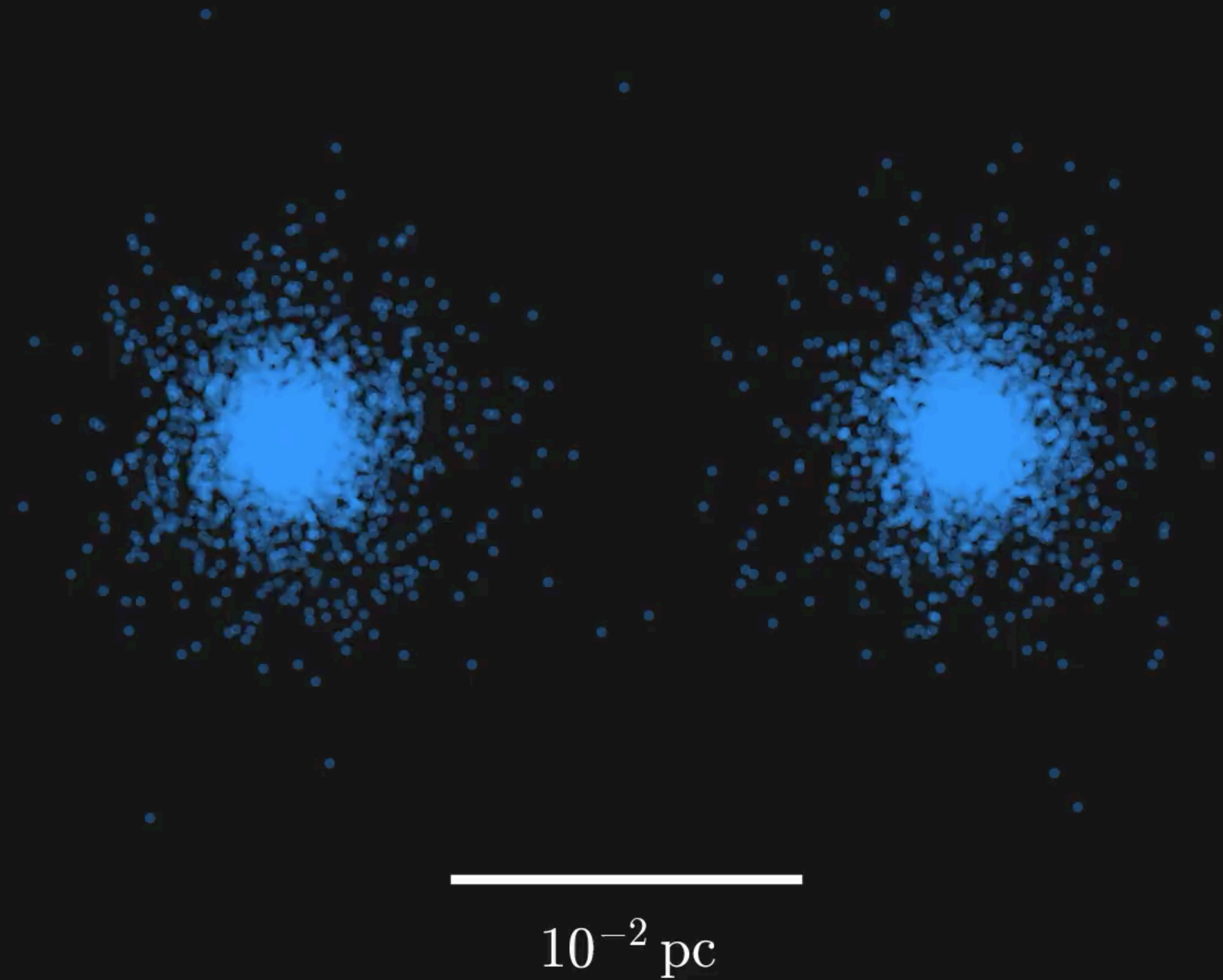


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[BJK, Gaggero & Bertone, [1805.09034](#)]

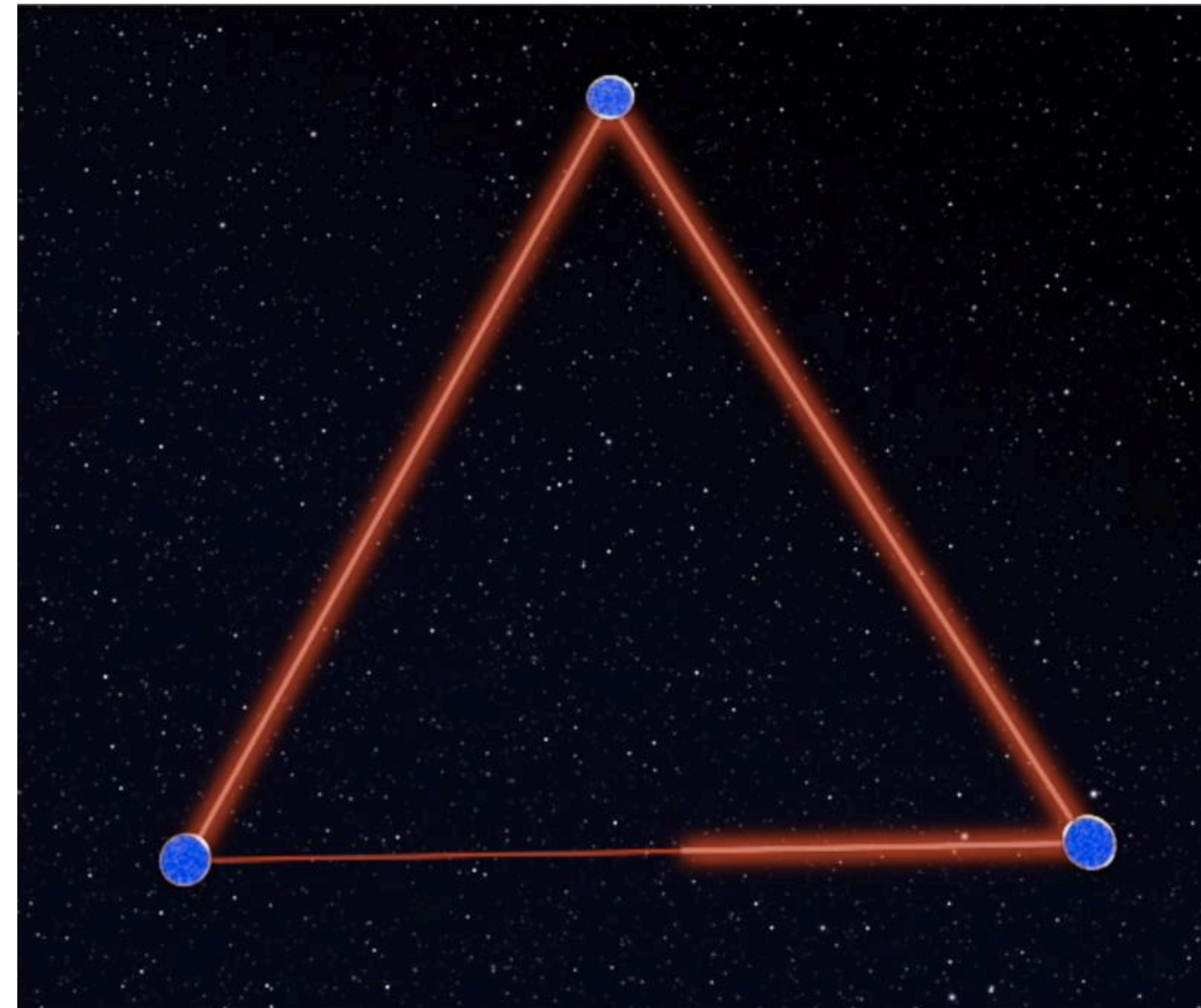
[Movies [here](#)]

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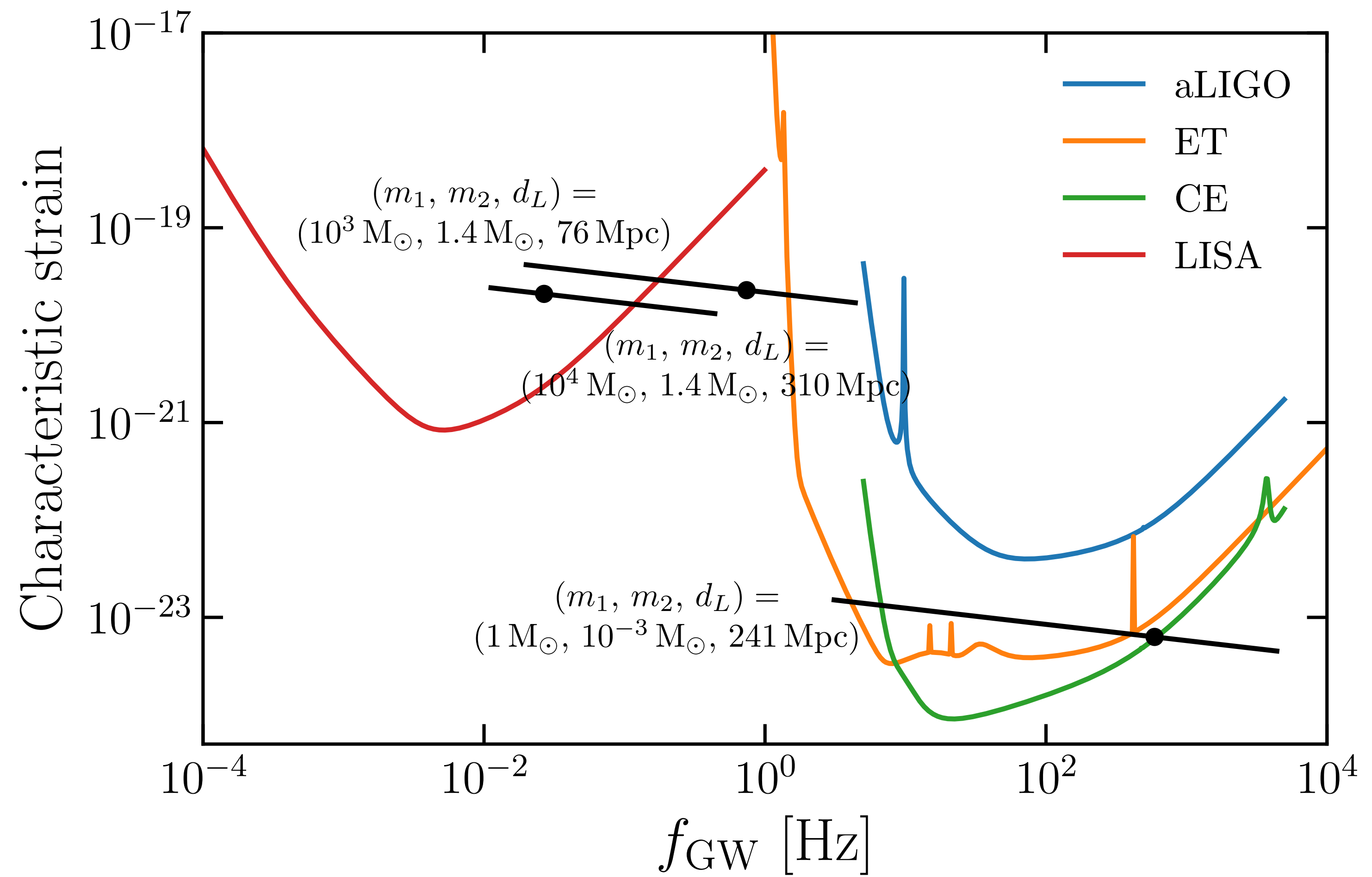
GW Sensitivity

© AEI / MM / exozet



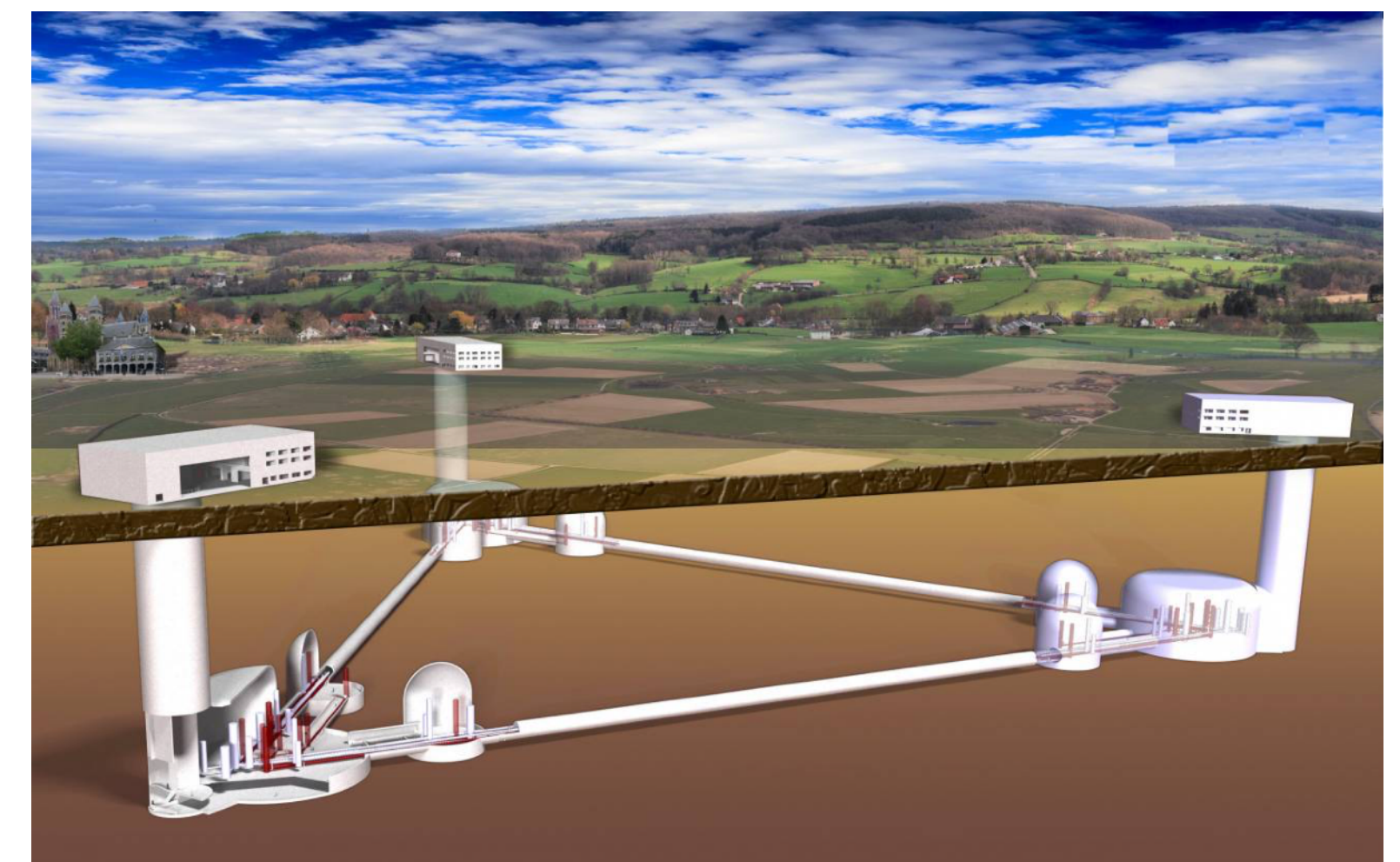
Laser Interferometer Space Antenna
(planned for the 2030s)

[\[1907.06482\]](#)

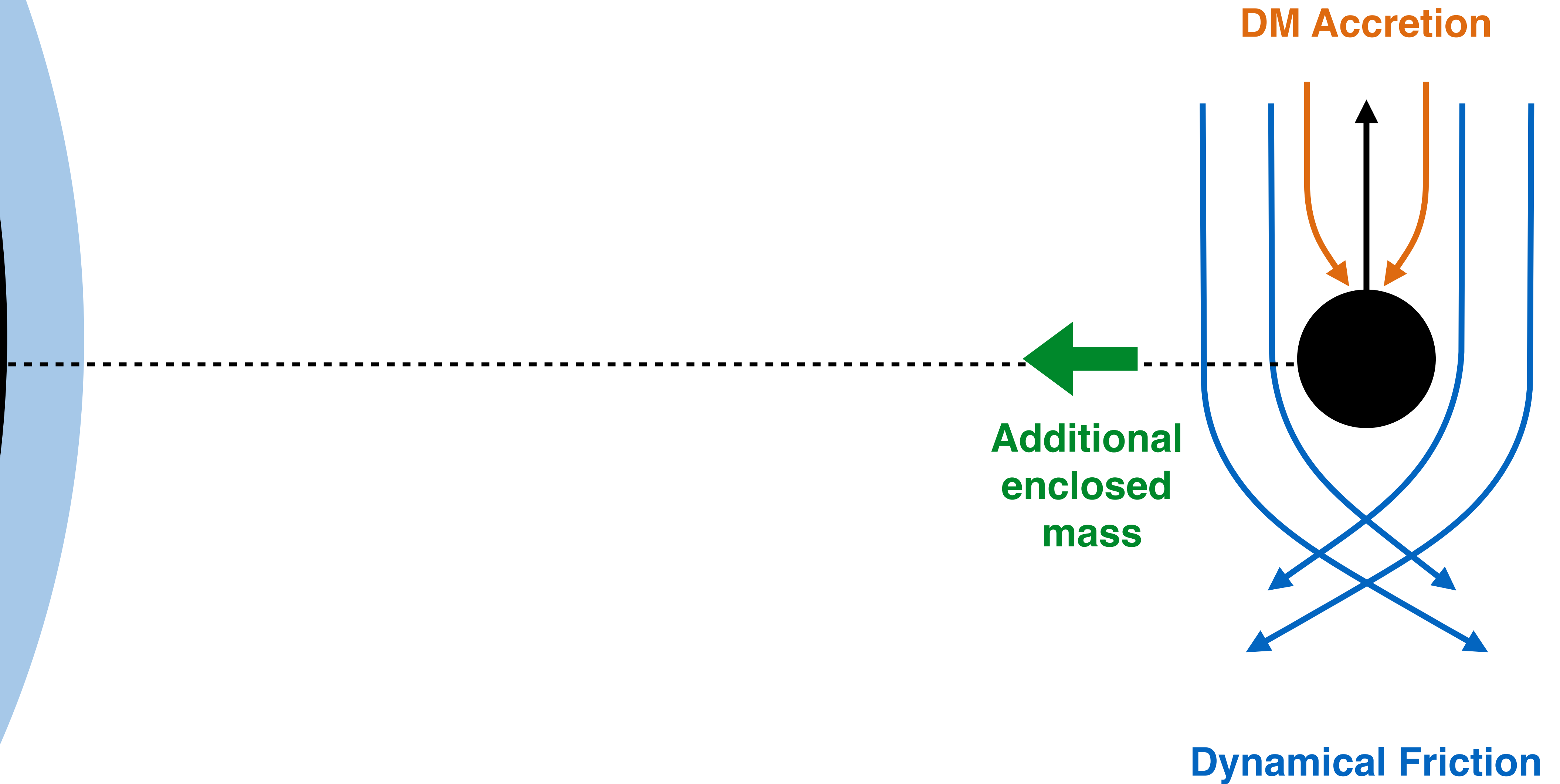
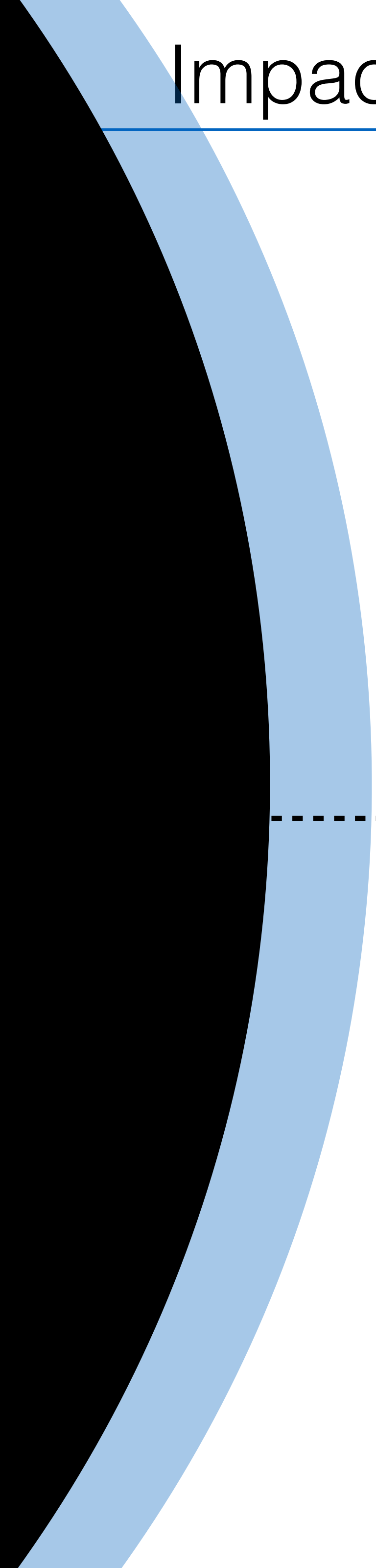


Einstein Telescope

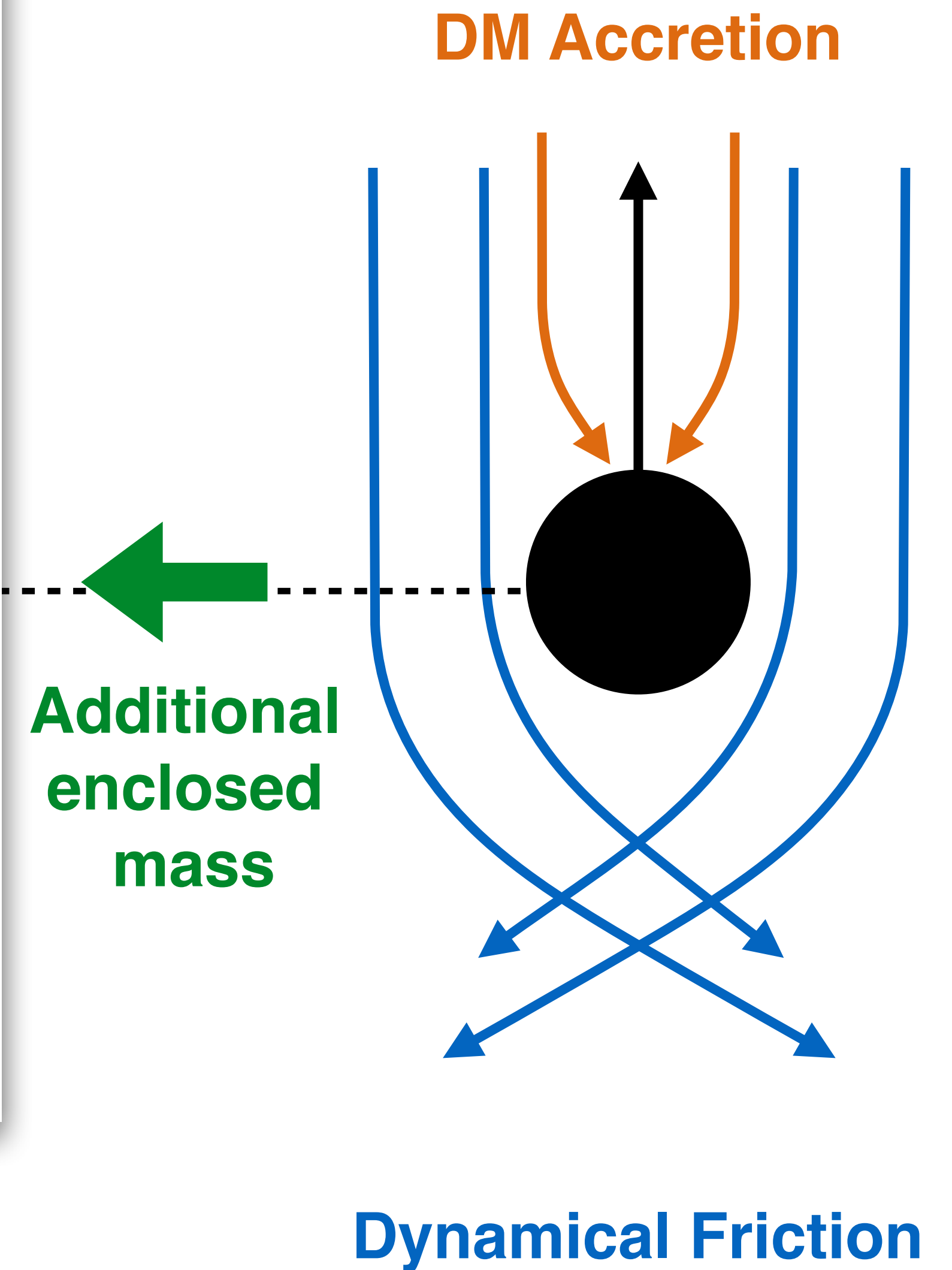
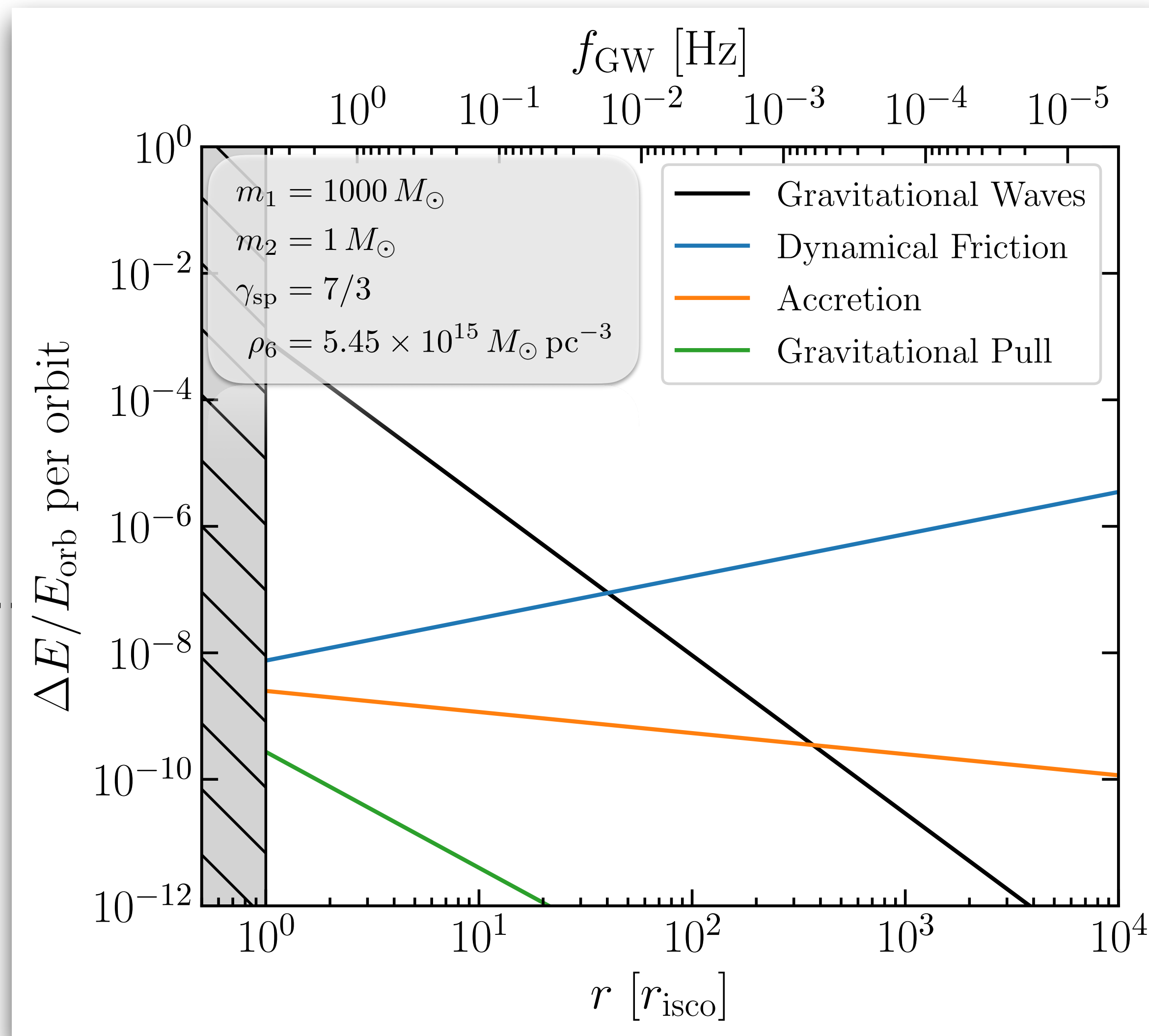
[\[1912.02622\]](#)



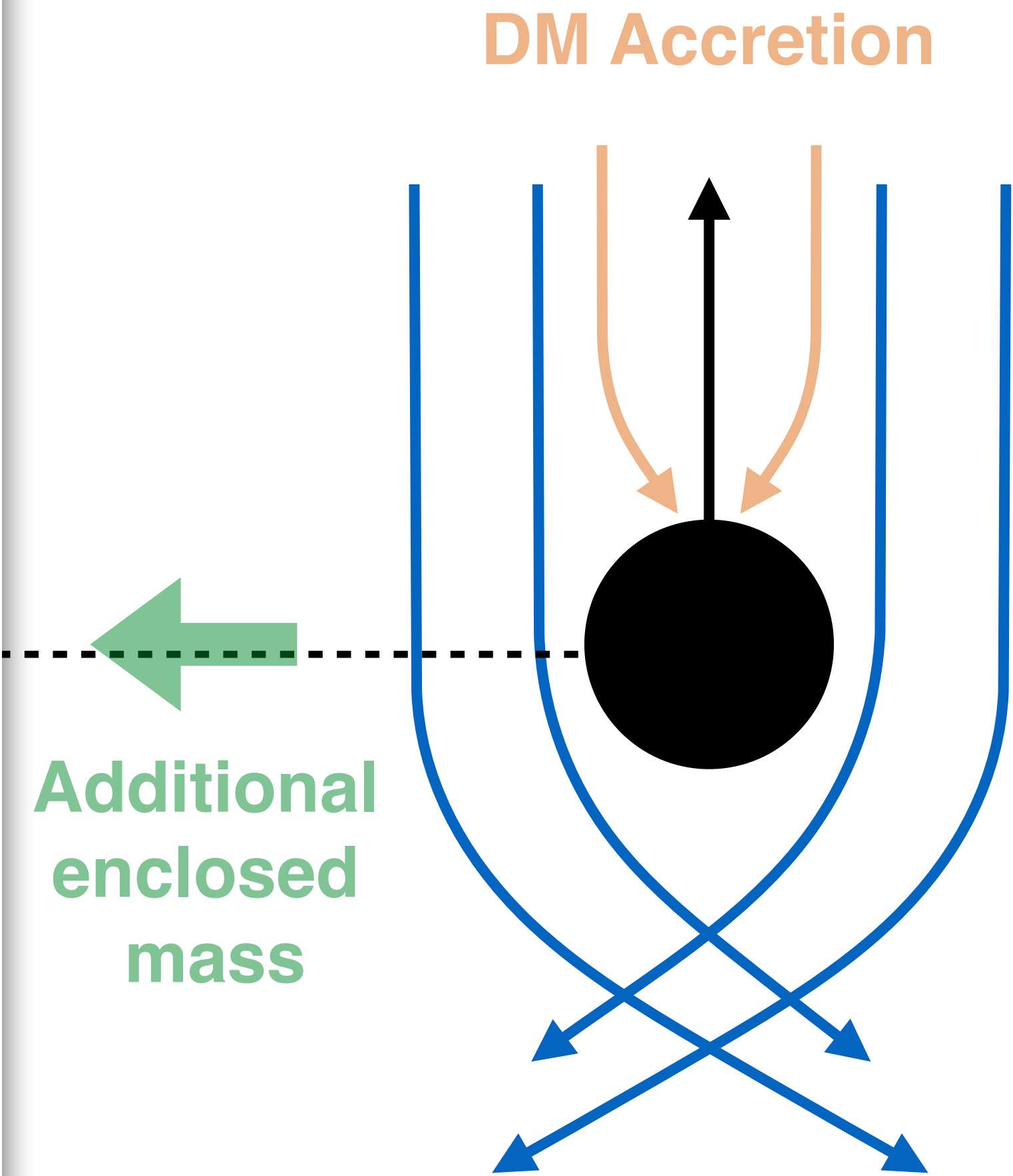
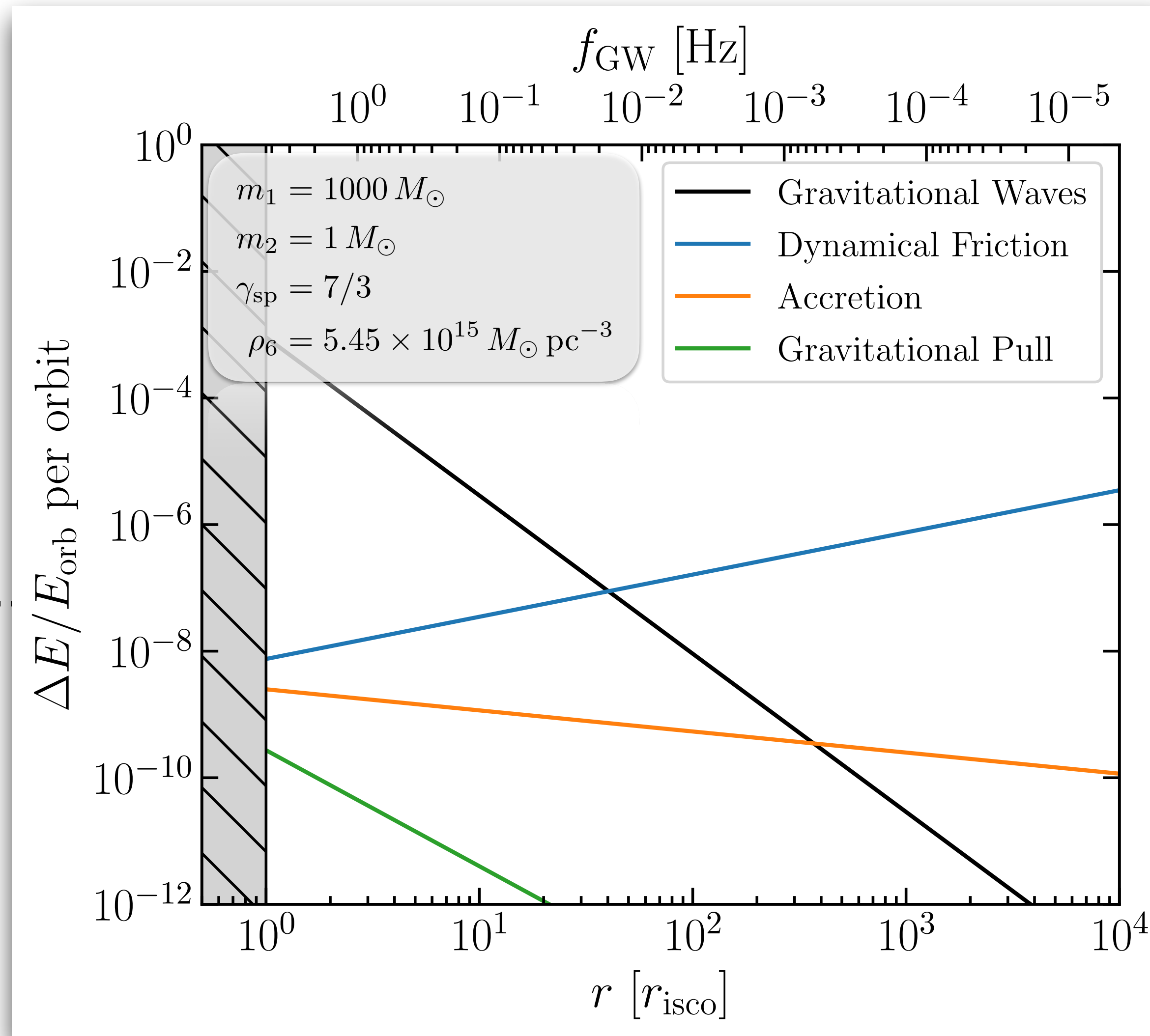
Impact of DM Spikes



Impact of DM Spikes



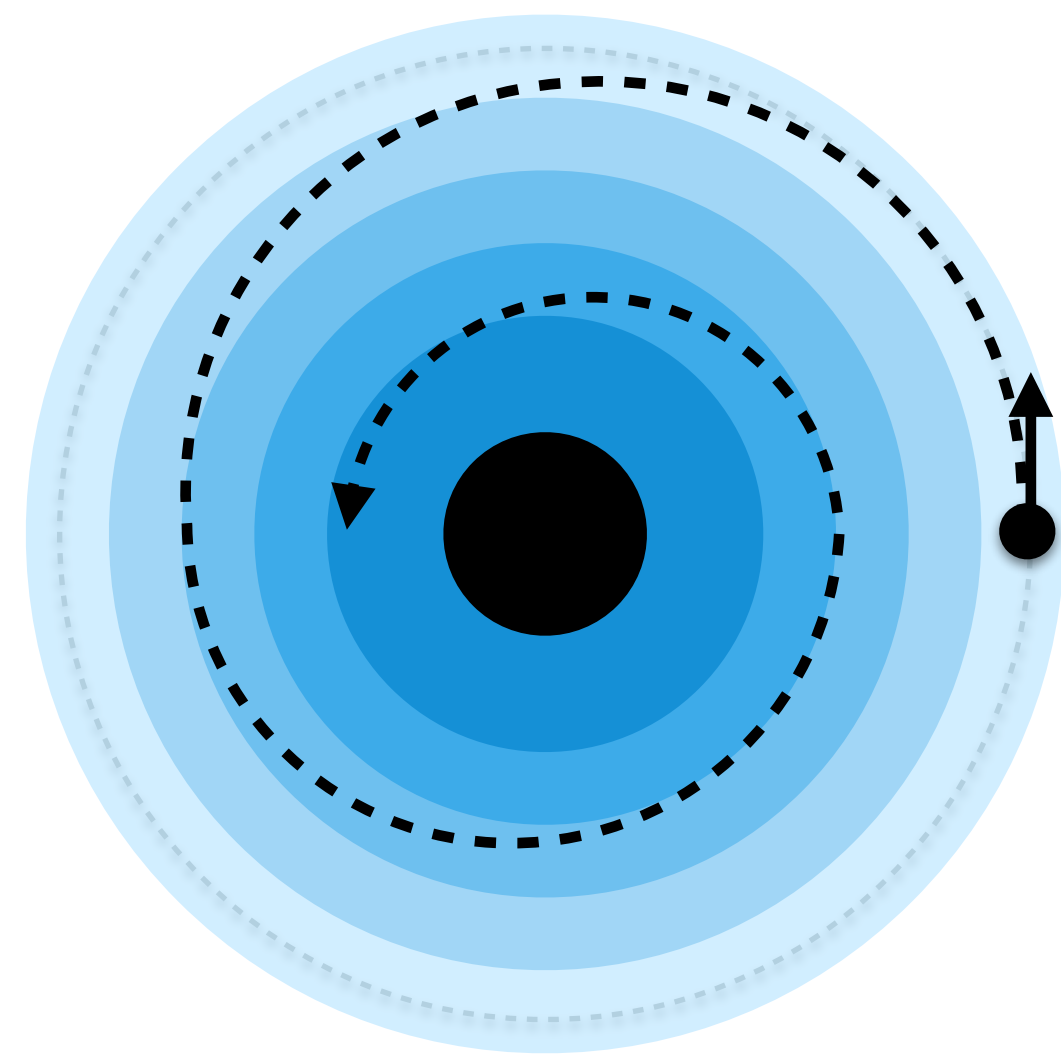
Impact of DM Spikes



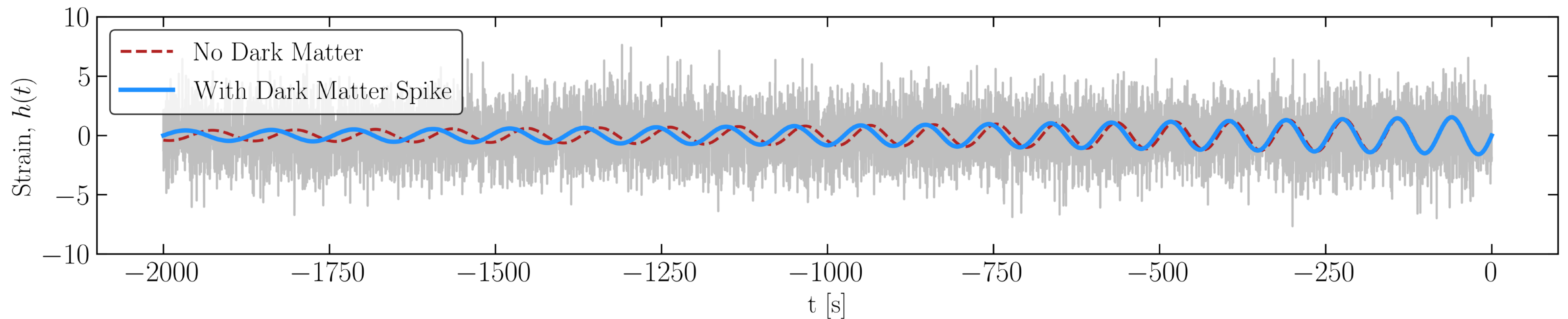
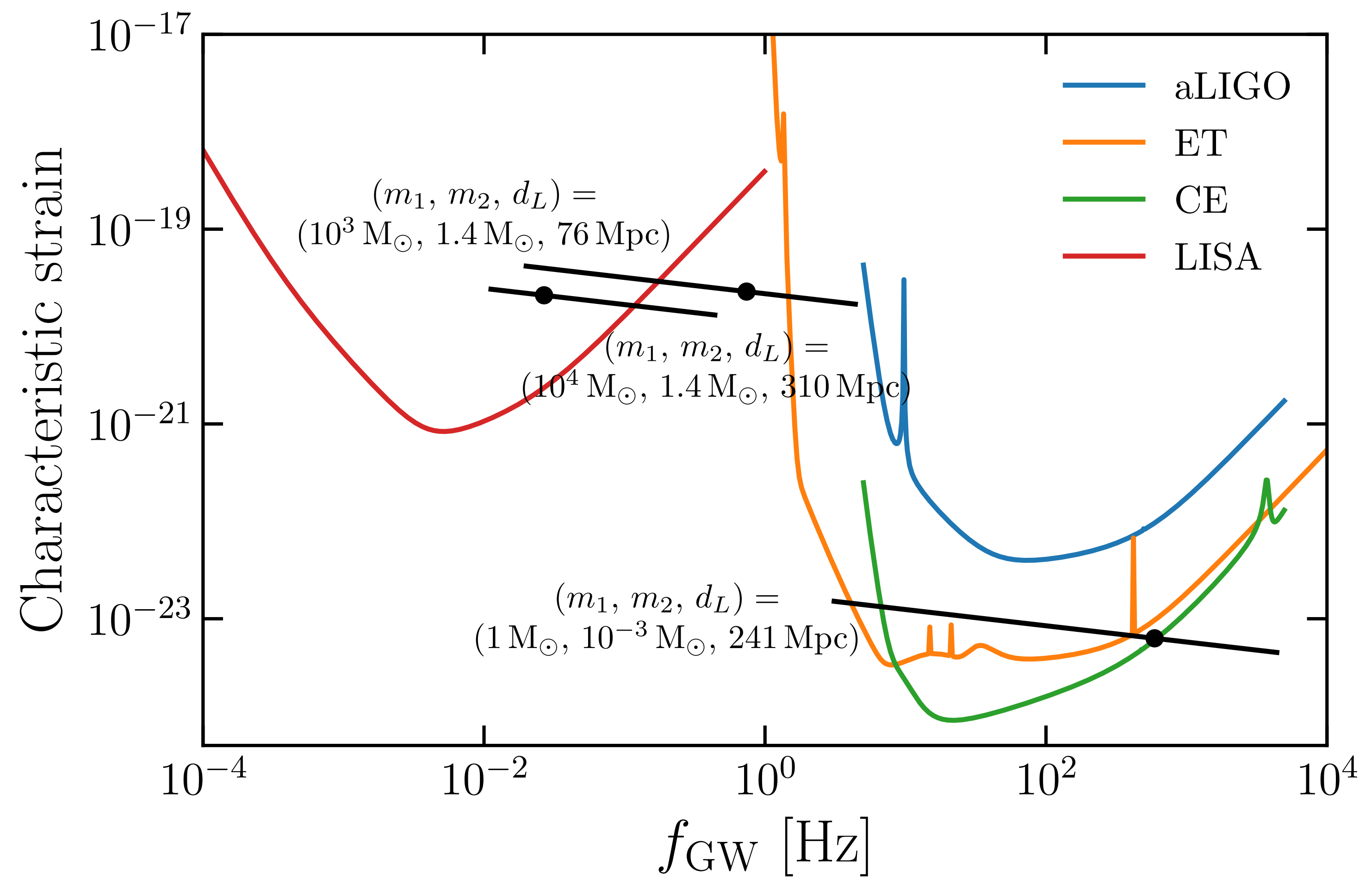
$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$

Dynamical Friction

Dephasing of IMRIs



$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$



Halo Feedback

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

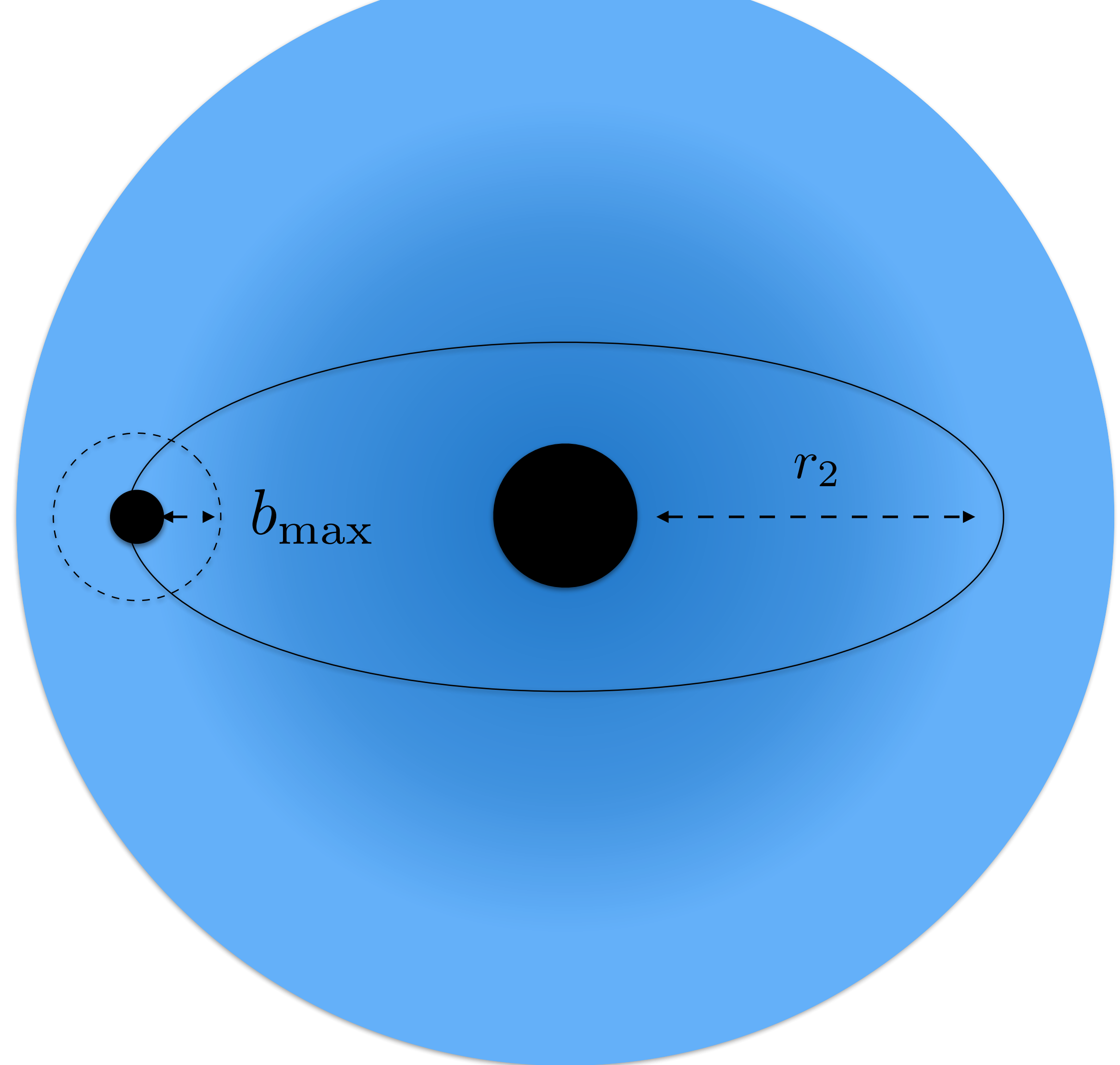
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a 'kick' through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$



Compact object scatters with all DM particles within 'torus' of influence over one orbit

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

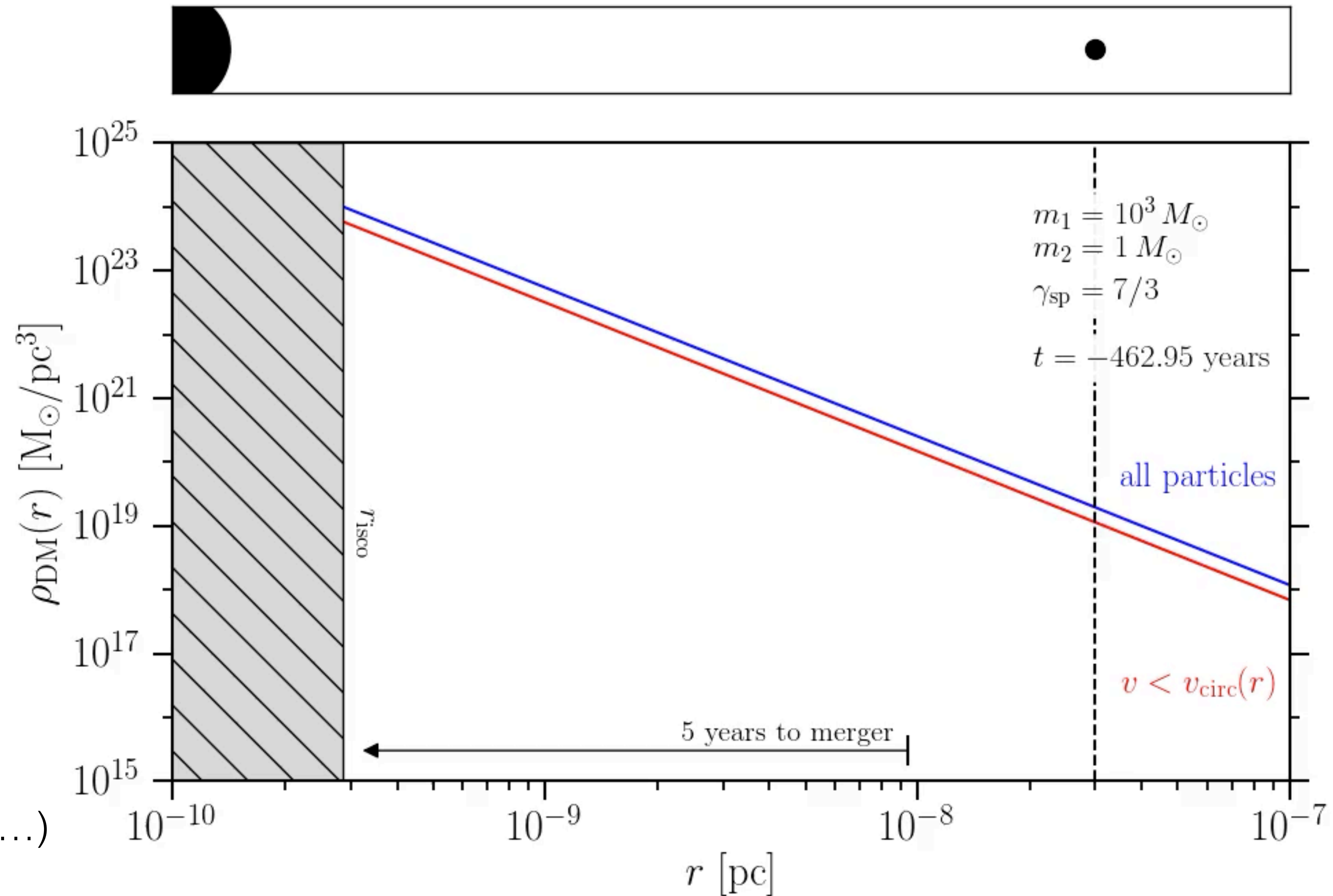
[Code available online:
github.com/bradkav/HaloFeedback]

Co-evolution

Newtonian motion of the binary, taking into account:

- GW emission
- Dynamical Friction
- **DM Halo Feedback**

Density of the DM spike is depleted (and replenished...)



This is one of the reasons we want to look at IMRIs/EMRIs...

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

[Movies: tinyurl.com/GW4DM]

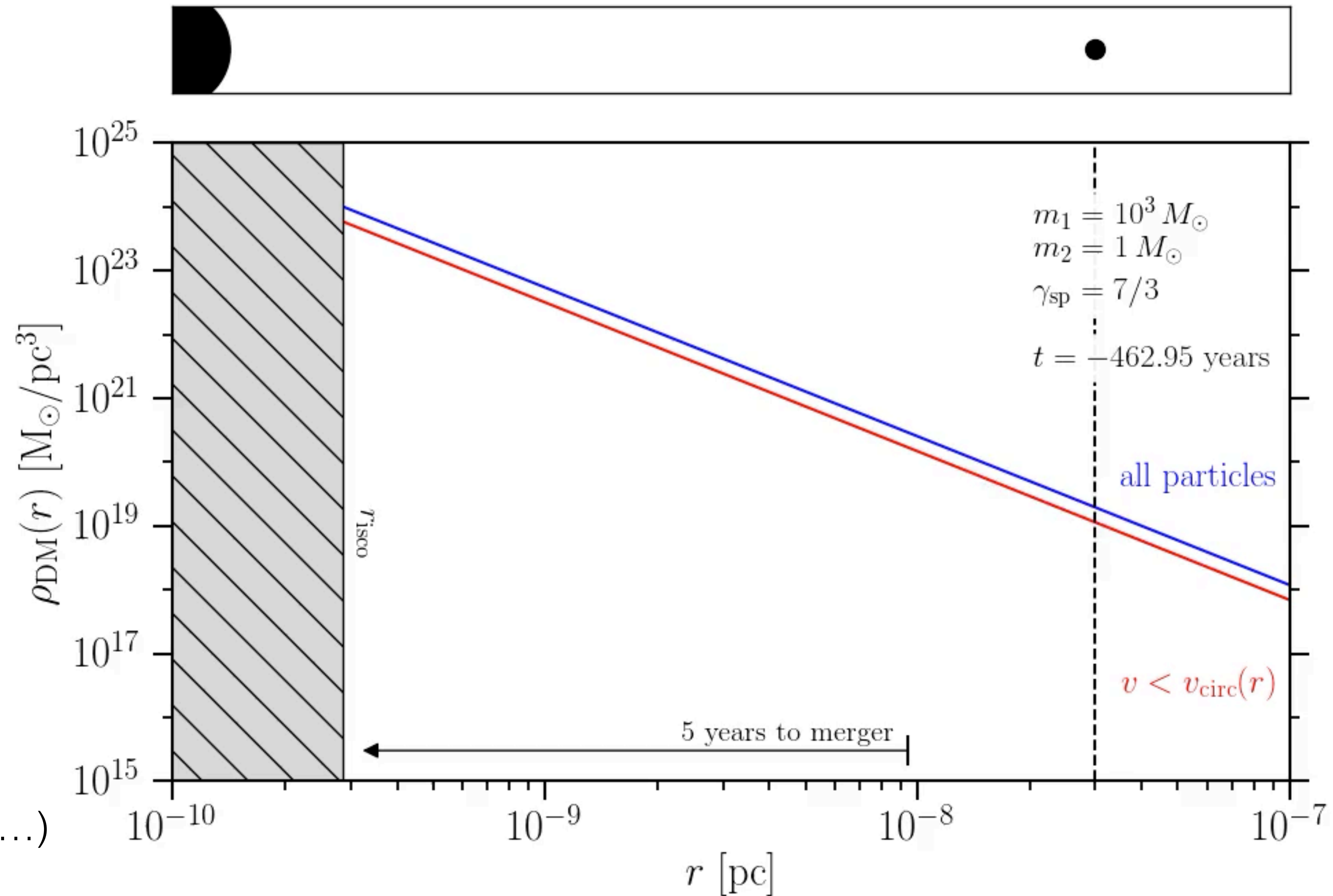
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[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

[Movies: tinyurl.com/GW4DM]

[Code: github.com/bradkav/HaloFeedback]

Self-consistent Dephasing

Consider our astro benchmark system, starting at some initial separation:

$$r_{\text{ini}} \sim 10^{-8} \text{ pc}$$



$$t_{\text{merge}}^{\text{vacuum}} \sim 5 \text{ yr}$$

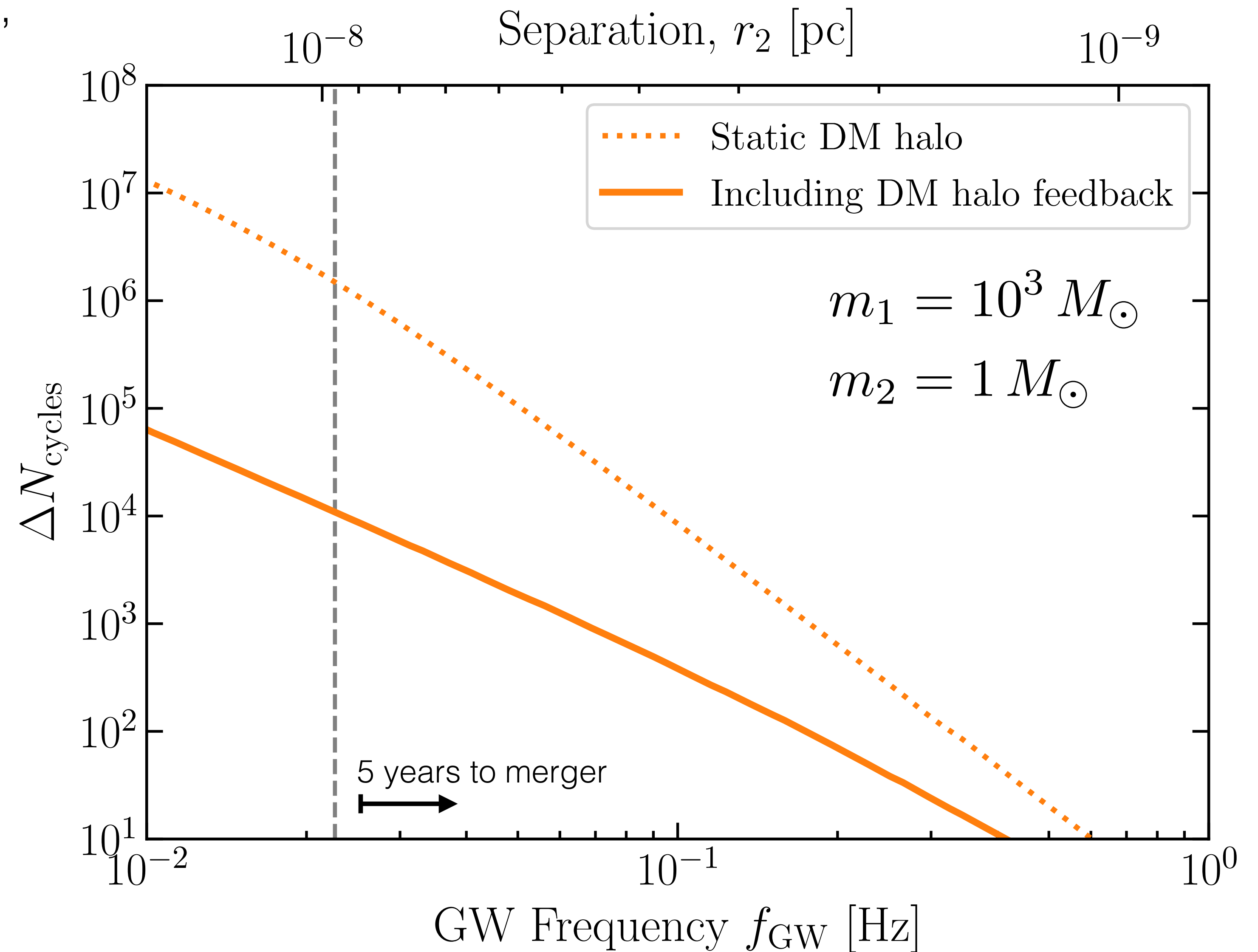
$$N_{\text{cycles}}^{\text{vacuum}} \sim 6 \times 10^6$$

DM dephasing



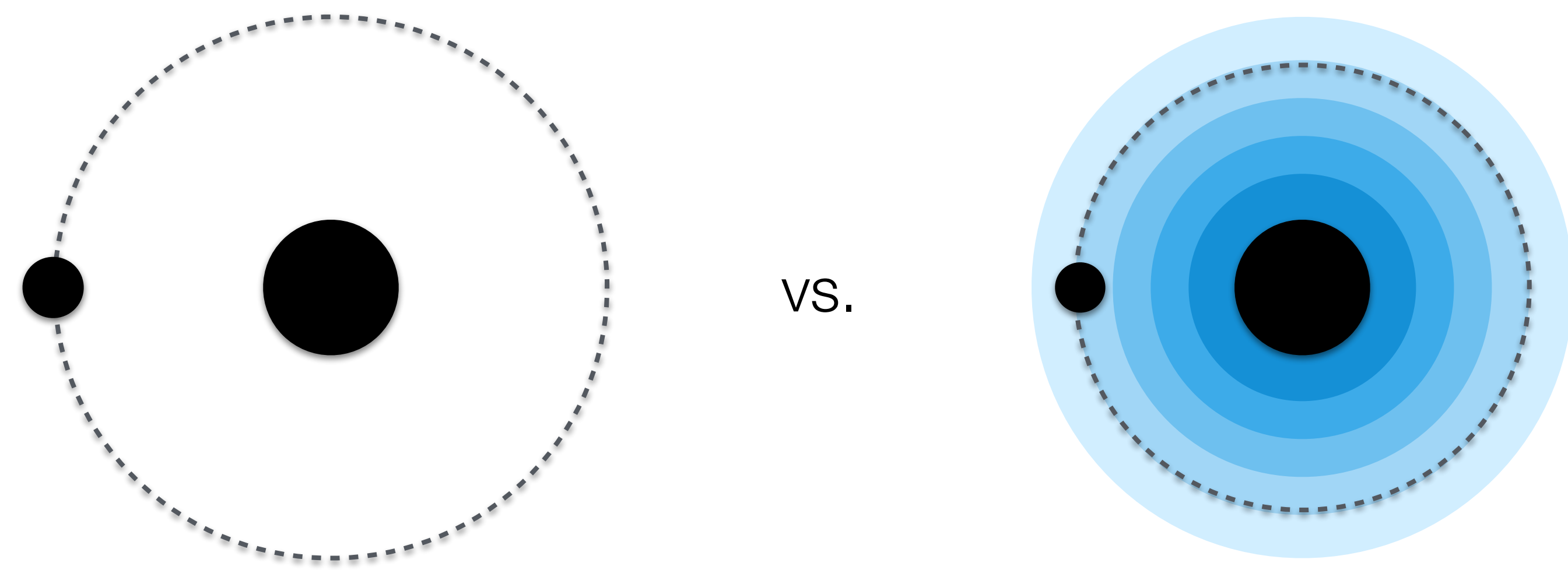
$$\Delta N_{\text{cycles}} \sim \mathcal{O}(10^4) \text{ cycles} \sim \% \text{ level}$$

Change in the number of GW cycles to merger, starting at some initial frequency/separation:

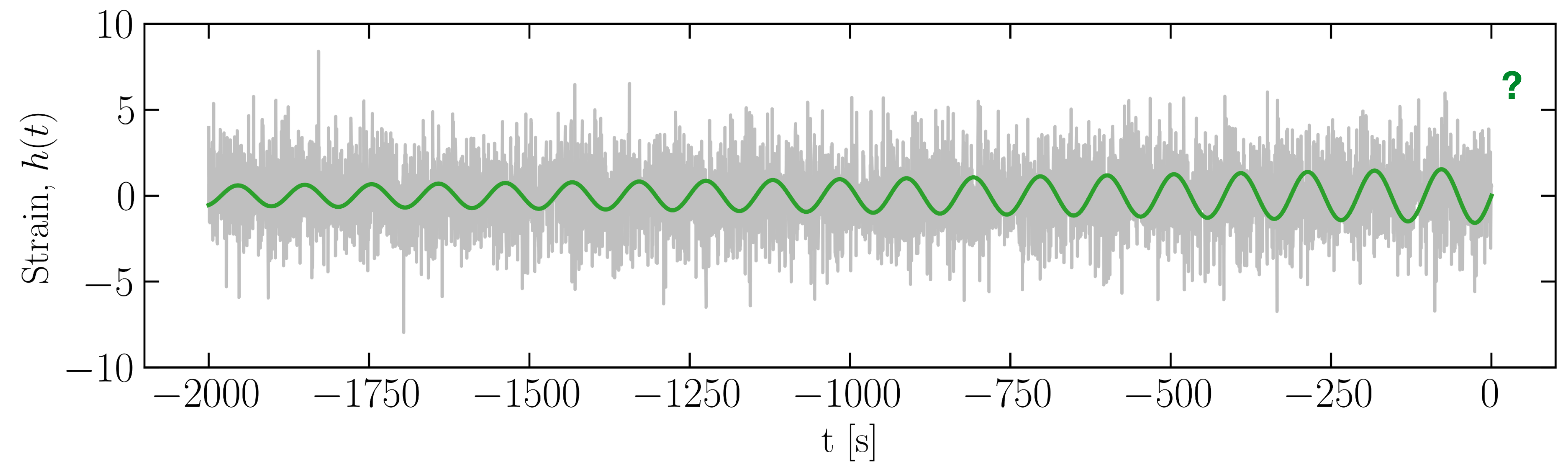


Detectability and more

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]



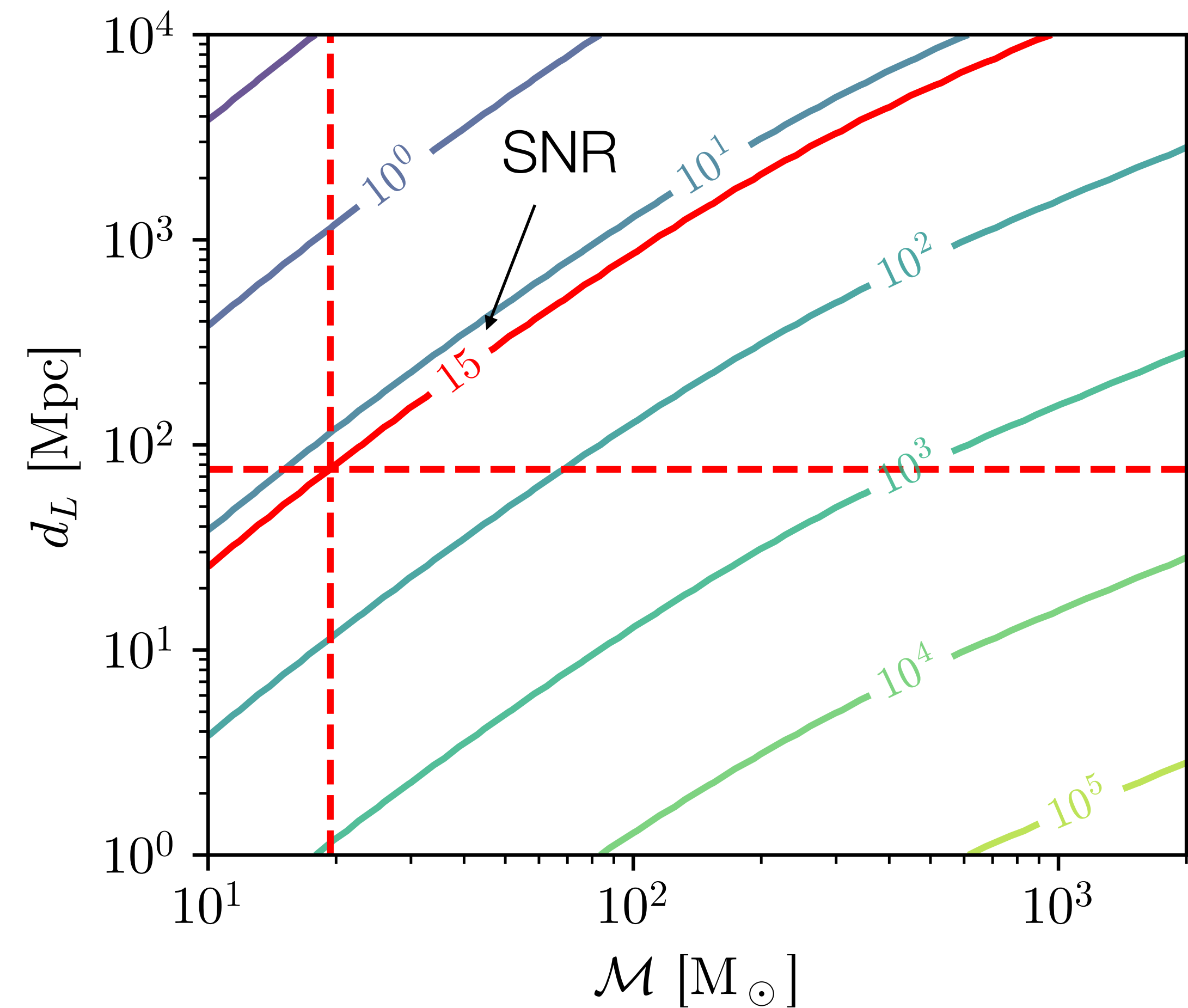
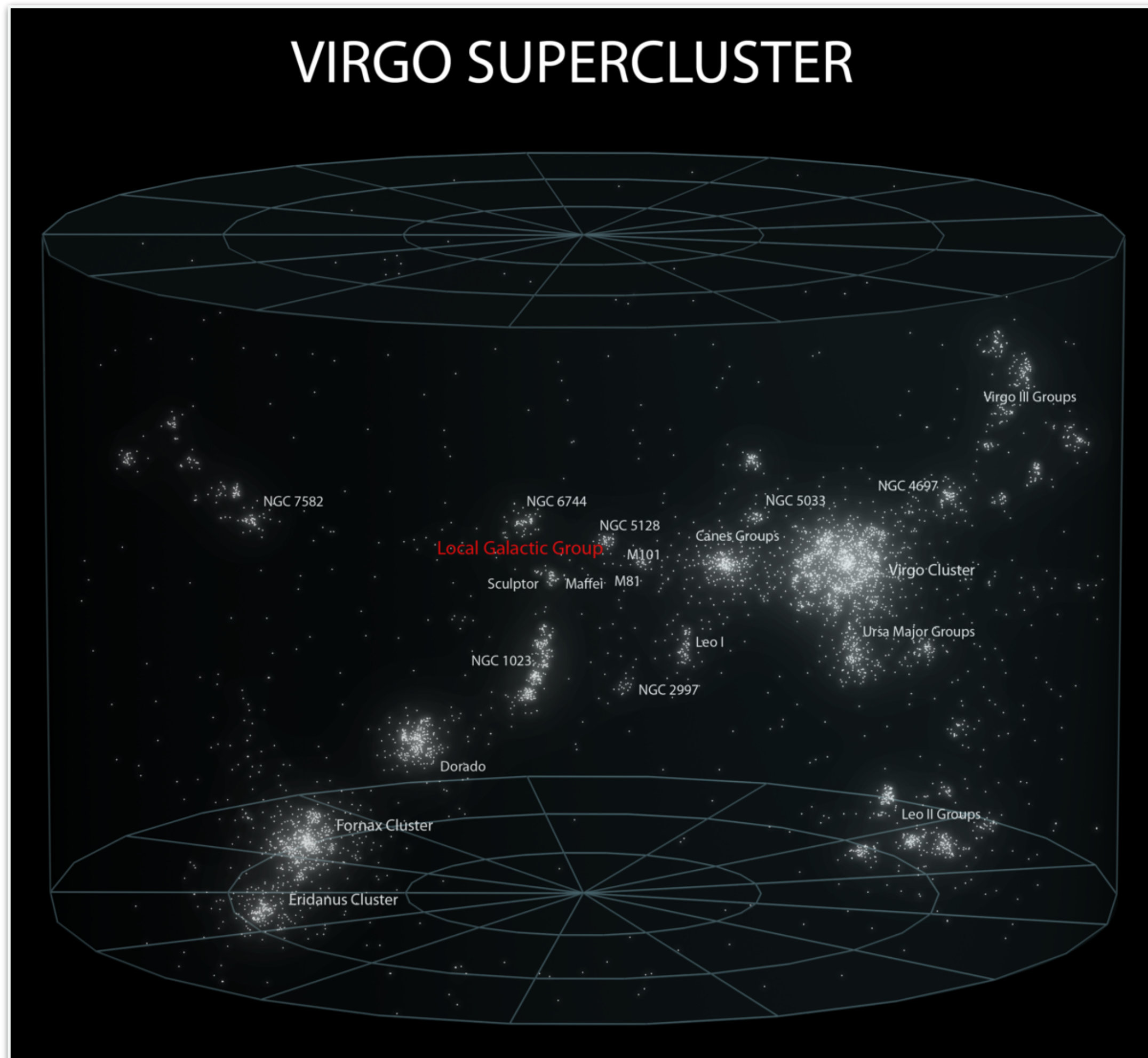
Detectability? Discoverability? Measurability?



Detectability

$$(m_1, m_2) = (10^3, 1) M_\odot$$

Assume a signal may be **detectable** with LISA using matched filtering with a signal-to-noise ratio (SNR) $\gtrsim 15$... [\[1905.11998\]](#)



[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]

Discoverability

$$(m_1, m_2) = (10^3, 1) M_\odot$$

$$q = m_2/m_1$$

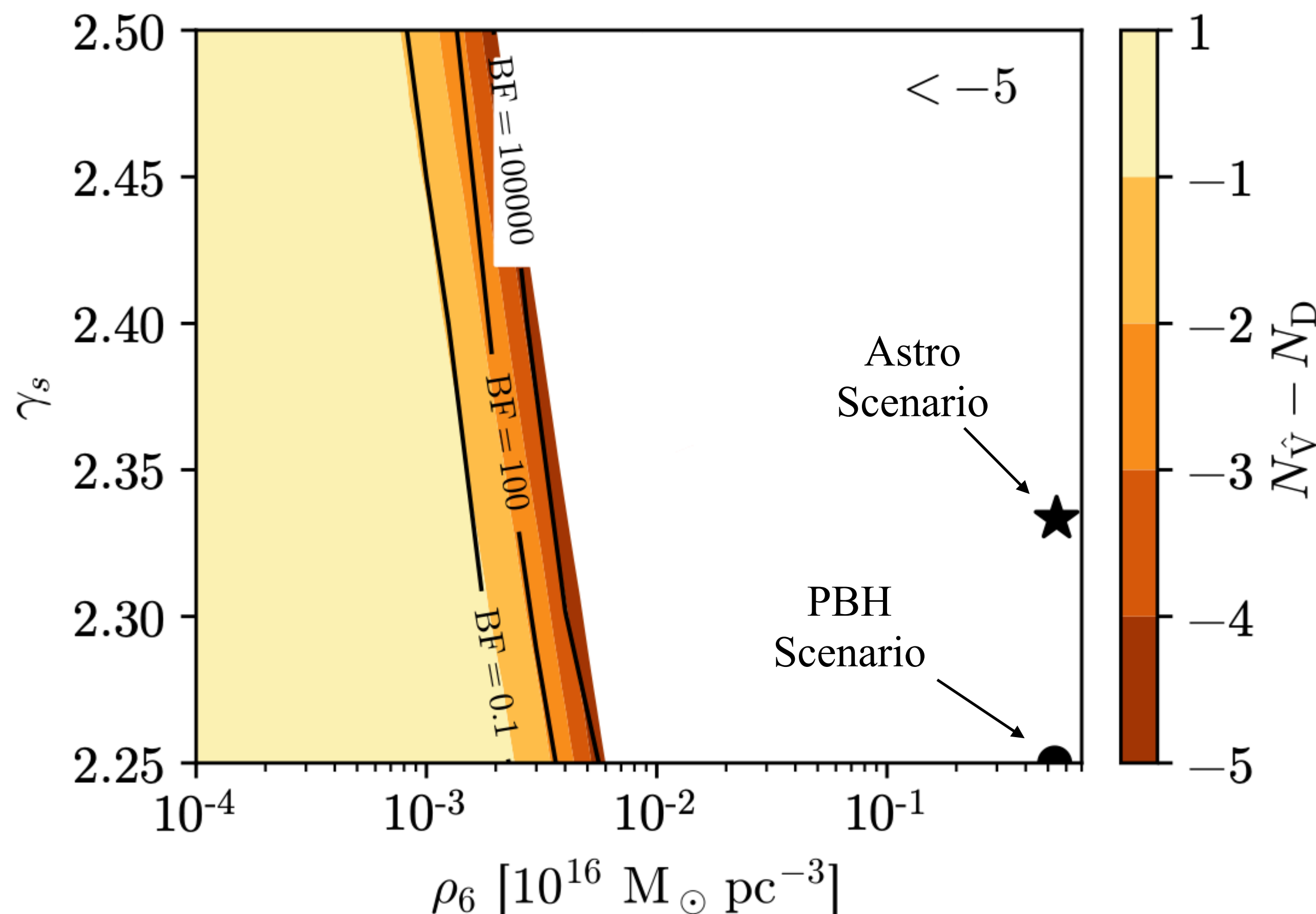
We'll call a DM spike **discoverable** if it can be distinguished from a GR-in-vacuum system.

Given the data d , compare Bayesian evidence $p(d)$ for **V**acuum and **D**ressed systems:

$$\theta_V = \{\mathcal{M}\} \quad \text{vs.} \quad \theta_D = \{\gamma_{\text{sp}}, \rho_6, \mathcal{M}, \log_{10} q\}$$

(maximising over extrinsic variables

$$\theta_{\text{ext}} \equiv \{D_L, \phi_c, \tilde{t}_c\})$$



$$p(d) = \int d\theta \mathcal{L}(\theta) p(\theta)$$

Likelihood

Prior

$$\text{BF}(d) \equiv \frac{p(d|D)}{p(d|V)}$$

Use an approximate waveform parametrisation in terms of θ_D

[Code available online:

<https://github.com/adam-coogan/pydd>]

[Coogan, Bertone, Gaggero, **BJK** & Nichols, 2108.04154]

Measurability

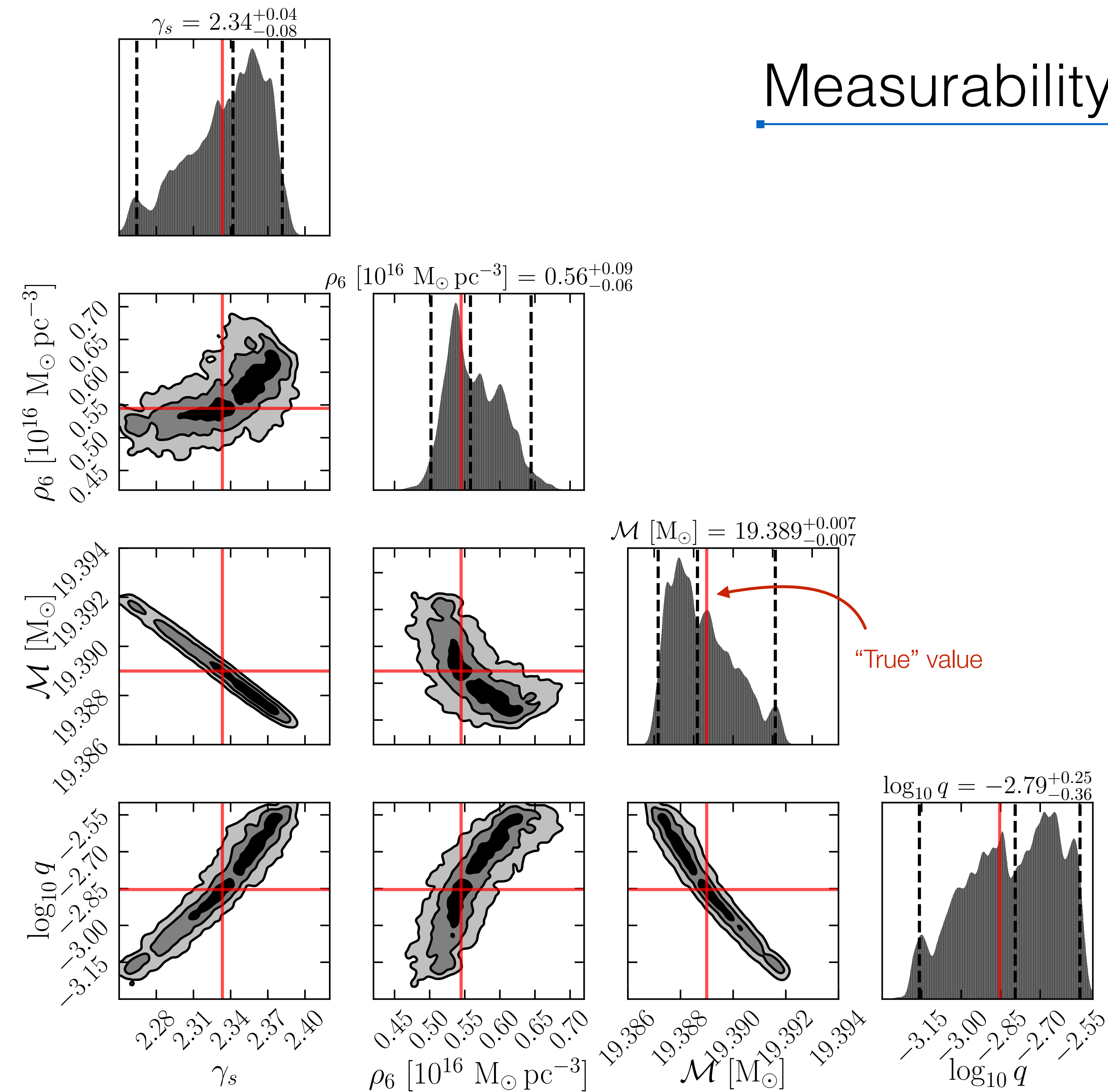
Astrophysical scenario

$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

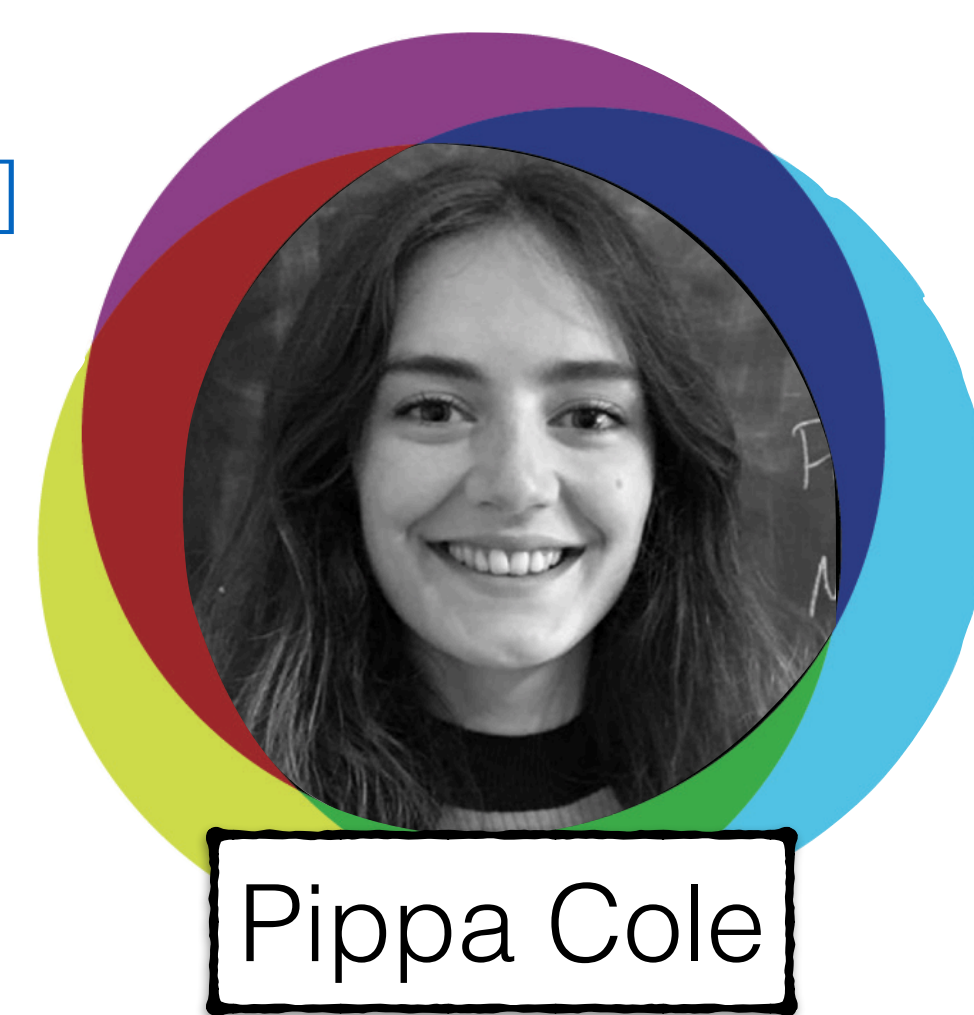
$$\gamma_{\text{sp}} = 7/3 \approx 2.3333 \dots$$

$$\rho_6 \approx 5.45 \times 10^{15} M_\odot \text{pc}^{-3}$$



Dressed PBHs

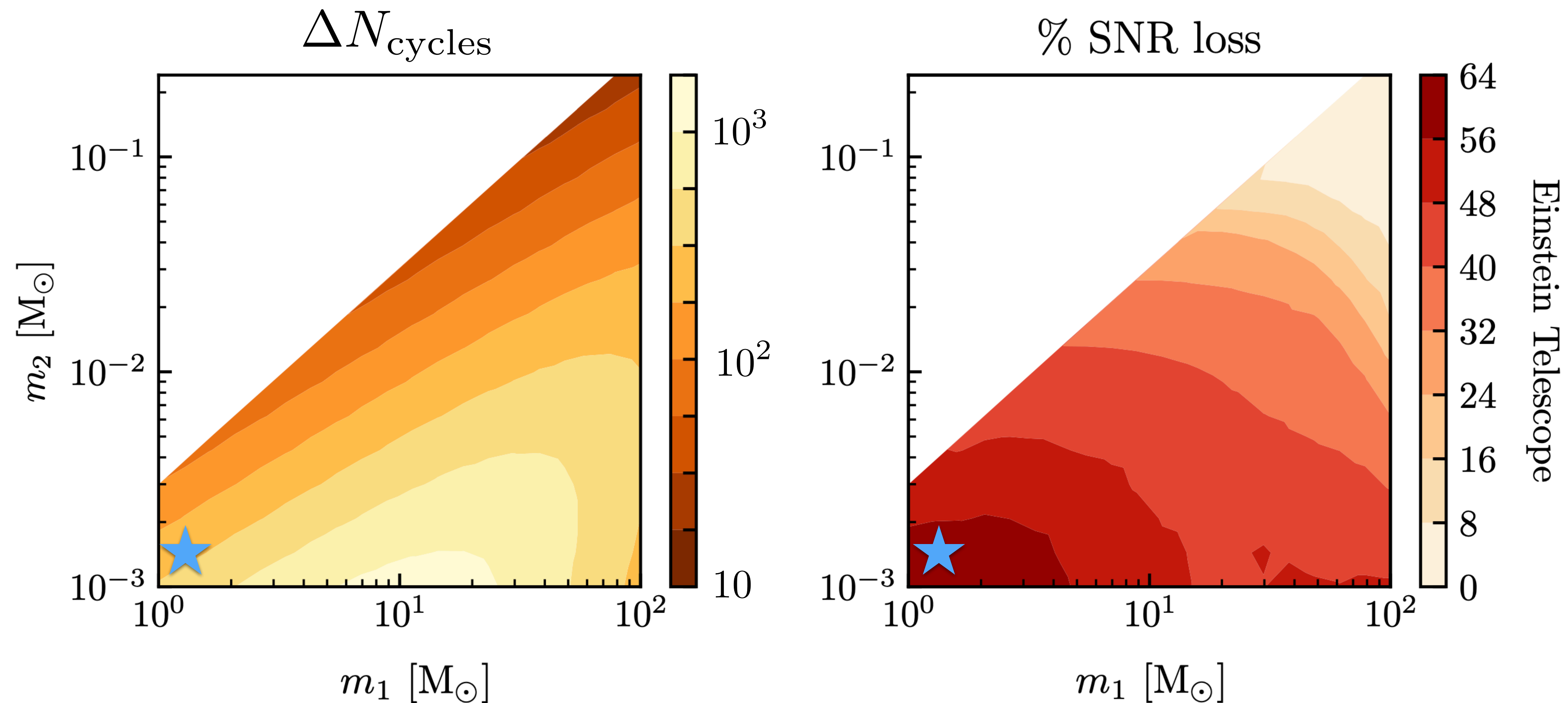
[Cole, Coogan, **BJK** & Bertone, [2207.07576](#)]



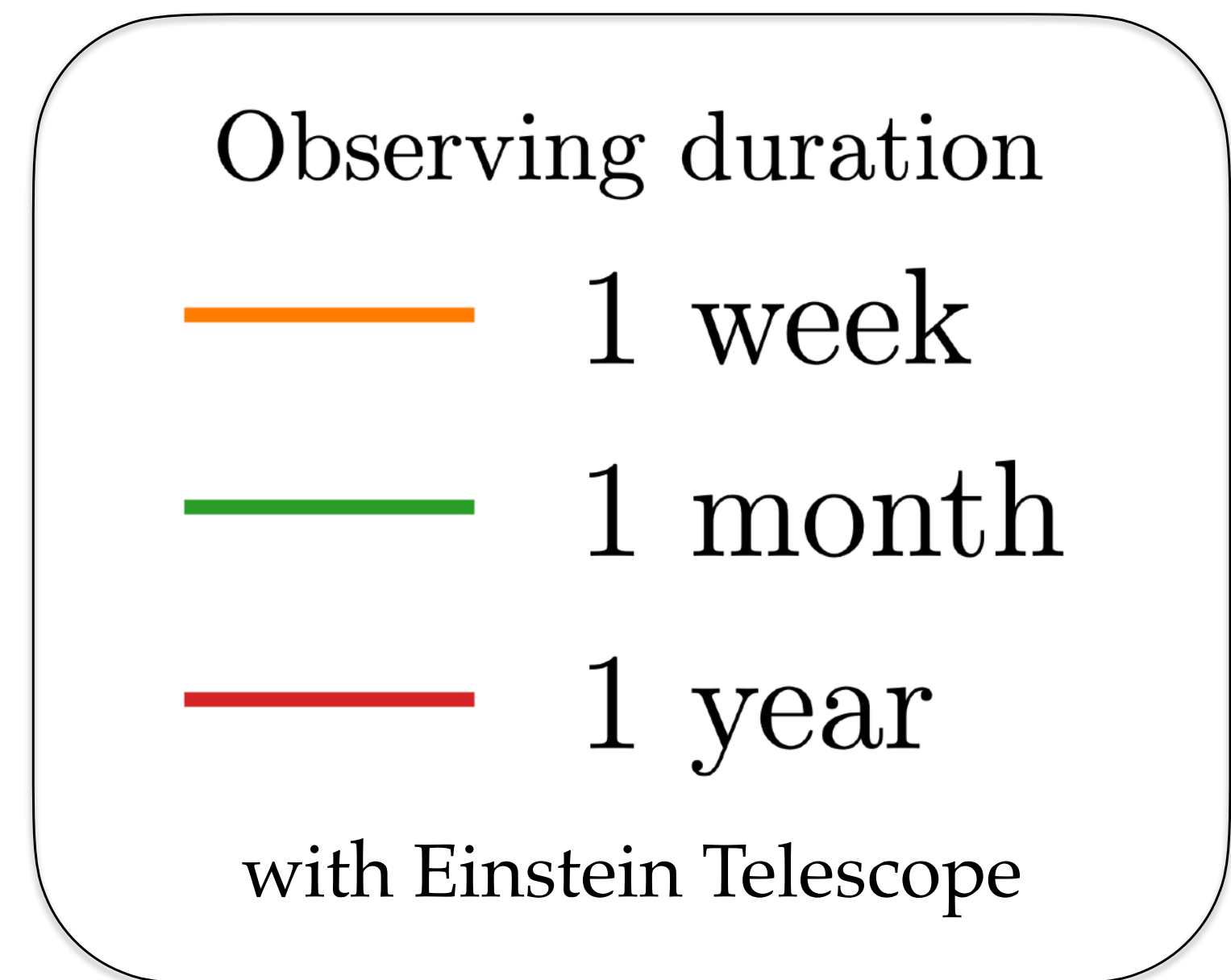
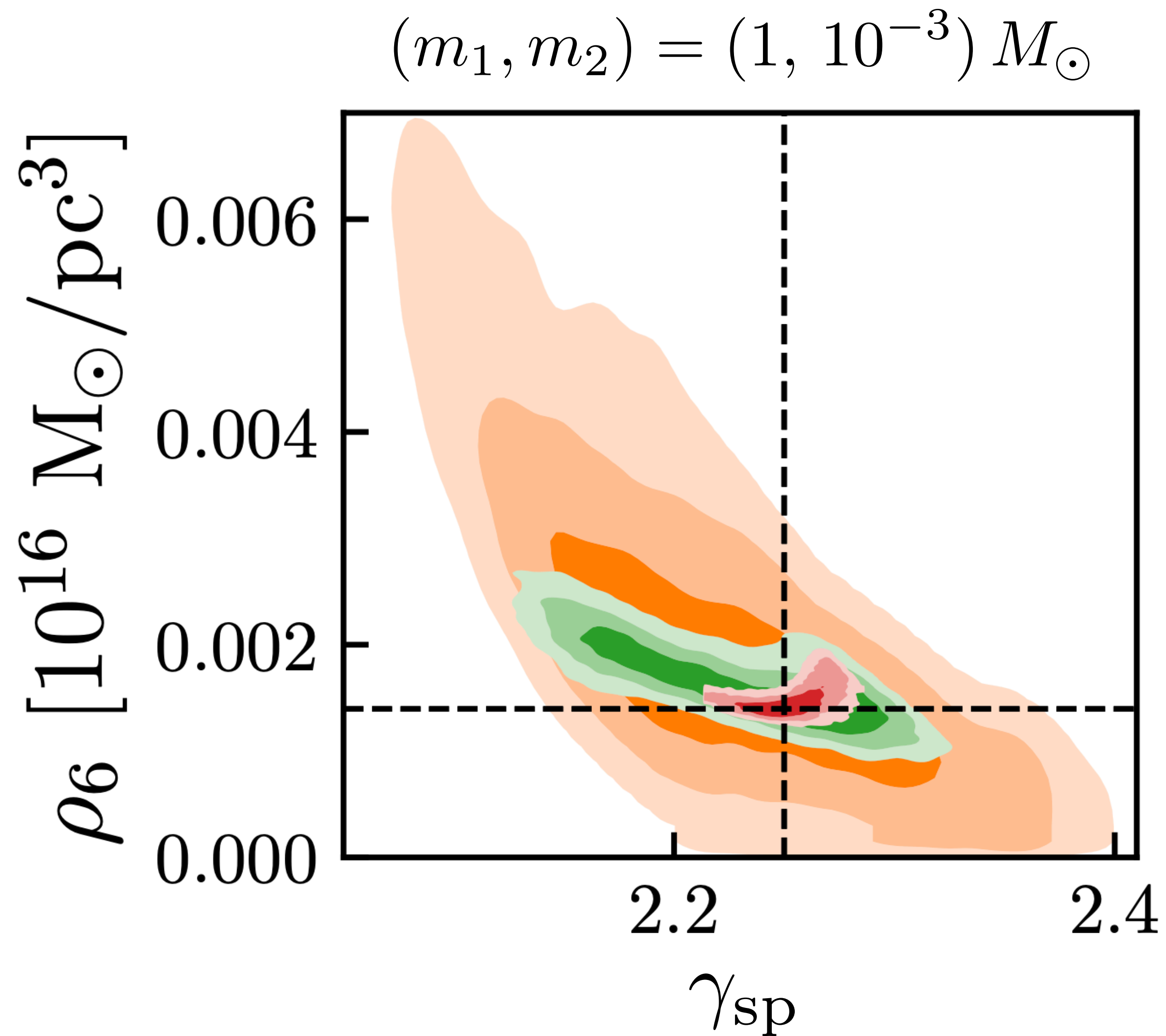
Consider now a binary of sub-solar mass PBHs, observed with future ground-based detectors:

★ $(m_1, m_2) = (1, 10^{-3}) M_\odot$

In vacuum, expect $\sim 10^7$ GW cycles during a one year observation.
But with a DM spike:



Spike properties

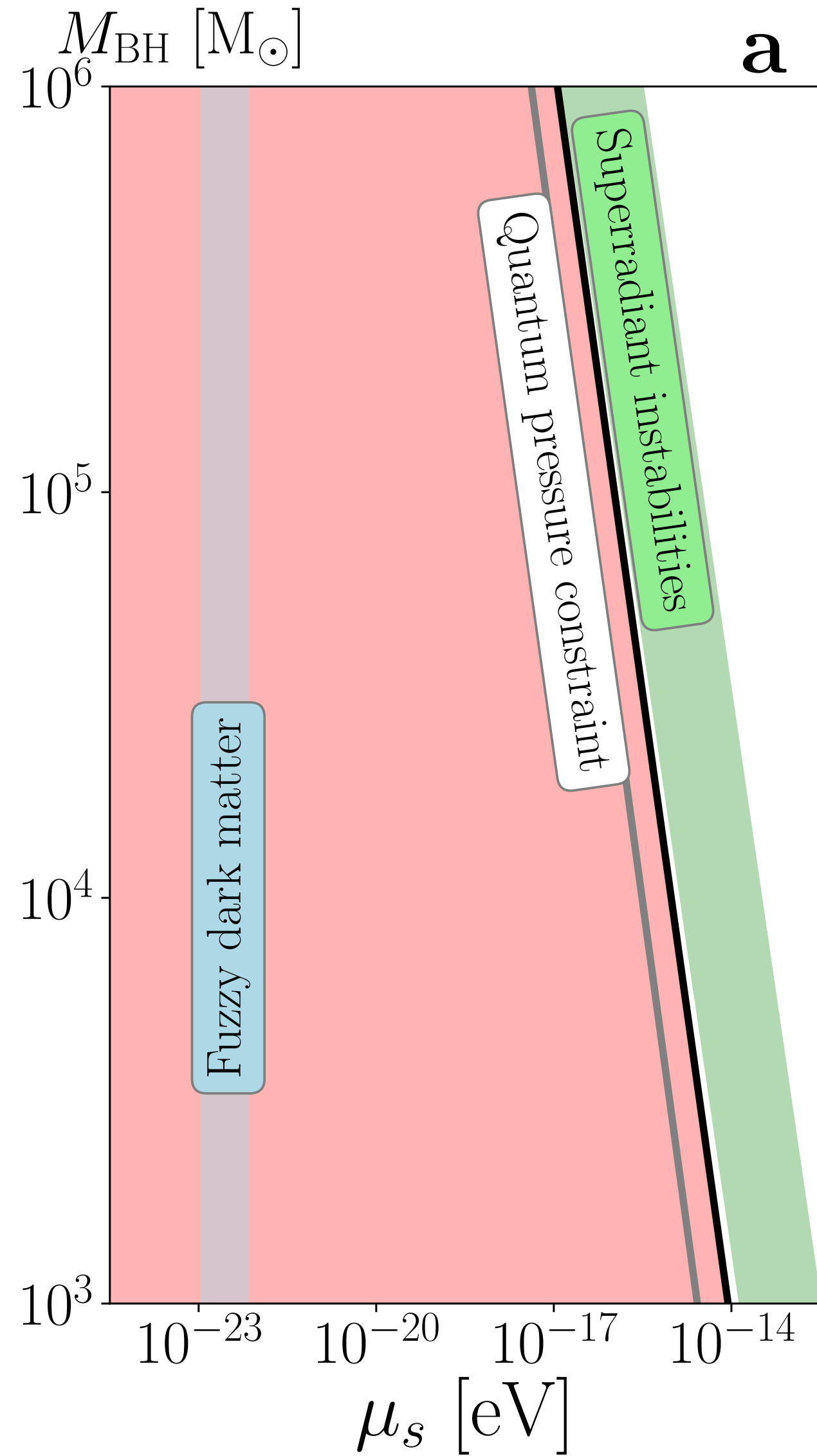


$$\rho_{\text{DM}}(r) = \rho_6 \left(\frac{r}{10^{-6} \text{ pc}} \right)^{-\gamma_{\text{sp}}}$$

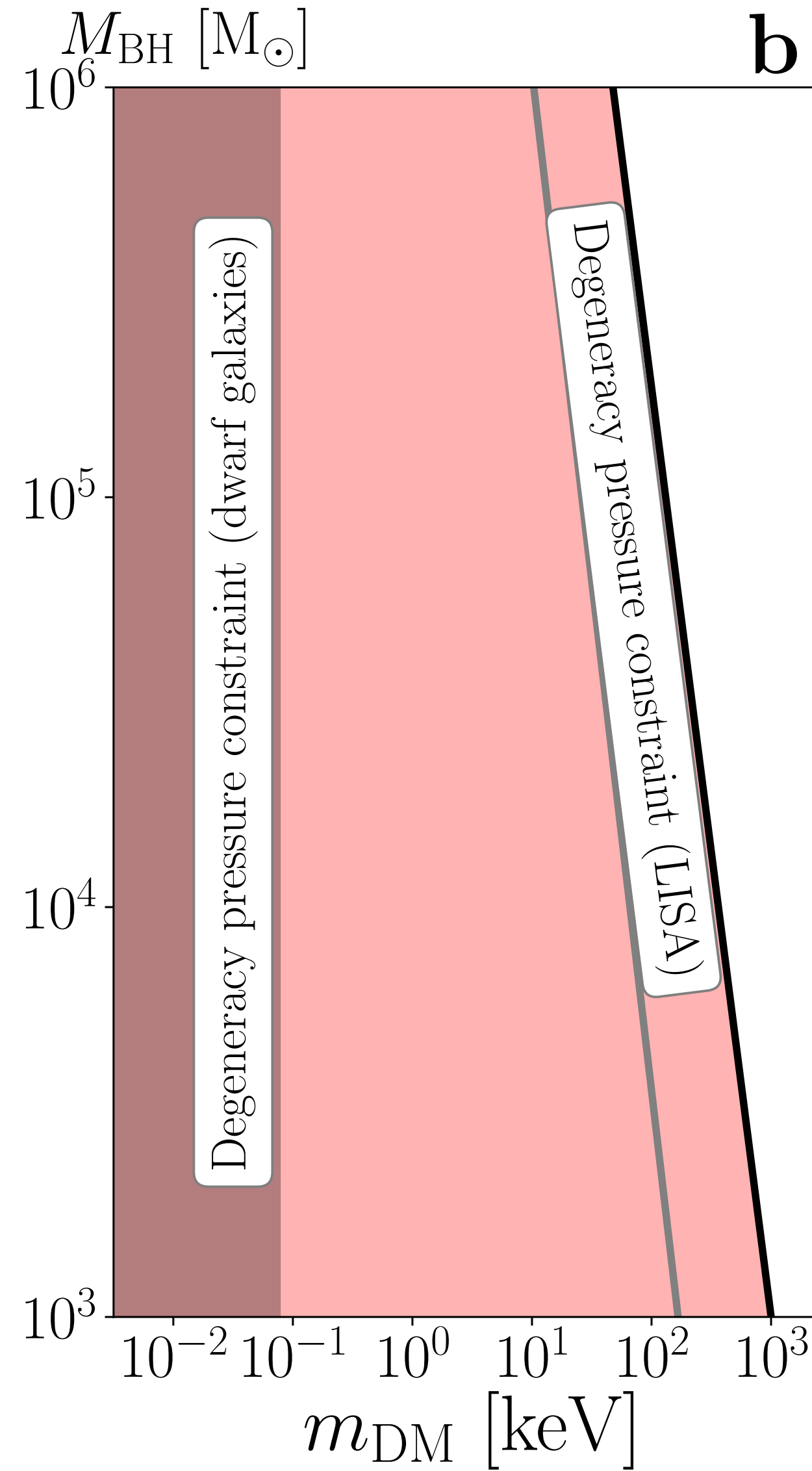
Nature of Dark Matter

Red regions would be ruled out by observation of a DM spike!

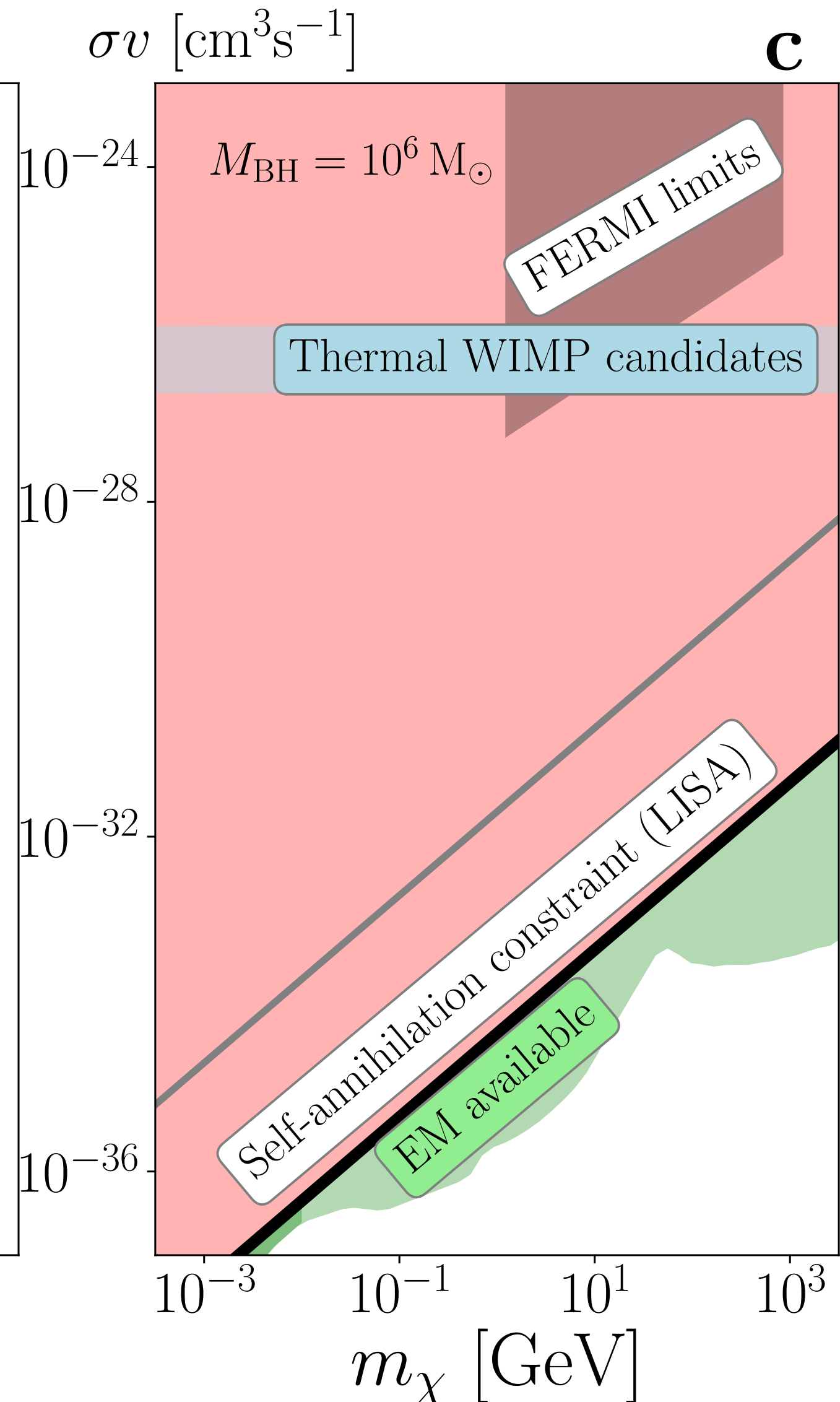
Ultralight bosonic DM



Fermionic DM



Self-annihilating DM



[Hannuksela, Ng & Li, [1906.11845](#)]

[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, [1905.01238](#)]

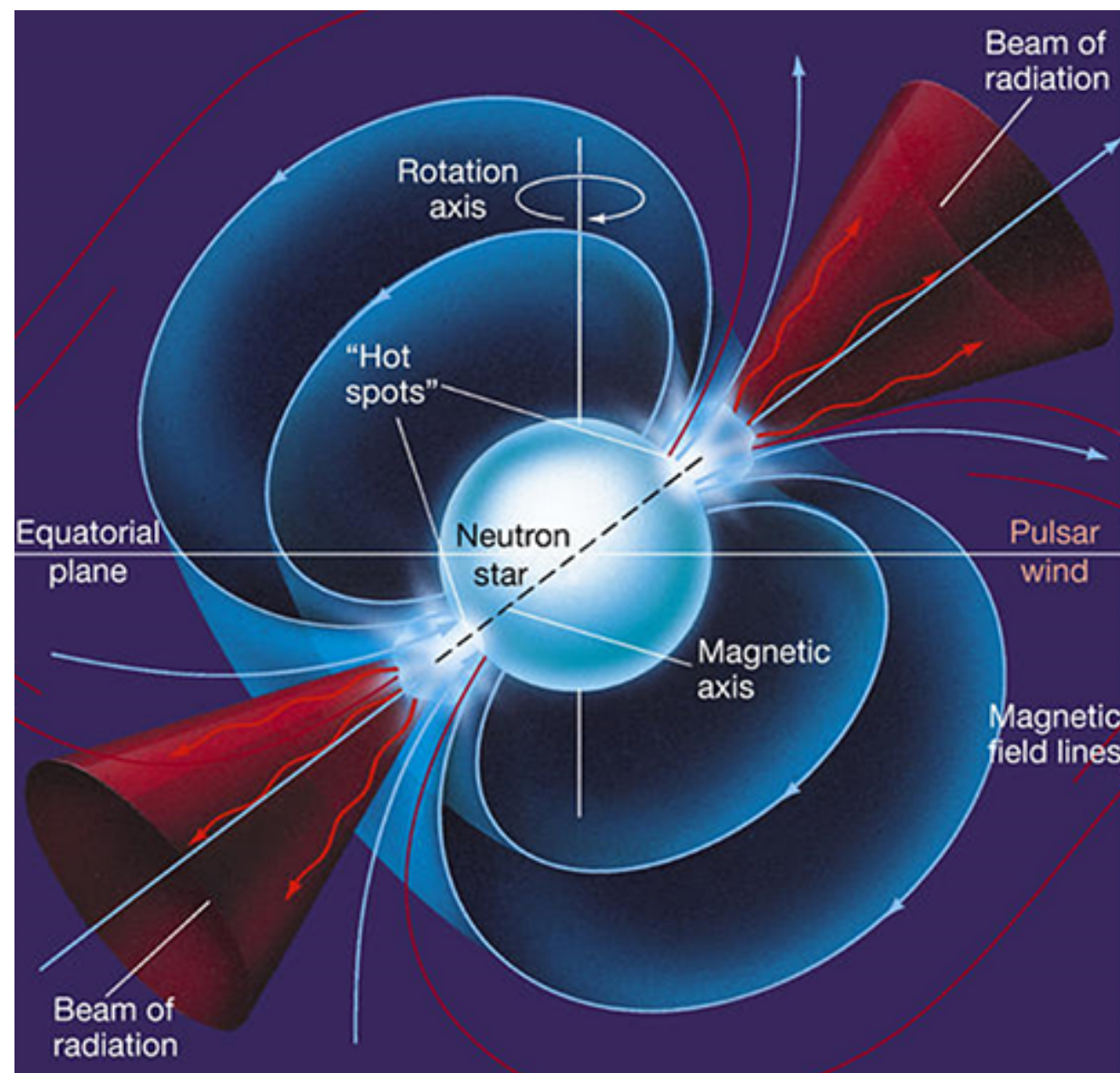
Multimessenger QCD Axions

Consider a DM spike consisting of QCD axions ($m_a \in [10^{-7}, 10^{-5}] \text{ eV}$):

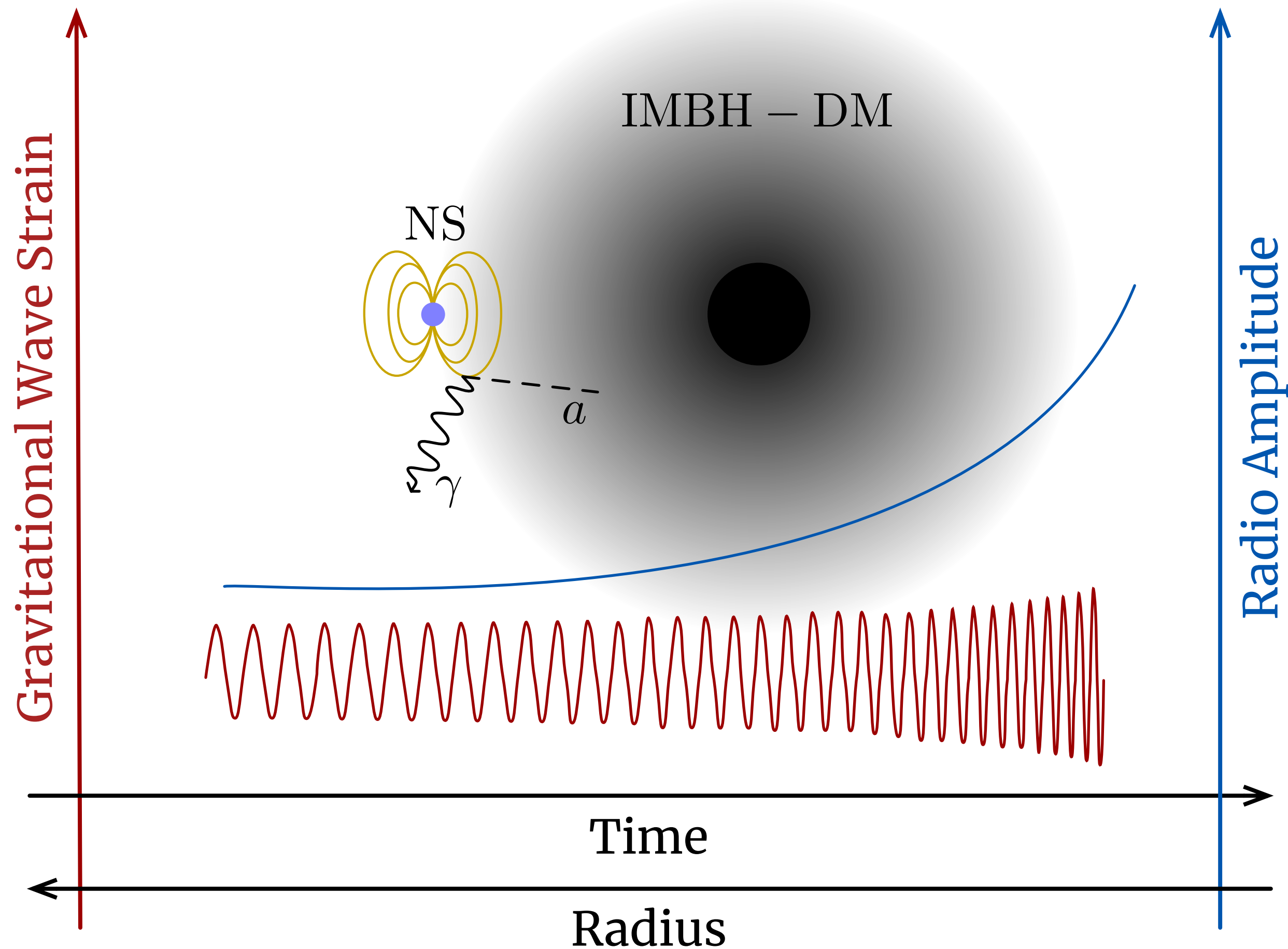
[Edwards, Chianese, **BJK**, Nissanke & Weniger, [1905.04686](#)]



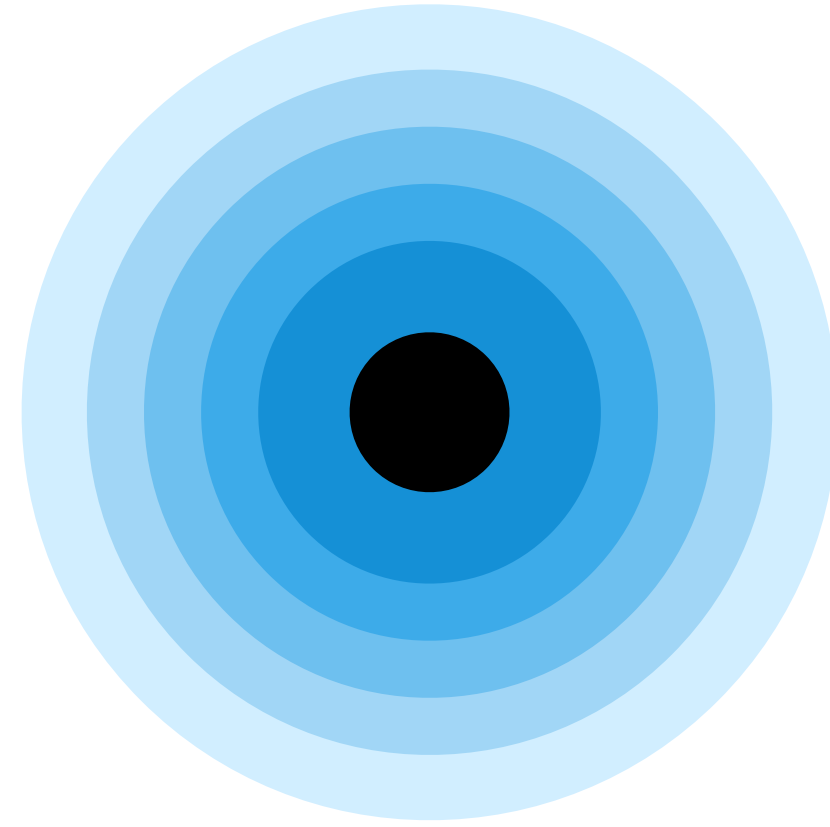
© 2005 Pearson Prentice Hall, Inc



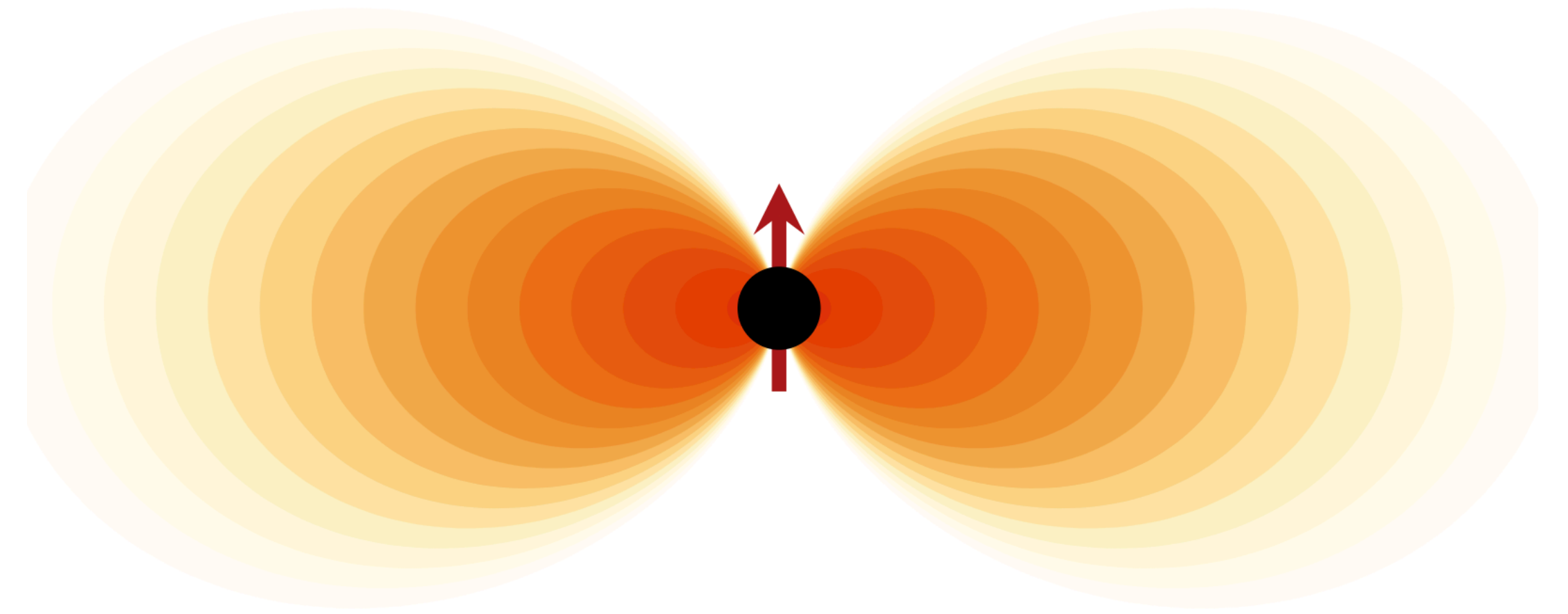
[See also [1803.08230](#), [1804.03145](#), [1811.01020](#), [1910.11907](#)]



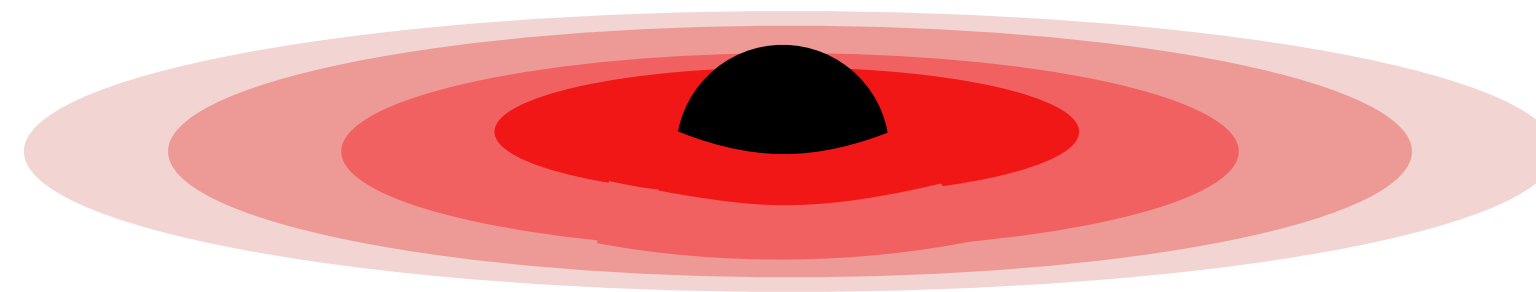
Black Hole Environments



Particle Dark Matter
'**Spikes**' or '**Dresses**'



'**Gravitational Atoms**' of
Ultralight Bosons



Baryonic
Accretion Disks

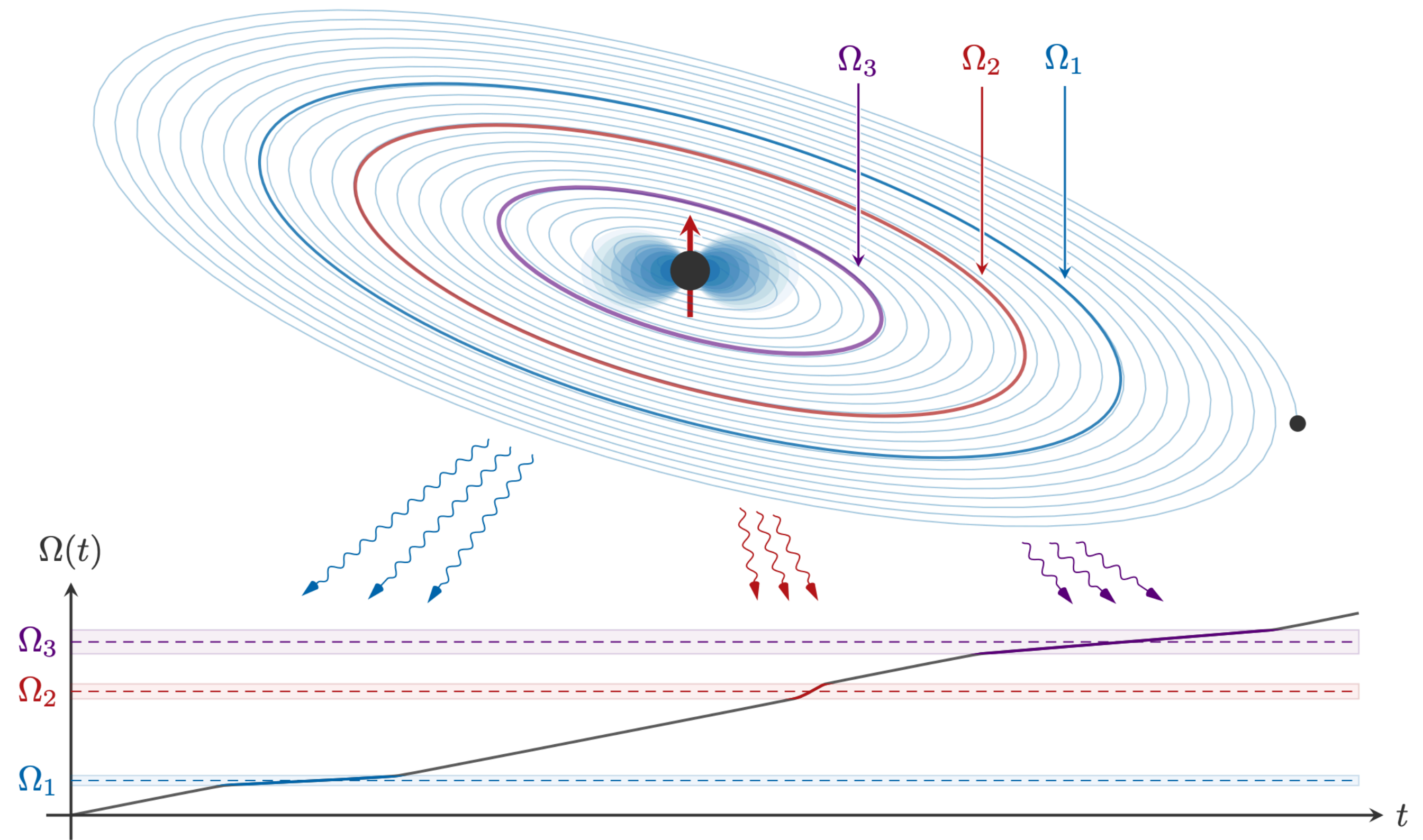
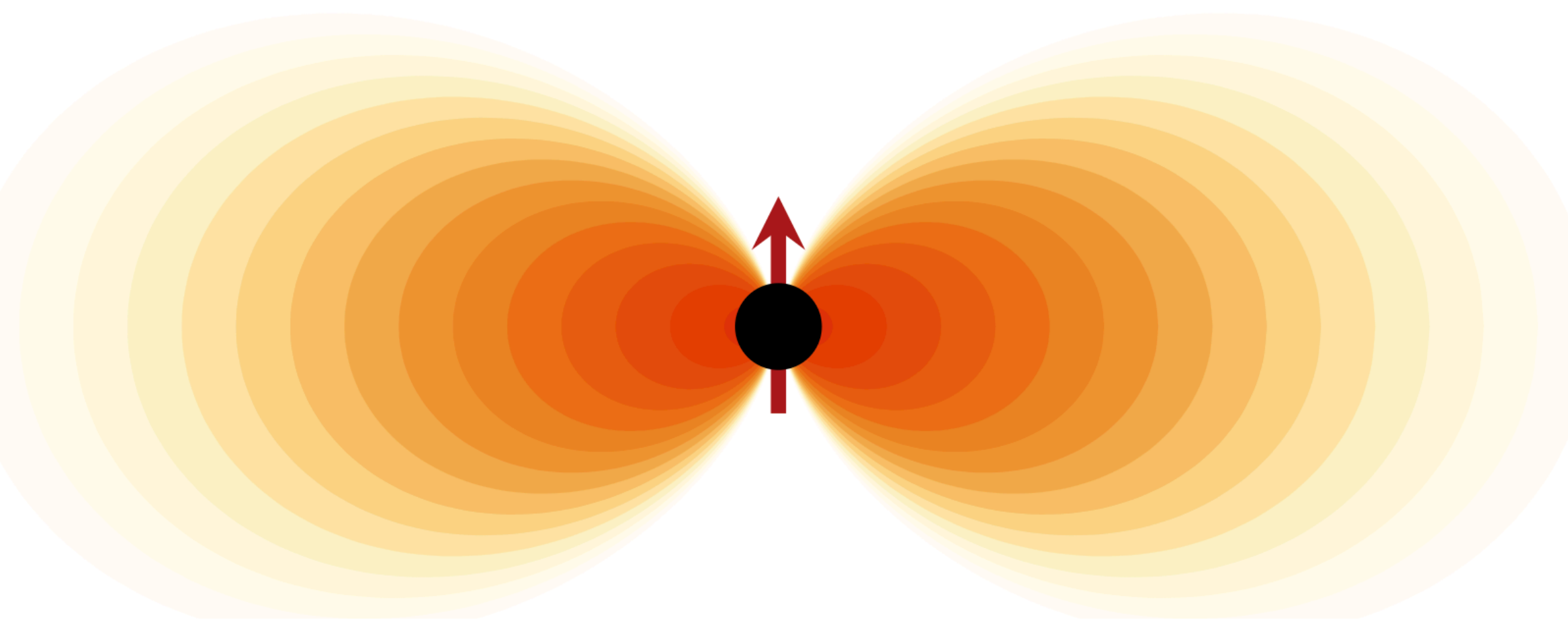
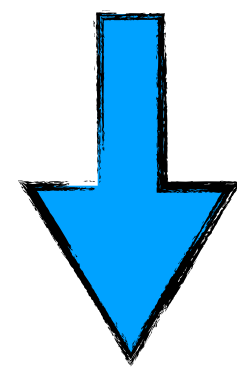
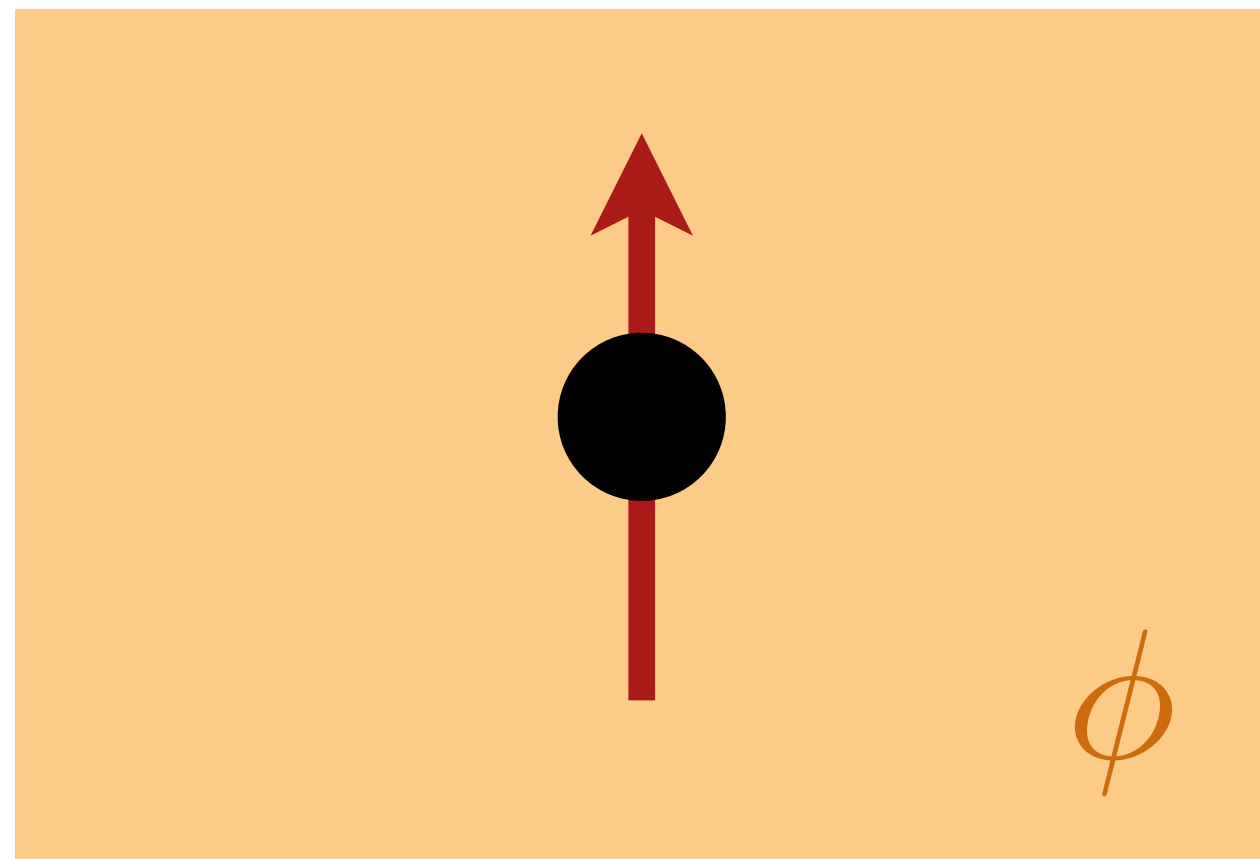
Black Holes (BHs) are extreme environments allowing us to probe high densities, long timescales...

Gravitational Atoms

[Baumann, Bertone, Stout & Tomaselli, [2206.01212](#), [2112.14777](#);
Tomaselli, Spieksma & Bertone, [2305.15460](#)]



Gimmy Tomaselli



[See also Baumann et al., [1804.03208](#), [1908.10370](#), [1912.04932](#)]

[See Gimmy's talk this afternoon]

Accretion Disks

Risk of confusion between New Physics and accretion disks?

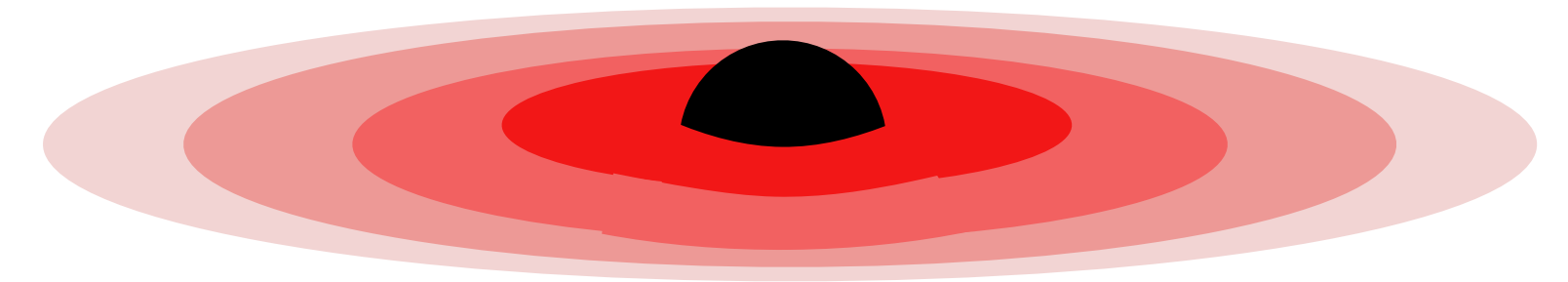
Many possible geometries and parameters,
but focus on dense, thin disks.

Dominant cause of dephasing is *not*
dynamical friction but *gas torques*.

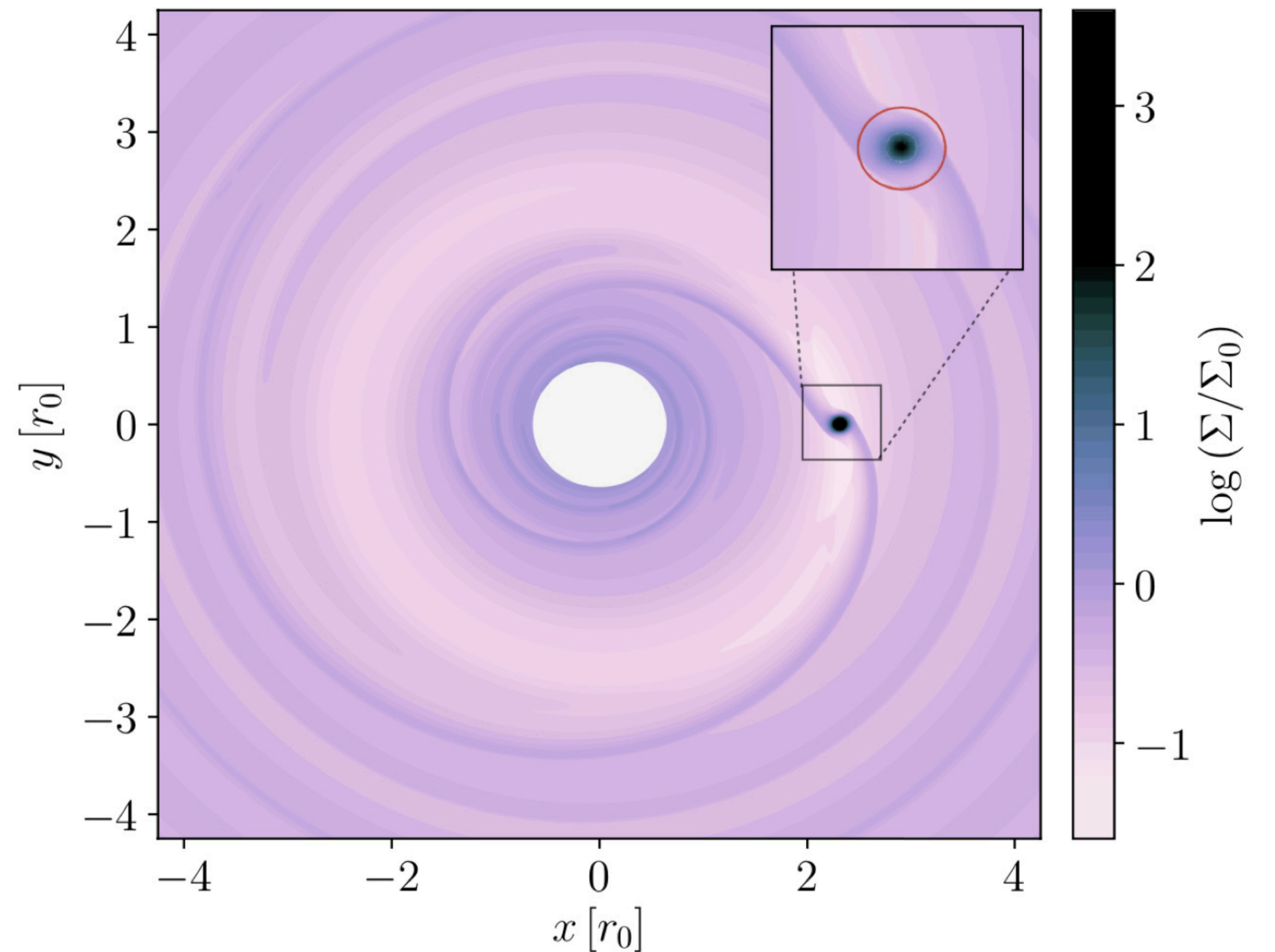
Perturbation of the disk leads to a build up of
gas inward or outward of the inspiraling BH.



Dependence of energy losses on BH
separation is *different* from DM spikes!

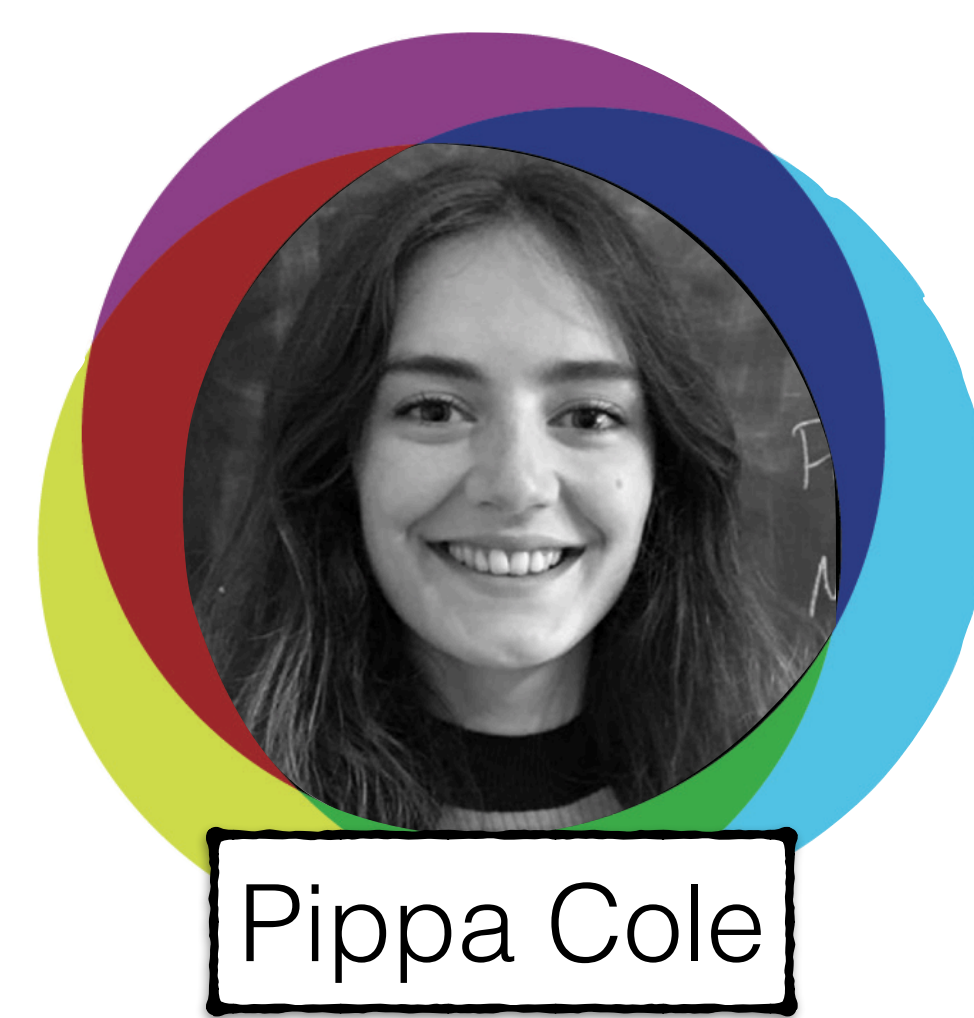


Baryonic
Accretion Disks

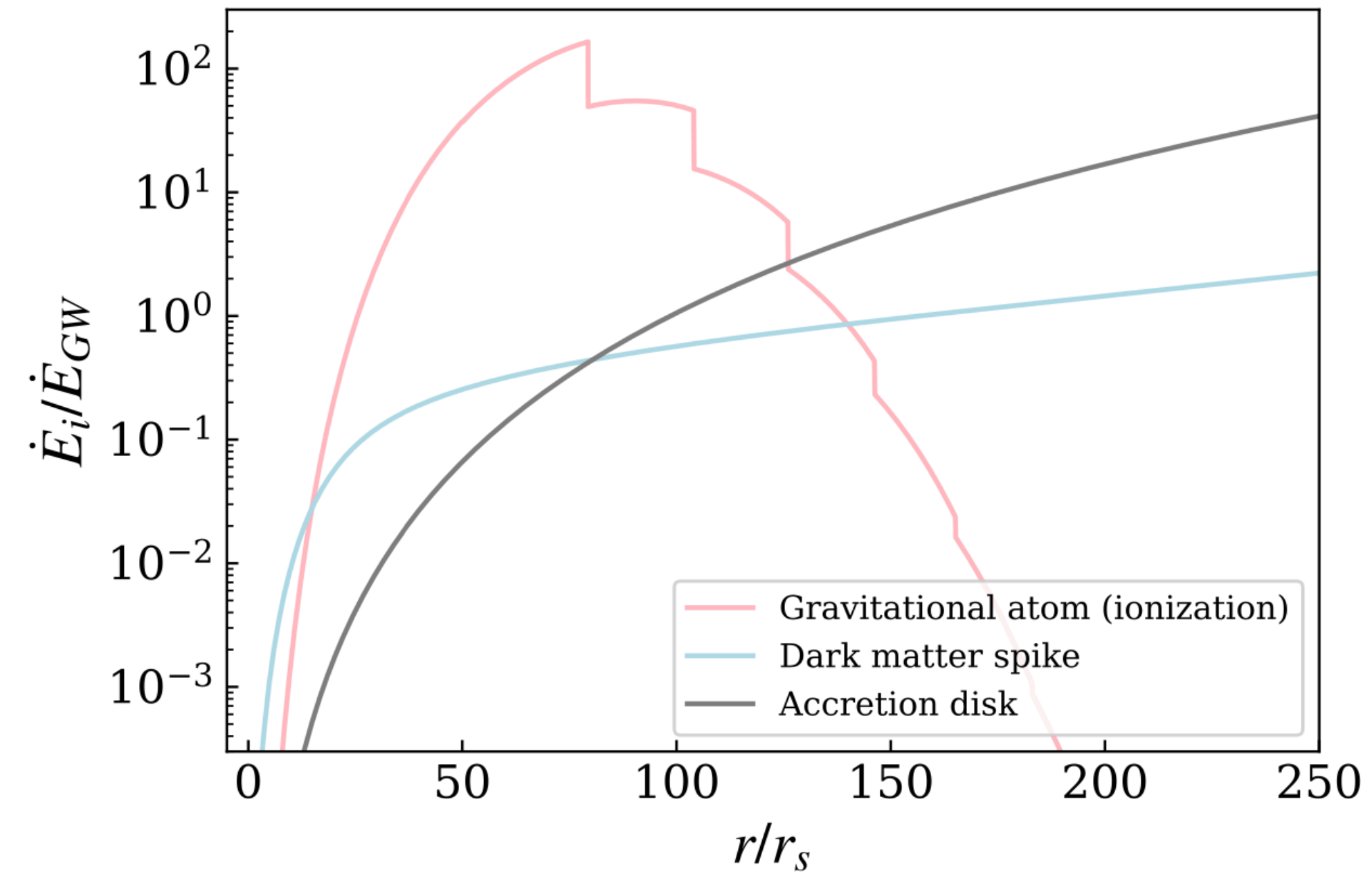
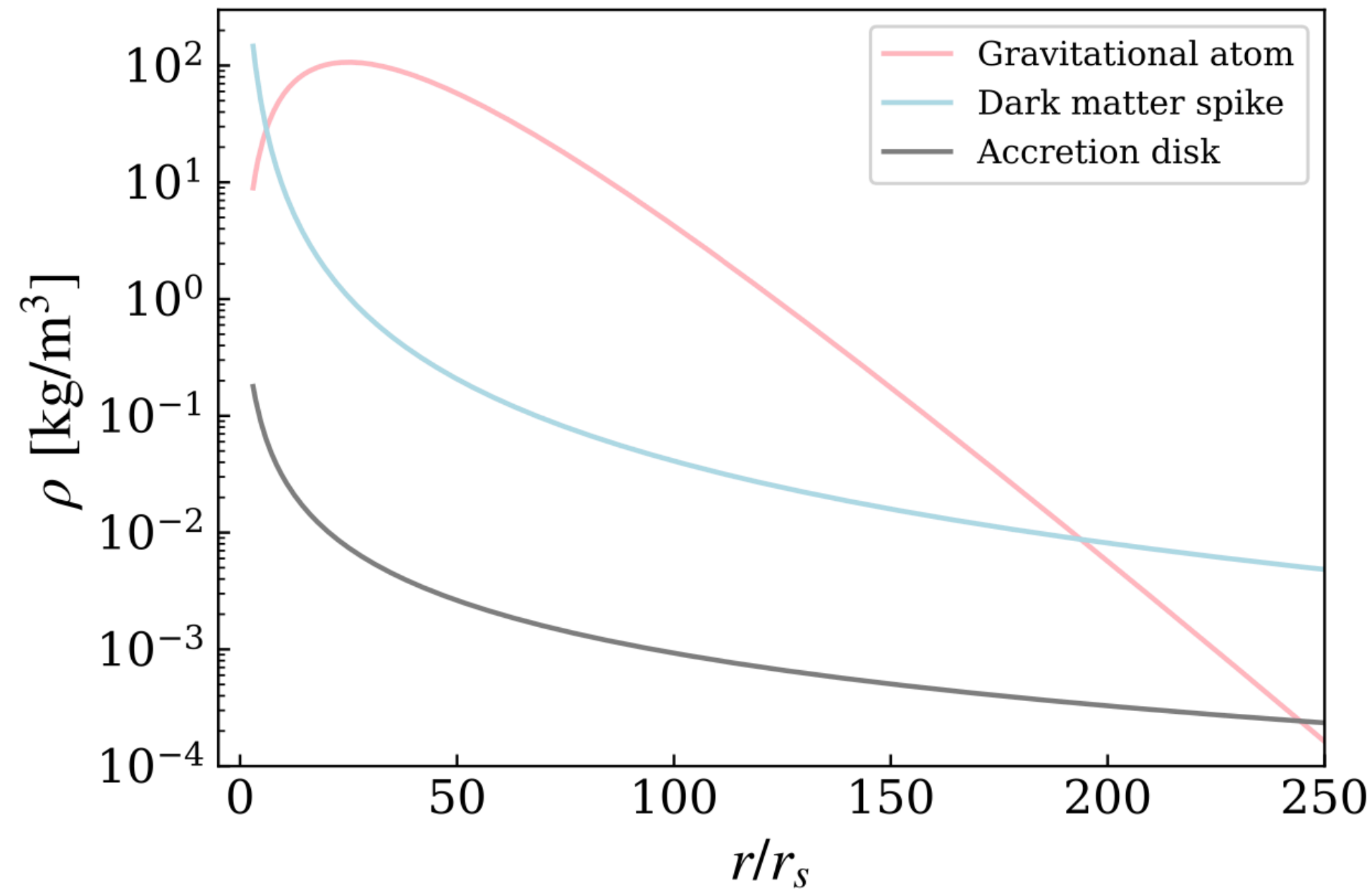


Discriminability

[Cole, Bertone, Coogan, Gaggero, Karydas, **BJK**, Spieksma, Tomaselli, [2211.01362](#), Nature Astronomy]



Pippa Cole

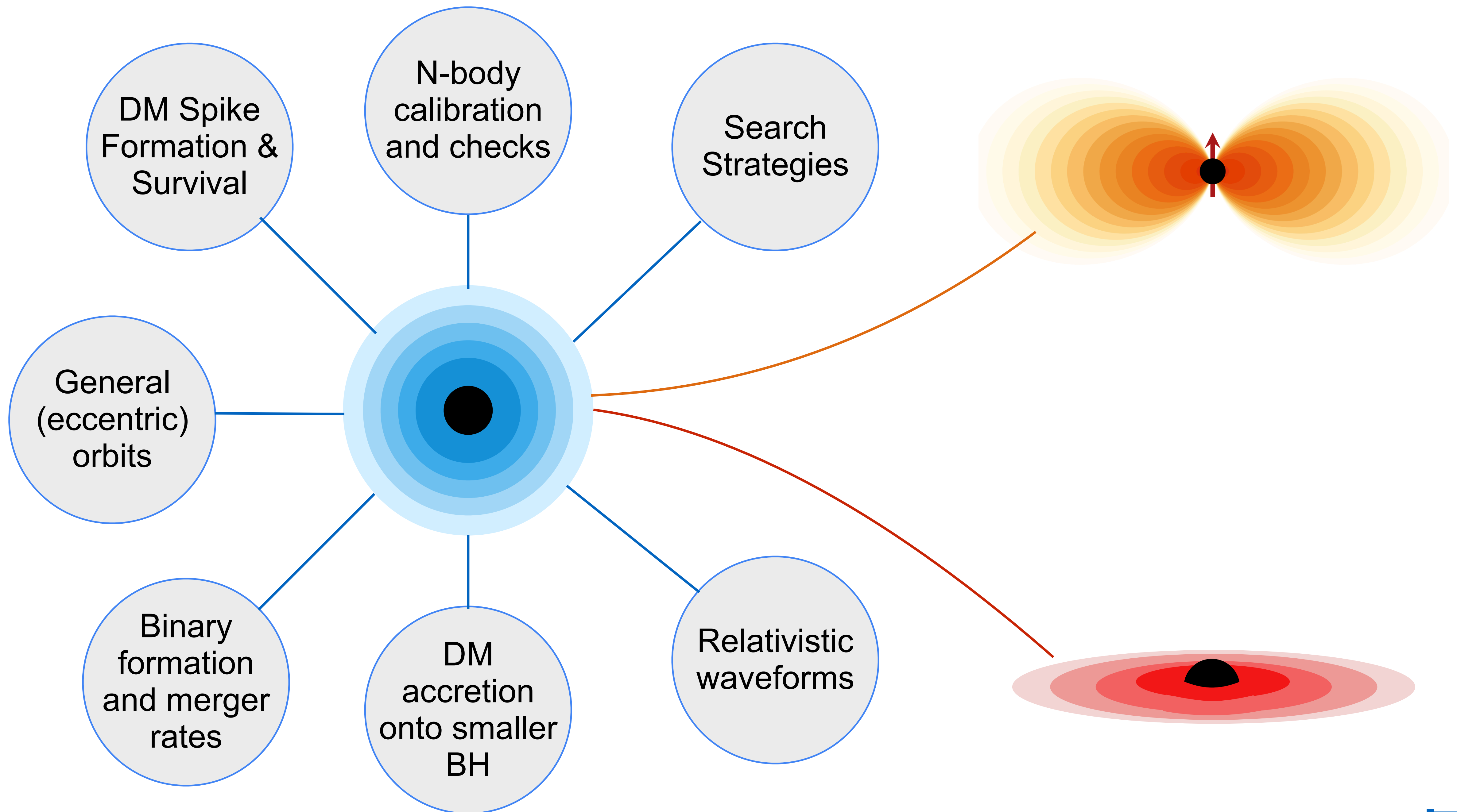


Signals **very hard to confuse** in 1 year of LISA data (huge Bayes factors!)

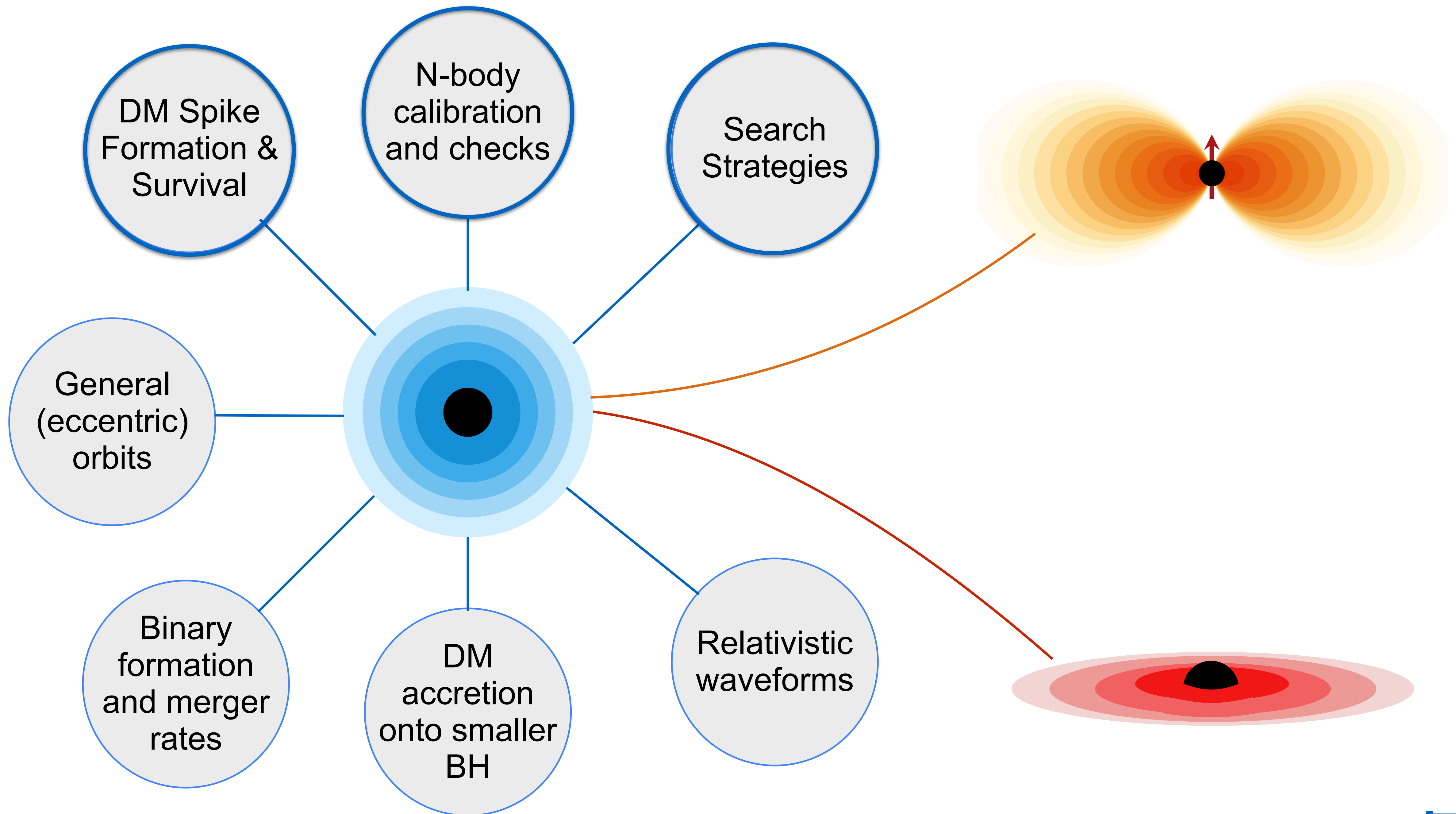
$\log_{10} \mathcal{B}$	Dark dress signal	Accretion disk signal	Gravitational atom signal
Vacuum template	34	6	39
Dark dress template	-	3	39
Accretion disk template	17	-	33
Gravitational atom template	24	6	-

[See Pippa's talk yesterday]

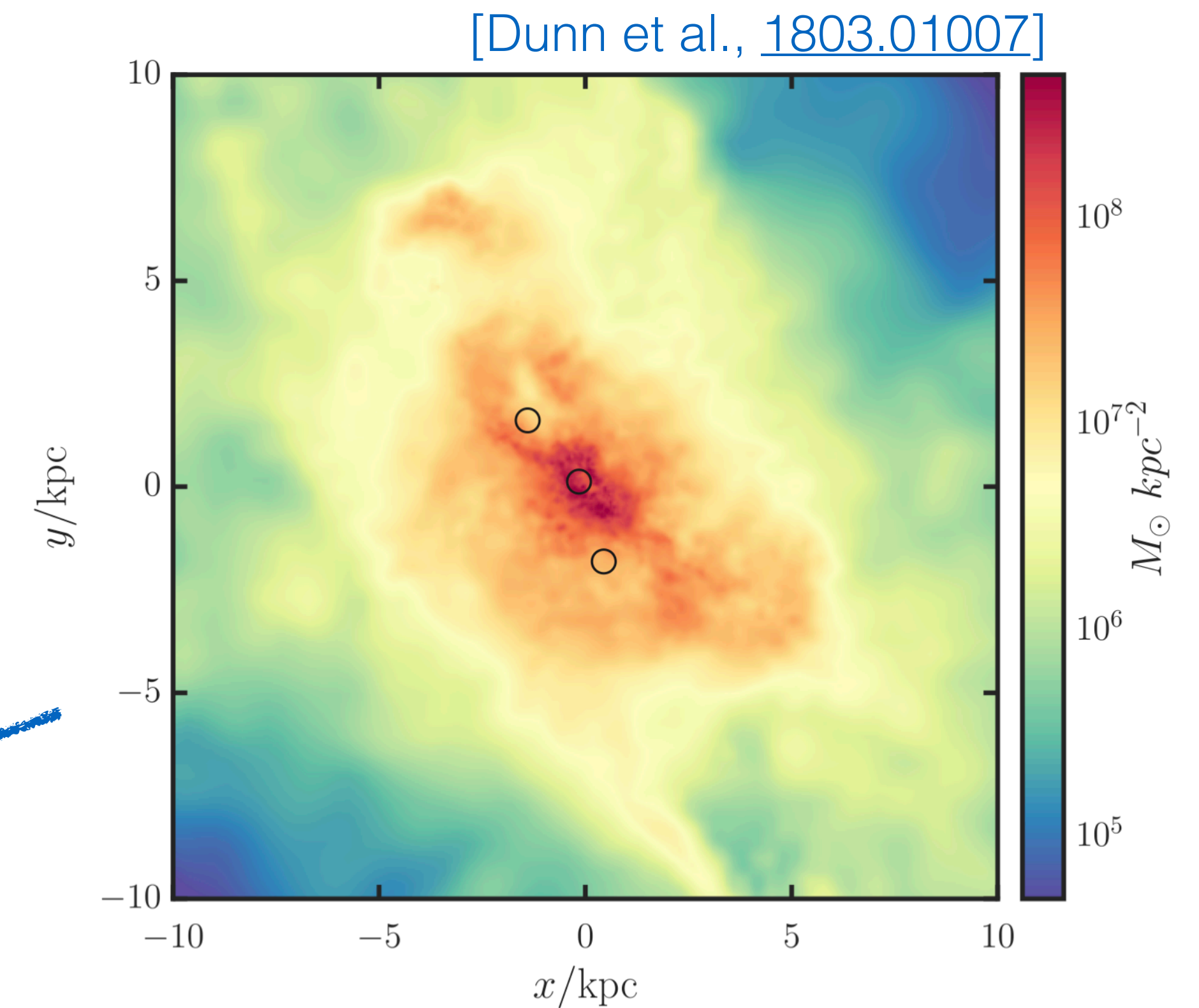
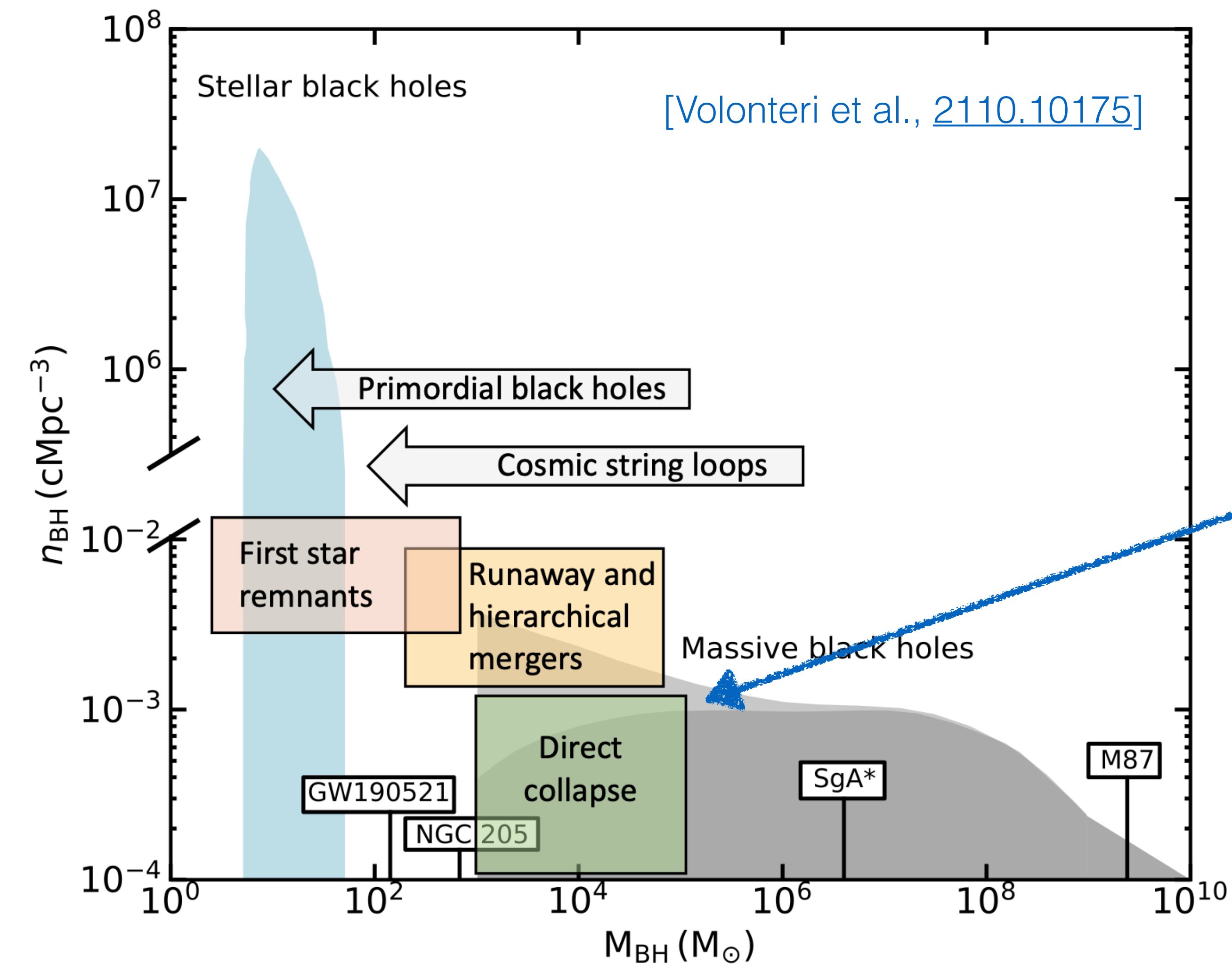
Many future directions



Many future directions



BH and Spike Formation



Carefully model the evolution of DM through the formation of the DCBH and subsequent growth by accretion.

Use semi-analytic galaxy formation models or simulations to study the properties of Direct Collapse Black Holes and the halos they form in.

How common are DM spikes around DCBHs, and what are their properties?

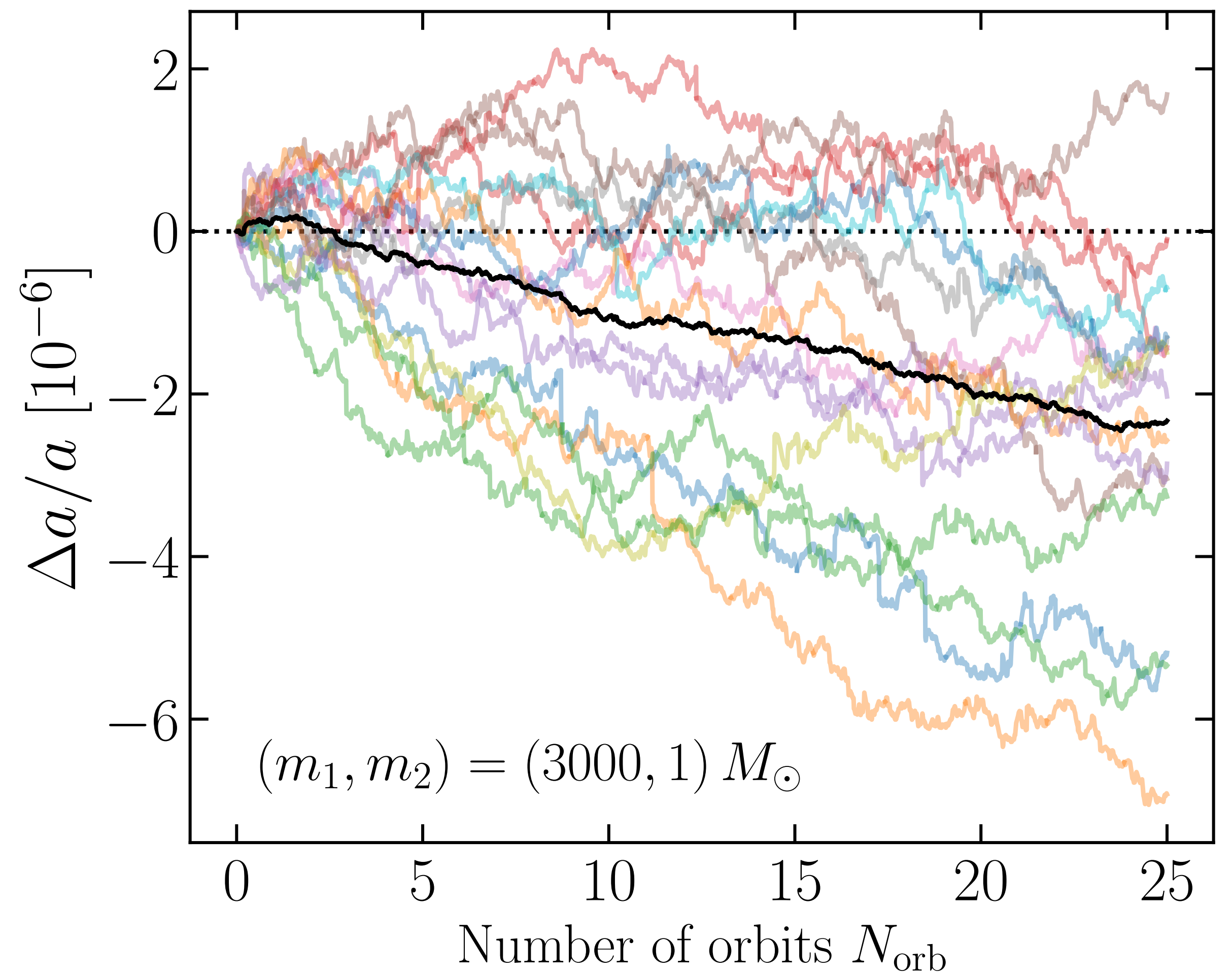
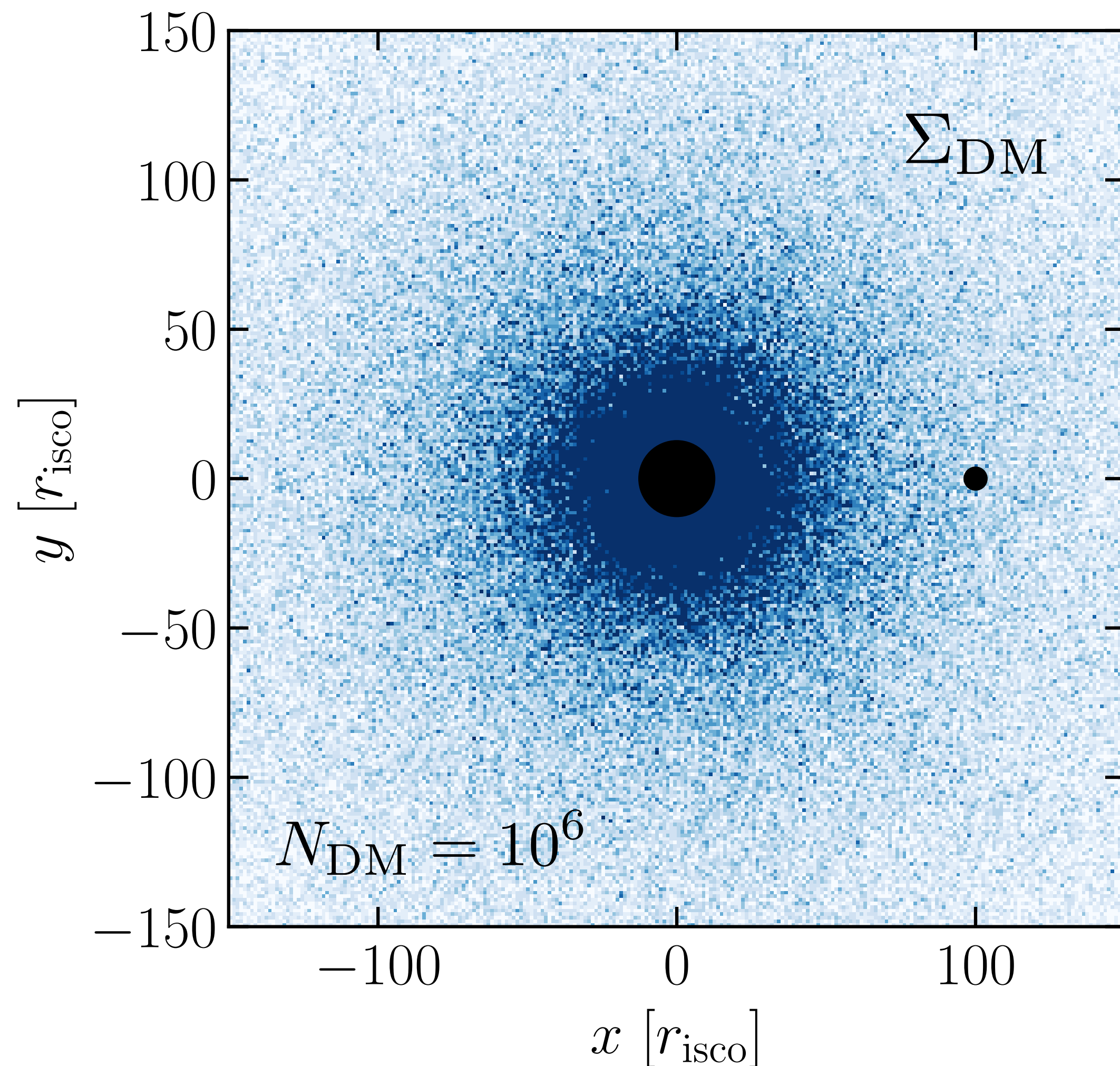
Simulations

Eventually need to expand and verify our description of dynamical friction and feedback in the DM spike (also include anisotropy, accretion, post-Newtonian corrections...)



NbodyIMRI: N-body solver tailored to DM spikes

[Code here: github.com/bradkav/NbodyIMRI]



Search Strategies

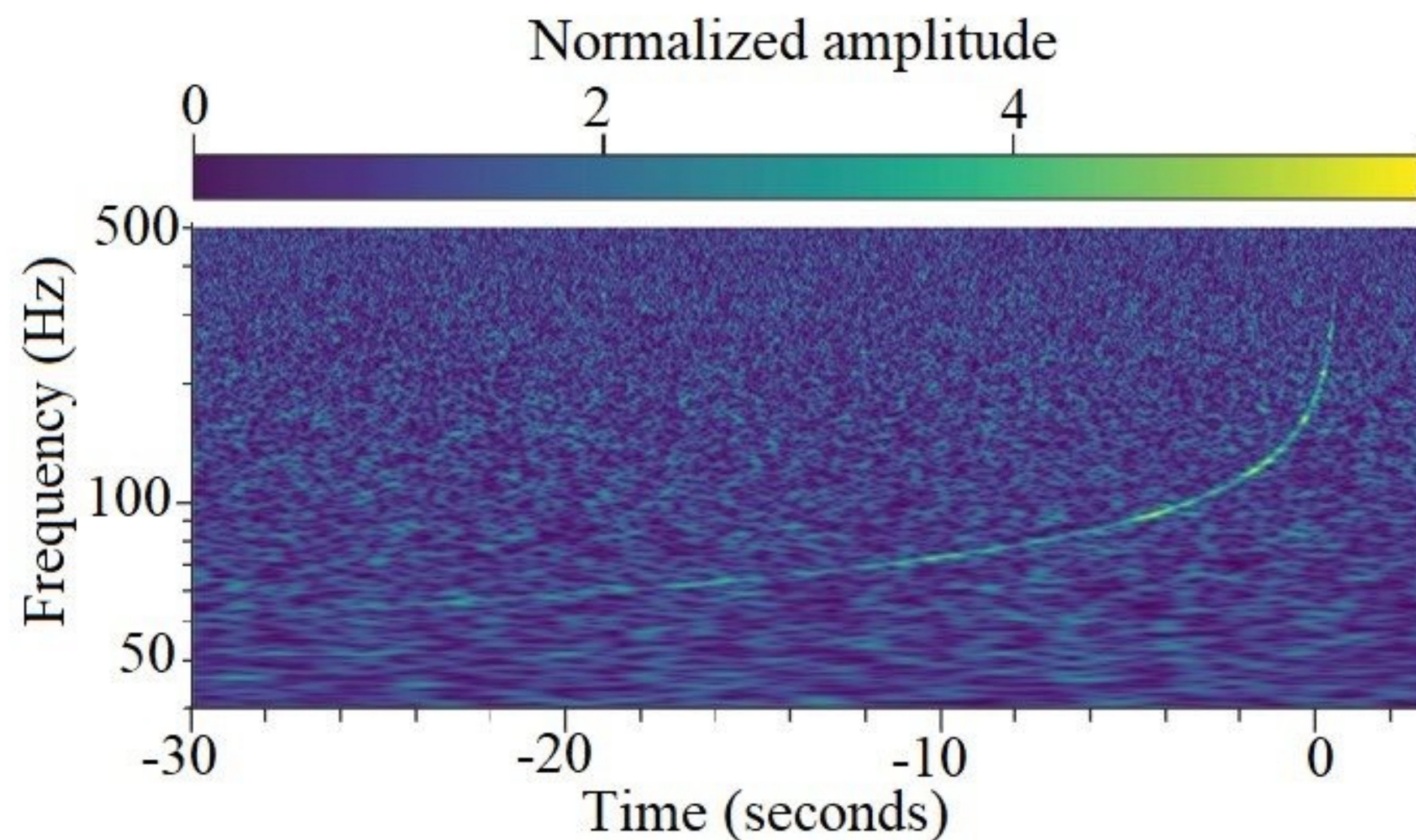
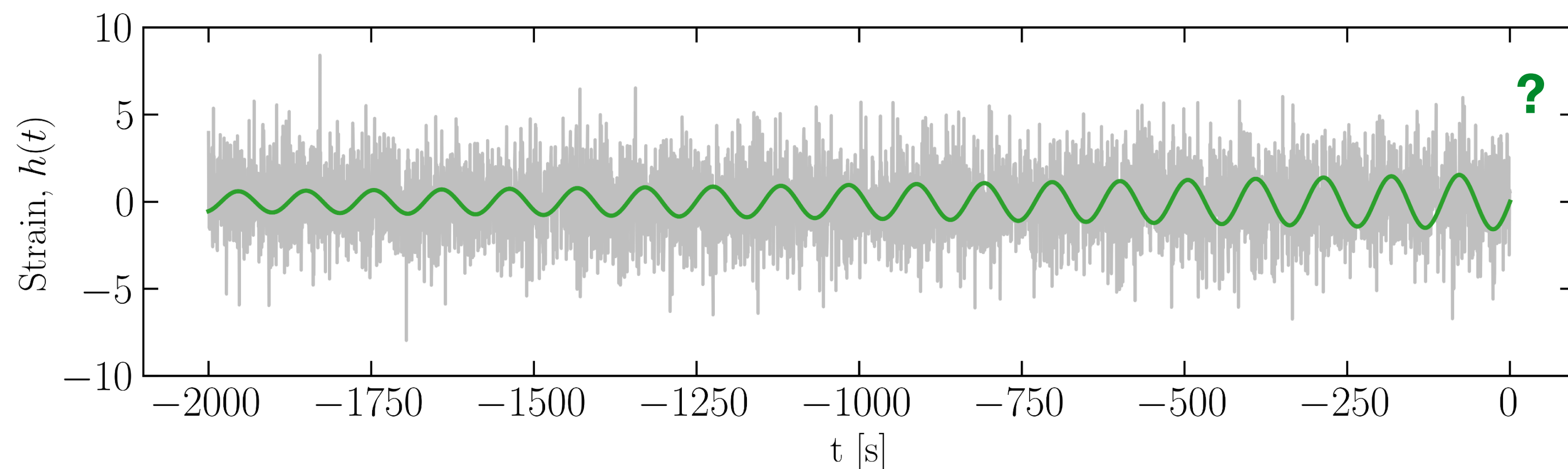
Possible options:

- Detect **GW signal close to merger** (where dephasing is small) then ‘track back’ through the data (as dephasing increases)
- Use ‘**generalised inspiral**’ waveforms to parametrise the dephasing

[Chia & Edwards, [2004.06729](#)]

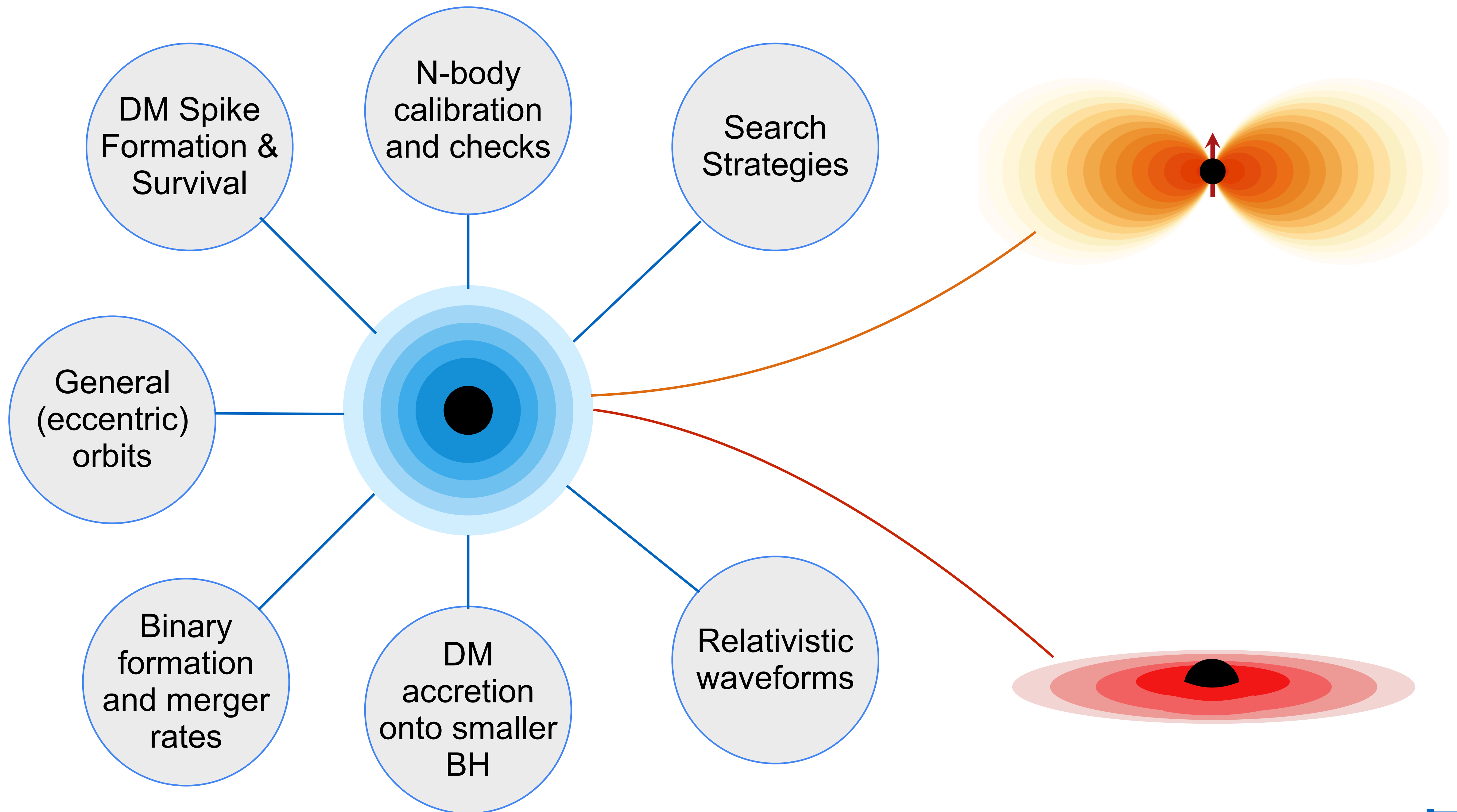
- Use tools from **continuous-wave searches**, breaking the inspiral into ‘chunks’ and looking for quasi-monochromatic signals in each chunk (e.g. Hough transform)

[Guo & Miller, [2205.10359](#)]



Map $f(t) \rightarrow (f, \dot{f})$?

Many future directions



A Collective Effort



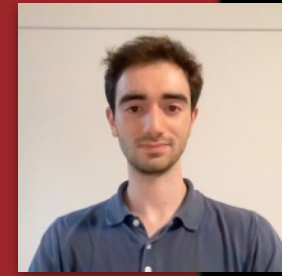
Gianfranco Bertone
(GRAPPA, Amsterdam)



Pippa Cole
(GRAPPA, Amsterdam)



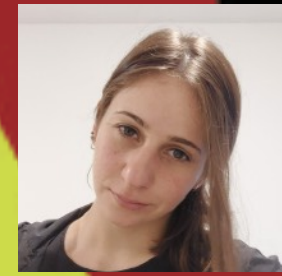
Samaya Nissanke
(GRAPPA, Amsterdam)



Jimmy Tomaselli
(GRAPPA, Amsterdam)



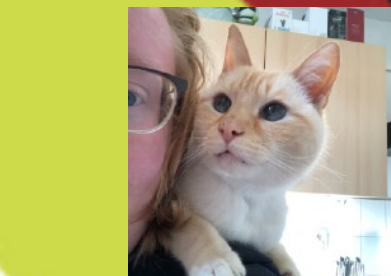
Alessandro Parisi
(GRAPPA, Amsterdam)



Ismini Andrianou
(GRAPPA, Amsterdam)



Daniel Baumann
(GRAPPA, Amsterdam)



Renske Wierda
(GRAPPA, Amsterdam)



Leon Kamermans
(GRAPPA, Amsterdam)



Adam Coogan
(Mila, Montreal)



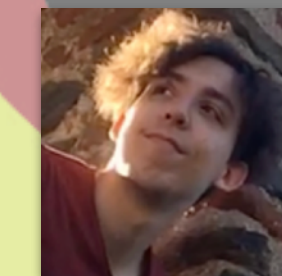
Tom Edwards
(Johns Hopkins)



Daniele Gaggero
(IFT, Madrid)



David Nichols
(U. Virginia)



Theophanes Karydas
(GRAPPA, Amsterdam)



Jose Maria Diego
(IFCA, Santander)



Francesca Scarcella
(IFT, Madrid)



Abram Perez Herrero
(IFCA, Santander)



Pratibha Jangra
(IFCA, Santander)

...and others...

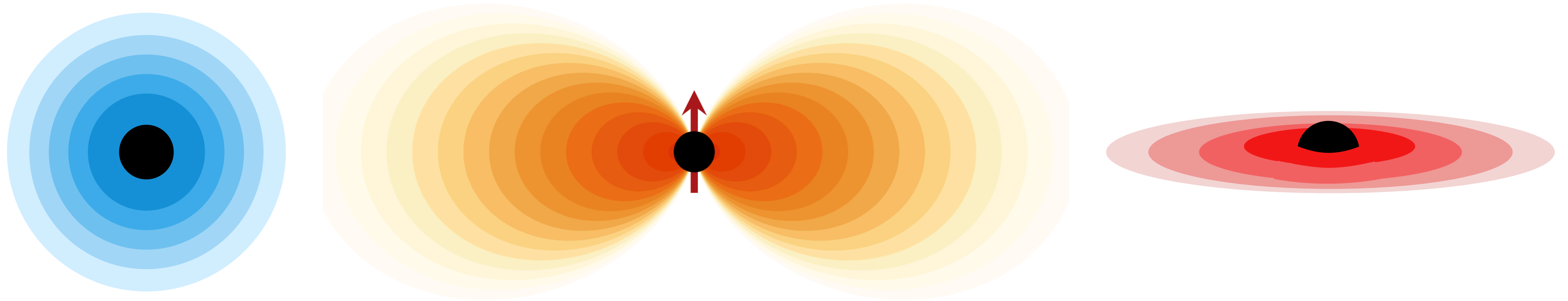
A Collective Effort



Fuel for Science



Conclusions

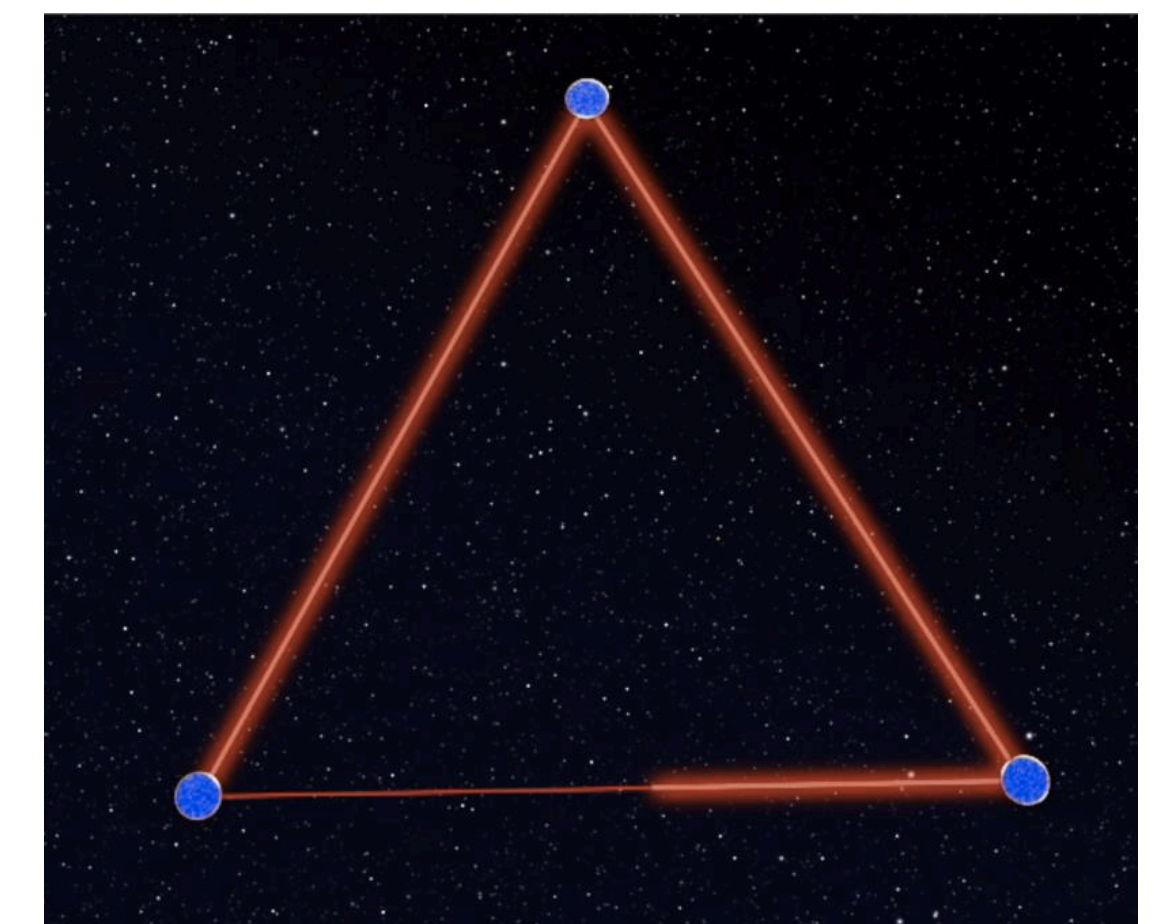
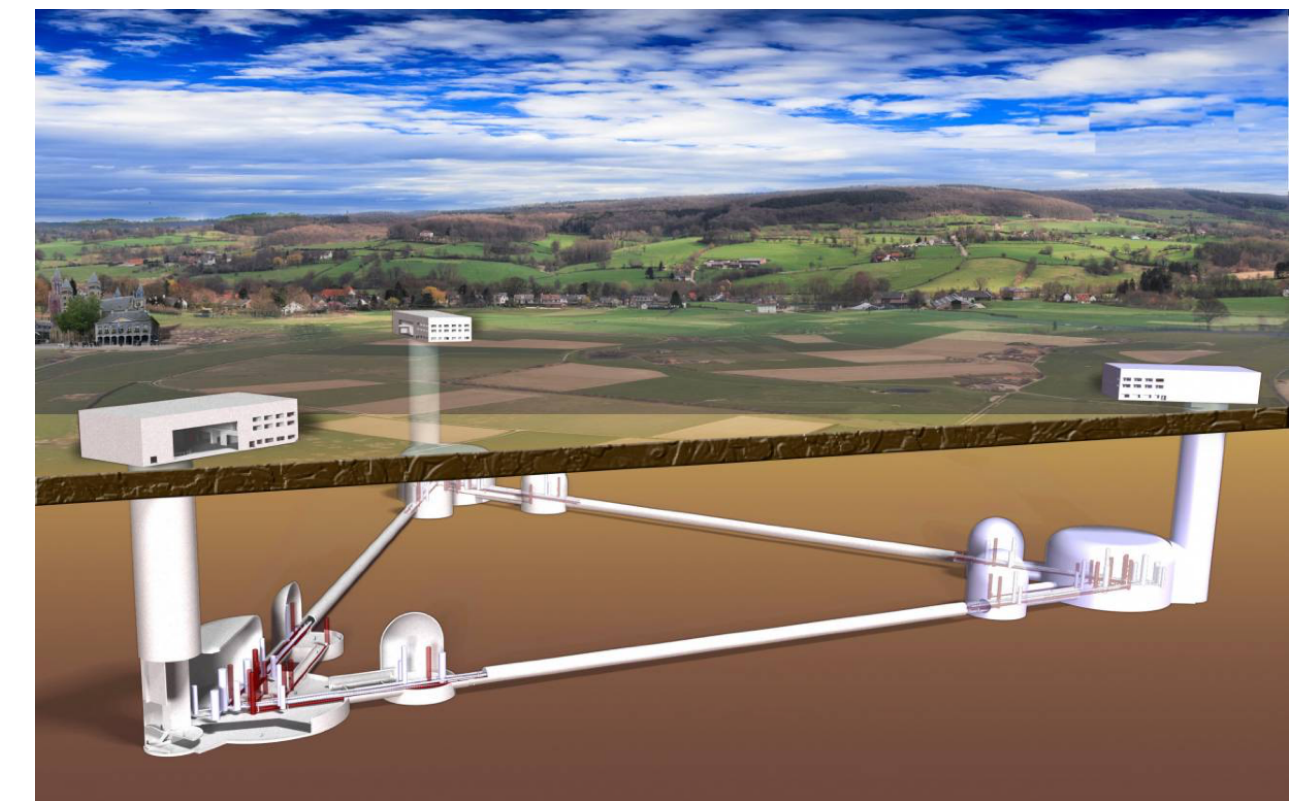


Understanding Black Hole environments represents both a **great challenge** and a **great opportunity** for future GW searches.

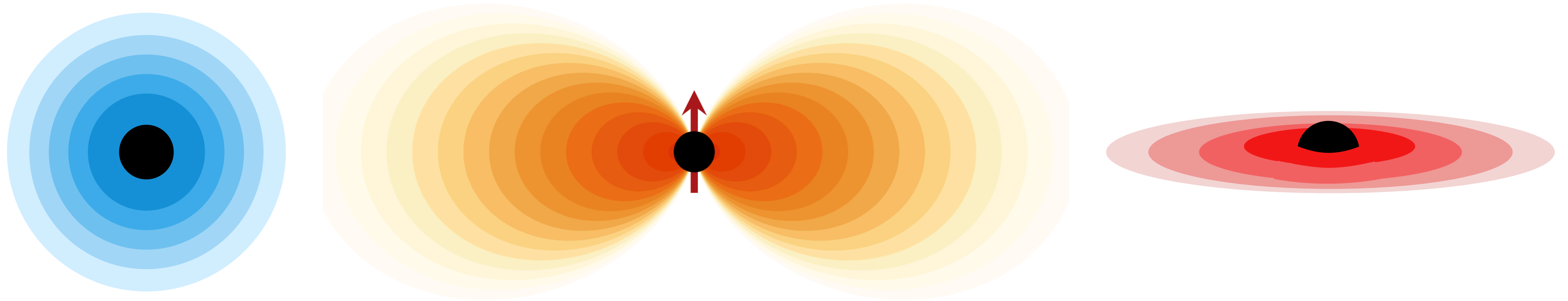
A true GRAPPA problem: General Relativity, Gravitational Waves, Accretion, Dark Matter, complex data analysis.

Many problems and questions still to tackle!

Many great people working hard with and within GRAPPA to solve them!



Conclusions



Understanding Black Hole environments represents both a **great challenge** and a **great opportunity** for future GW searches.

A true GRAPPA problem: General Relativity, Gravitational Waves, Accretion, Dark Matter, complex data analysis.

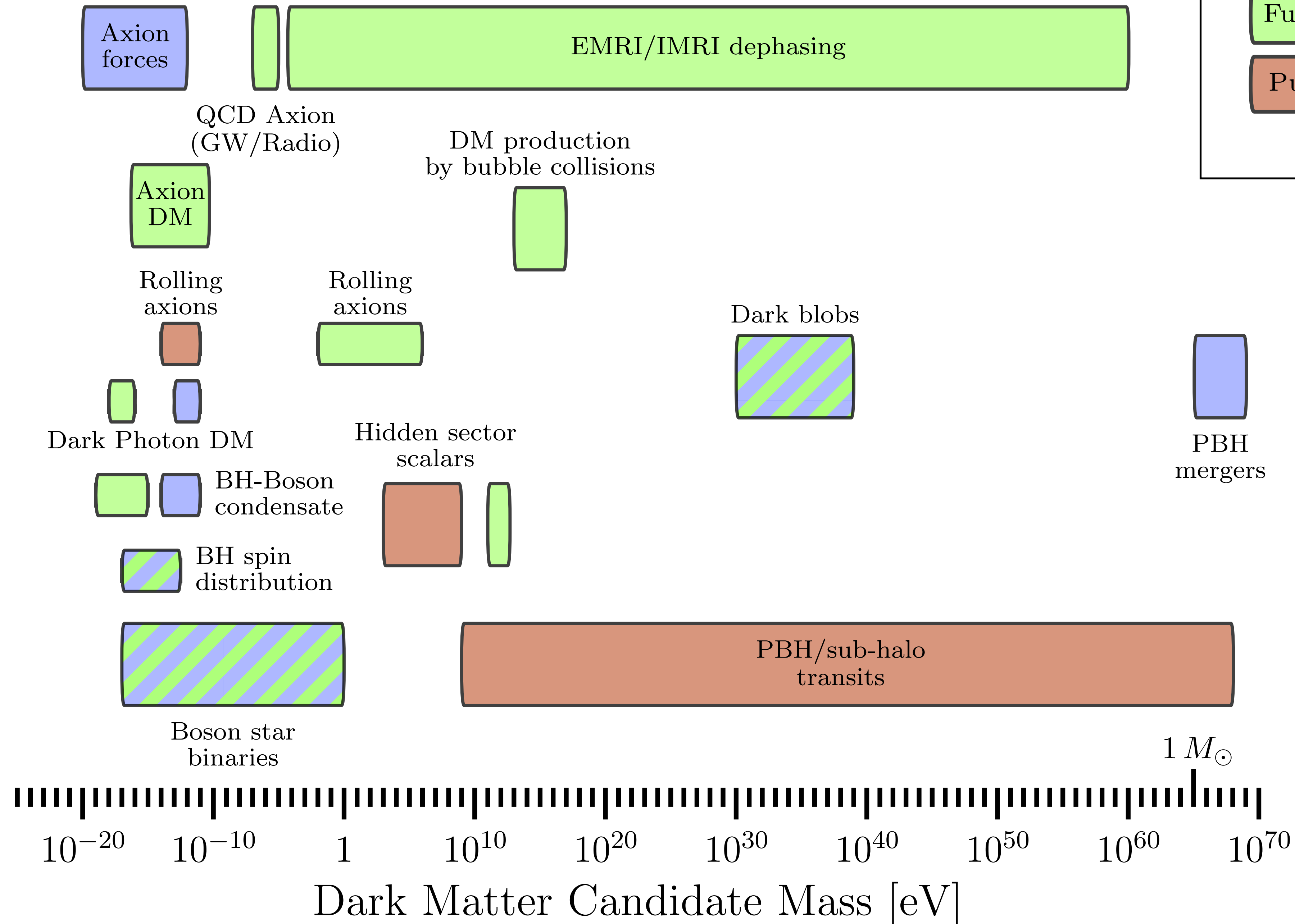
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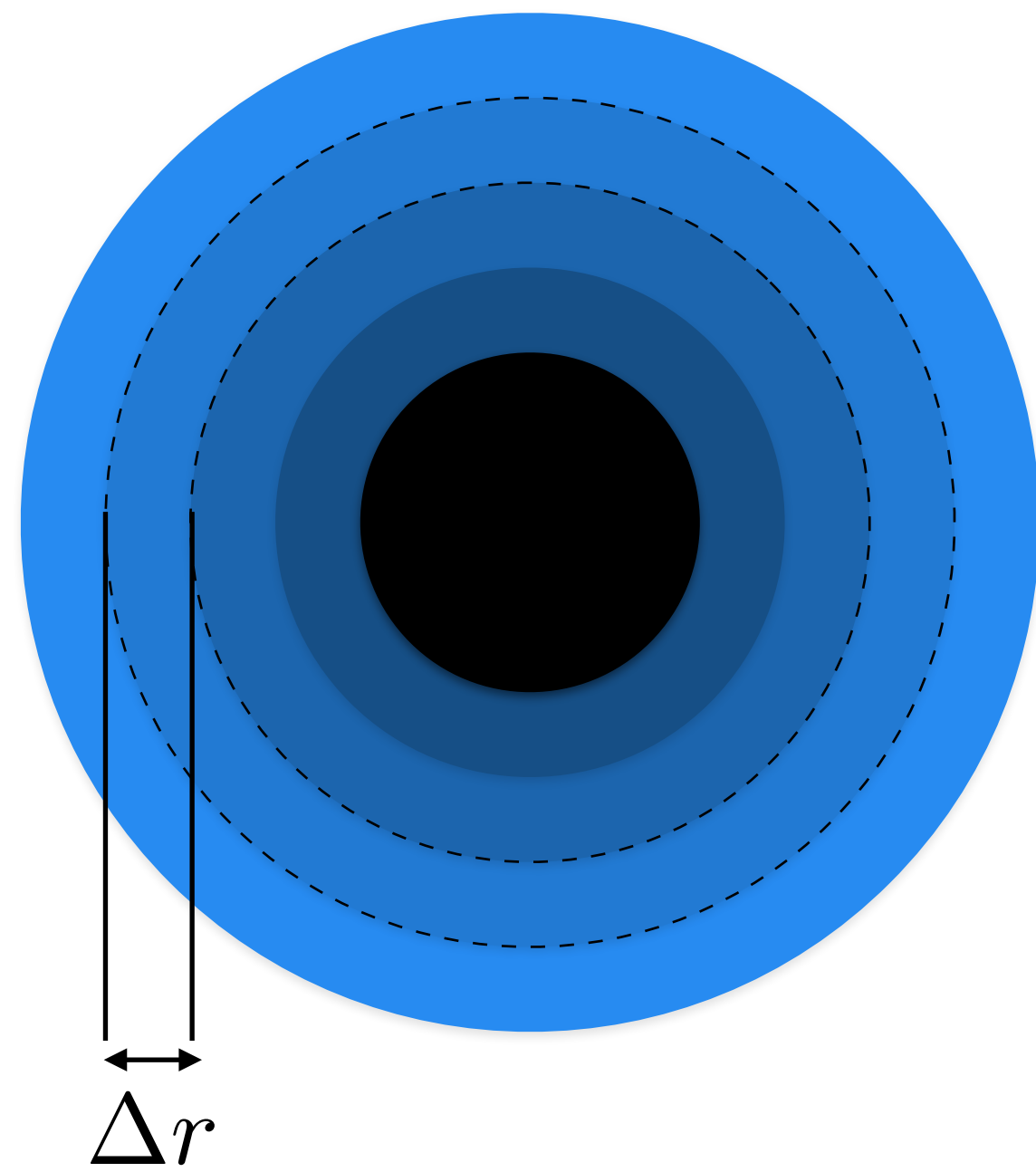
Backup Slides

GW Probes of Dark Matter

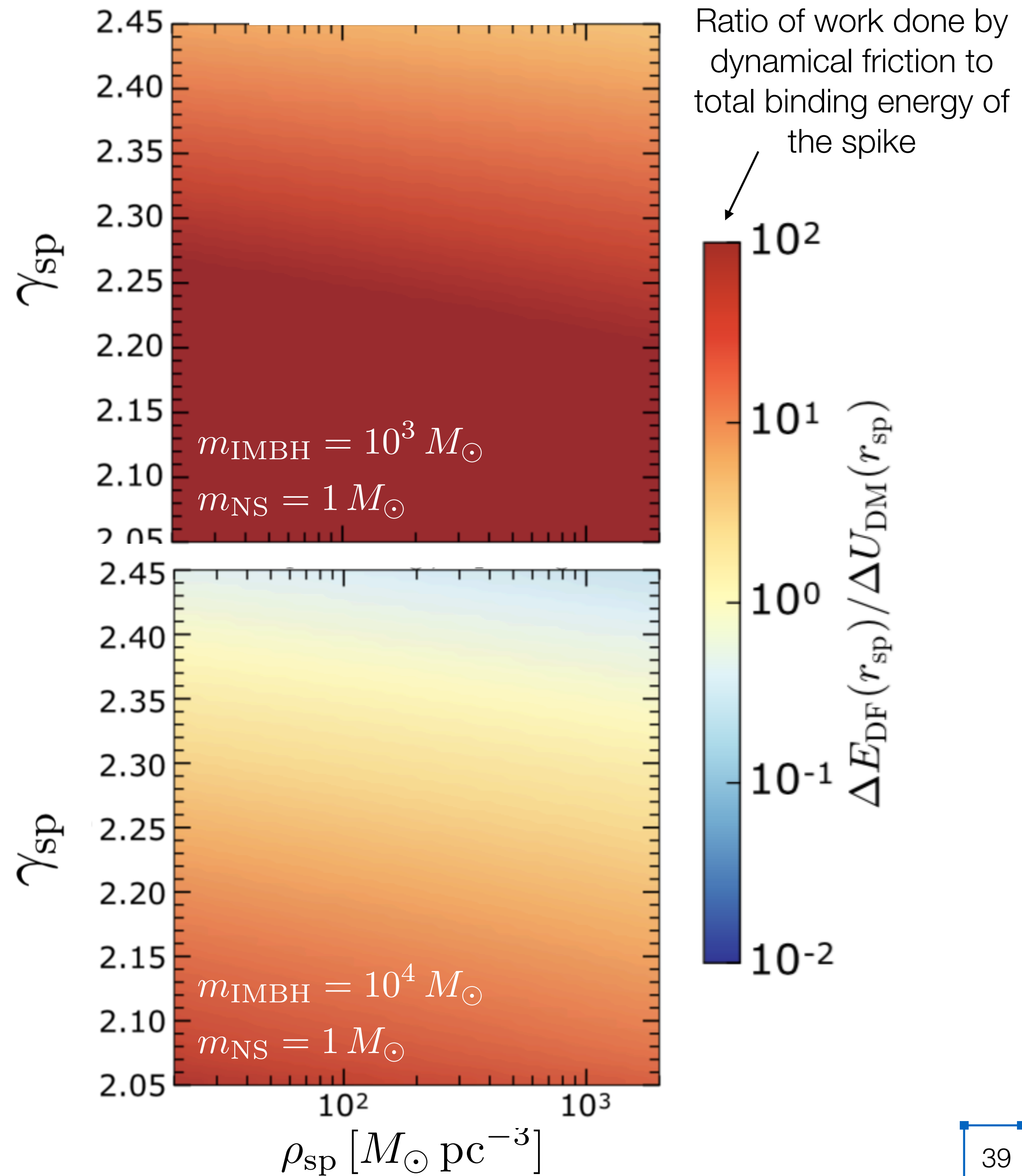


DM Spikes Energy Conservation

Q: How much energy is *available* for dynamical friction?



A: Binding energy of DM
 ΔU_{DM} over radius Δr



Self-consistent Evolution

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{df(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E} - \Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy \mathcal{E} to scatter and receive a 'kick' $\Delta\mathcal{E}$

$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$ - total probability for a particle with energy \mathcal{E} to scatter

Self-consistent Evolution

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Particles scattering from
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

Particles scattering from
 $\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

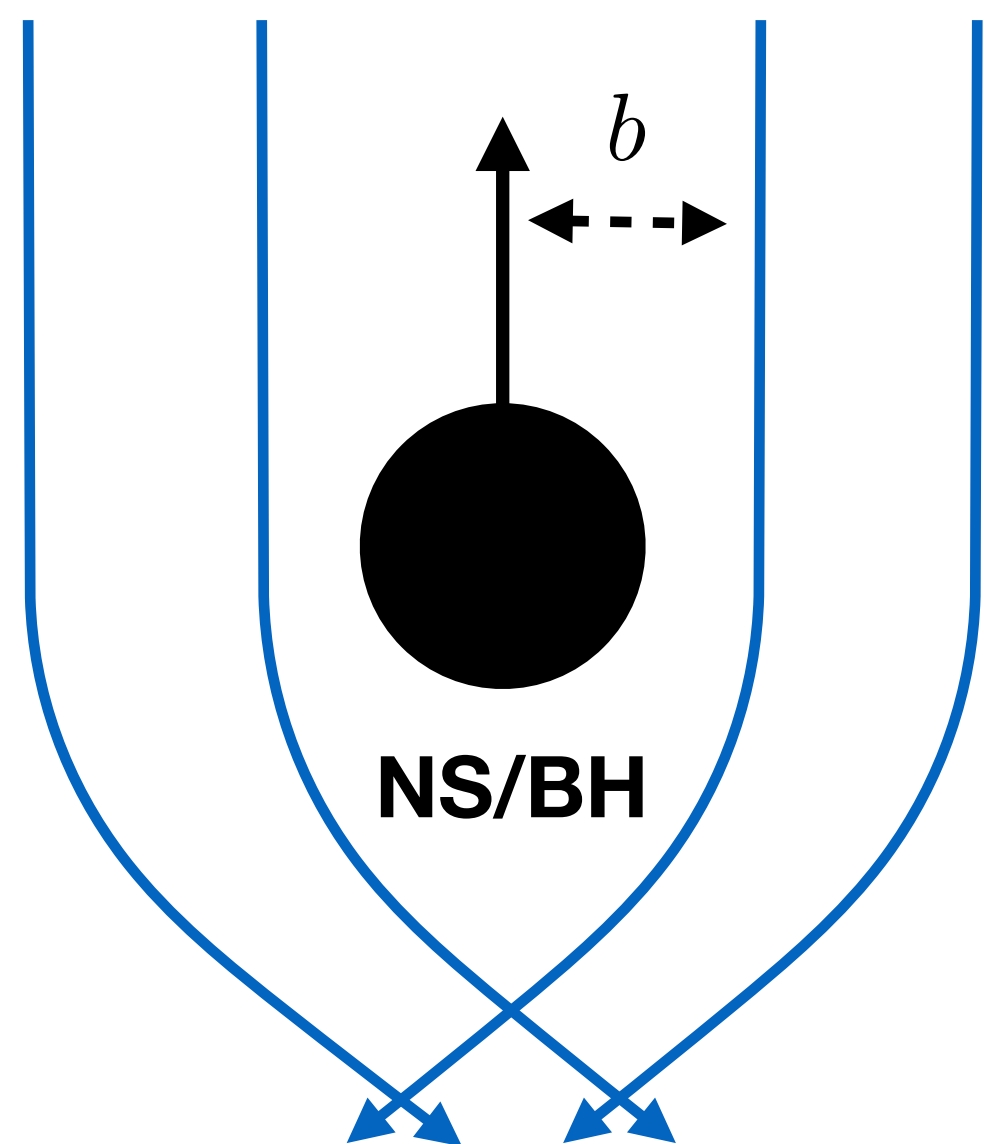
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Scattering Probability

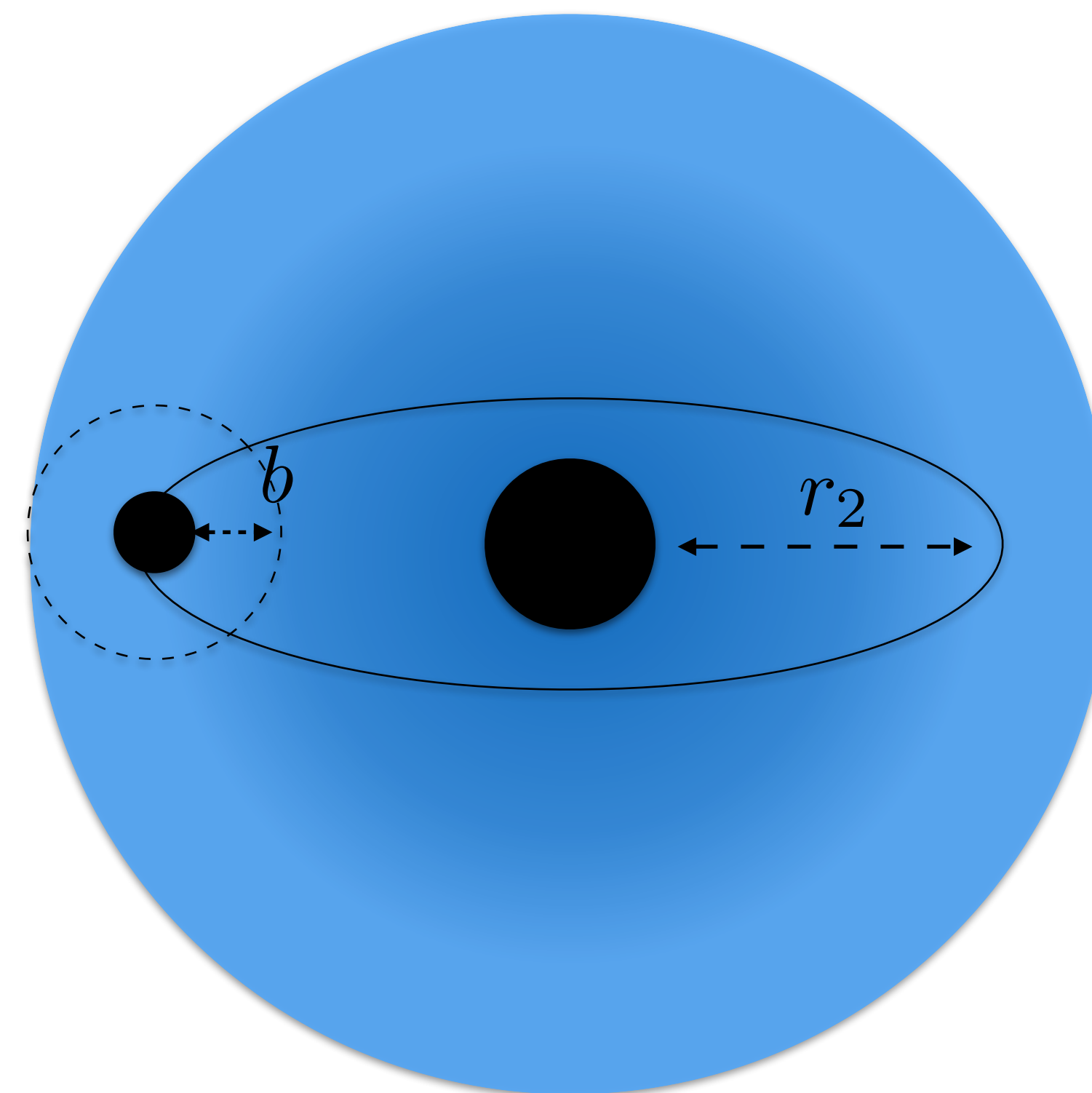
Two body scattering problem relates energy exchange to impact parameter:

$$\Delta\mathcal{E}(b) = -2v_0^2 \left[1 + \frac{b^2 v_0^4}{G^2 m_2^2} \right]^{-1}$$



Scattering probability becomes a *geometric* problem:

$$P_{\mathcal{E}}(\Delta\mathcal{E}) \propto P(b|\mathcal{E})$$



Code available online: github.com/bradkav/HaloFeedback
(See also <https://github.com/DMGW-Goethe/imripy>)