New directional signatures from non-relativistic effective field theory
or
'Who ordered all these operators?'

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NewDark

Based on arXiv:1505.07406

Overview

We typically assume a particular particle physics interaction when calculating the rate in directional detectors.

Question #1:
What is the rate in directional detectors if we consider more general interactions?

Question #2:
Can we use directional detection to distinguish between different particle physics?

Answer these questions within the framework of the non-relativistic effective theory of interactions…

Full Disclosure

Dark matter directional detection in non-relativistic effective theories

1 [hep-ph] 24 May 2015

[arXiv:1505.06441]

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Abstract. We extend the formalism of dark matter directional detection to arbitrary one-body dark matter-nucleon interactions. The new theoretical framework generalizes the one currently used, which is based on 2 types of dark matter-nucleon interaction only. It includes 14 dark matter-nucleon interaction operators, 8 isotope-dependent nuclear response functions, and the Radon transform of the first 2 moments of the dark matter velocity distribution. We

[arXiv:1505.07406]

SACLAY-t15/093

New directional signatures from the non-relativistic effective field theory of dark matter

27 May 2015

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The framework of non-relativistic effective field theory (NREFT) aims to generalise the standard analysis of direct detection experiments in terms of spin-dependent (SD) and spin-independent (SI) interactions. We show that a number of NREFT operators lead to distinctive new directional signatures, such as prominent ring-like features in the directional recoil rate, even for relatively low mass WIMPs. We discuss these signatures and how they could affect the interpretation of future results from directional detectors. We demonstrate that considering a range of possible operators introduces a factor of 2 uncertainty in the number of events required to confirm the median recoil direction of the signal. Furthermore, using directional detection, it is possible to distinguish the more general NREFT interactions from the standard SI/SD interactions at the 2σ level with $\mathcal{O}(100-500)$ events. In particular, we demonstrate that for certain NREFT operators, directional sensitivity provides the only method of distinguishing them from these standard operators, highlighting the importance of directional detectors in probing the particle physics of dark matter.

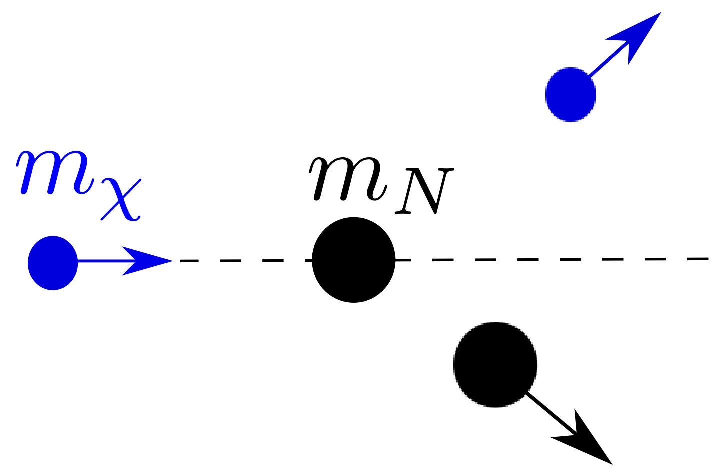
I. INTRODUCTION

for DM-nucleus interactions. The non-relativistic effective field theory (NREFT; introduced by Fan et al. [22] and extended in Refs. [23–25]) considers all possible non-

The standard framework

The Directional Spectrum

Recoil distribution for WIMP-nucleus recoils in direction $\hat{q}$ with fixed WIMP speed $\vec{v}$:



$$\mu_{\chi N} = \frac{m_\chi m_N}{m_\chi + m_N}$$

$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}$$

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} = \frac{\rho_0 v}{m_\chi} \frac{\langle |\mathcal{M}|^2 \rangle}{32\pi m_N^2 m_\chi^2 v^2} \frac{v\,\delta\left(\vec{v}\cdot\hat{q} - v_{\min}\right)}{2\pi}$$

WIMP flux — Cross section — Kinematics

For standard SI and SD interactions: $\langle |M|^2 \rangle \sim v^0 q^0$

Radon Transform

For standard SI/SD, for fixed WIMP speed:

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} \propto \delta\left(\vec{v}\cdot\hat{q} - v_{\mathrm{min}}\right)$$

So integrating over all WIMP speeds:

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} \propto \int_{\mathbb{R}^3} f(\vec{v})\delta\left(\vec{v}\cdot\hat{q} - v_{\mathrm{min}}\right)\,\mathrm{d}^3\vec{v} \equiv \hat{f}(v_{\mathrm{min}}, \hat{q})$$

'Radon Transform' (RT)

For the SHM:

$$f(\vec{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}}\exp\left[-\frac{(\vec{v}-\vec{v}_{\mathrm{lag}})^2}{2\sigma_v^2}\right]$$

$$\hat{f}(v_{\mathrm{min}}, \hat{q}) = \frac{1}{\sqrt{2\pi\sigma_v^2}}\exp\left[-\frac{(v_{\mathrm{min}}-\vec{v}_{\mathrm{lag}}\cdot\hat{q})^2}{2\sigma_v^2}\right]$$

Non-relativistic EFT

Non-relativistic effective field theory (NREFT)

The interaction is (ultra-)non-relativistic, so we can write down all possible non-relativistic (NR) WIMP-*nucleon* operators which can mediate the *elastic* scattering.

[Fan et al - 1008.1591, Fitzpatrick et al. - 1203.3542]

The building blocks of these operators are:

$$\vec{S}_n \qquad \vec{S}_\chi \qquad \frac{\vec{q}}{2m_n} \qquad \vec{v}_\perp$$

The WIMP velocity operator is not Hermitian, so it can appear only through the Hermitian *transverse velocity*:

$$\vec{v}_\perp = \vec{v} + \frac{\vec{q}}{2\mu_{\chi n}} \qquad \Rightarrow \vec{v}_\perp \cdot \vec{q} = 0$$

NREFT Operators

Write down all operators which are Hermitian, Galilean invariant and time-translation invariant:

SI

$$\mathcal{O}_1 = 1$$

$$\mathcal{O}_3 = i\vec{S}_n \cdot (\frac{\vec{q}}{m_n} \times \vec{v}^\perp)$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_n$$

SD

$$\mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_n} \times \vec{v}^\perp)$$

$$\mathcal{O}_6 = (\vec{S}_\chi \cdot \vec{q})(\vec{S}_n \cdot \vec{q})$$

$$\mathcal{O}_7 = \vec{S}_n \cdot \vec{v}^\perp$$

$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$$

$$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_n \times \vec{q})$$

$$\mathcal{O}_{10} = i\vec{S}_n \cdot \vec{q}$$

$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \vec{q}$$

$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \vec{q}$$

$$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_n \times \vec{v}^\perp)$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_n \cdot \frac{\vec{q}}{m_n})$$

$$\mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_n})(\vec{S}_n \cdot \vec{v}^\perp)$$

$$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_n})((\vec{S}_n \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_n}) \,.$$

NB: two sets of operators, one for protons and one for neutrons…

[1308.6288]

NREFT event rate

The matrix element for operator *i* can now be written as:

$$\langle|\mathcal{M}_i|^2\rangle = |\langle c_i \mathcal{O}_i \rangle_{\mathrm{nucleus}}|^2 = c_i^2 F_{i,i}(v_\perp^2, q^2)$$

[Assuming for now: $c^p = c^n$]

The nuclear response functions $F_{i,i}(v_\perp^2, q^2)$ are the expectation values of the operators summed over all nucleons in the nucleus. They depend only on $v_\perp^2$ and q^2 .

$$\frac{\mathrm{d}R_i}{\mathrm{d}E_R \mathrm{d}\Omega_q} = \frac{\rho_0}{64\pi^2 m_N^2 m_\chi^3} c_i^2 \int_{\mathbb{R}^3} F_{i,i}(v_\perp^2, q^2) f(\vec{v})\, \delta\left(\vec{v} \cdot \hat{q} - v_{\mathrm{min}}\right) \mathrm{d}^3\vec{v}$$

Framework previously applied to non-directional direct detection and solar capture [1211.2818, 1406.0524, 1503.03379, 1503.04109 and others].

Nuclear response functions

$$F_{1,1} = F_M$$

$$F_{3,3} = \frac{1}{8}\frac{q^2}{m_n^2}\left(v_\perp^2 F_{\Sigma'} + 2\frac{q^2}{m_n^2}F_{\Phi''}\right)$$

$$F_{4,4} = \frac{C(j_\chi)}{16}\left(F_{\Sigma'} + F_{\Sigma''}\right)$$

$$F_{5,5} = \frac{C(j_\chi)}{4}\frac{q^2}{m_n^2}\left(v_\perp^2 F_M + \frac{q^2}{m_n^2}F_\Delta\right)$$

$$F_{6,6} = \frac{C(j_\chi)}{16}\frac{q^4}{m_n^4}F_{\Sigma''}$$

$$F_{7,7} = \frac{1}{8}v_\perp^2 F_{\Sigma'}\,,$$

$$F_{8,8} = \frac{C(j_\chi)}{4}\left(v_\perp^2 F_M + \frac{q^2}{m_n^2}F_\Delta\right)$$

$$F_{9,9} = \frac{C(j_\chi)}{16}\frac{q^2}{m_n^2}F_{\Sigma'}$$

$$F_{10,10} = \frac{1}{4}\frac{q^2}{m_n^2}F_{\Sigma''}$$

$$F_{11,11} = \frac{1}{4}\frac{q^2}{m_n^2}$$

$$F_{12,12} = \frac{C(j_\chi)}{16}\left(v_\perp^2\left(F_{\Sigma''} + \frac{1}{2}F_{\Sigma'}\right) + \frac{q^2}{m_n^2}\left(F_{\tilde{\Phi}'} + F_{\Phi''}\right)\right)$$

$$F_{13,13} = \frac{C(j_\chi)}{16}\frac{q^2}{m_n^2}\left(v_\perp^2 F_{\Sigma''} + \frac{q^2}{m_n^2}F_{\tilde{\Phi}'}\right)$$

$$F_{14,14} = \frac{C(j_\chi)}{32}\frac{q^2}{m_n^2}v_\perp^2 F_{\Sigma'}$$

$$F_{15,15} = \frac{C(j_\chi)}{32}\frac{q^4}{m_n^4}\left(v_\perp^2 F_{\Sigma'} + 2\frac{q^2}{m_n^2}F_{\Phi''}\right)$$

$F_{M,\Sigma',\Sigma'',\tilde{\Phi}',\Phi'',\Delta}(q^2)$ are standard form factors encoding the distribution of nucleons in the nucleus - suppression at high q.

Coupling to q^2 does not affect the intrinsic directional rate.

But, each term in the response function is proportional to either $(v_\perp)^0$ or $(v_\perp)^2$.

Transverse Radon Transform

For response functions coupling to $(v_\perp)^2$ we need to calculate the *Transverse* Radon Transform (TRT):

$$\hat{f}^T(v_{\min}, \hat{q}) = \int_{\mathbb{R}^3} \frac{(v_\perp)^2}{c^2} f(\vec{v})\,\delta\left(\vec{v}\cdot\hat{q} - v_{\min}\right)\,\mathrm{d}^3\vec{v}$$

In the case of a Maxwell-Boltzmann distribution (e.g. SHM):

$$\hat{f}^T(v_{\min}, \hat{q}) = \frac{\left(2\sigma_v^2 + v_{\mathrm{lag}}^2 - (\vec{v}_{\mathrm{lag}}\cdot\hat{q})^2\right)}{\sqrt{2\pi}\sigma_v c^2} \exp\left[-\frac{(v_{\min} - \vec{v}_{\mathrm{lag}}\cdot\hat{q})^2}{2\sigma_v^2}\right]$$

If we measure recoil angles θ from the mean recoil direction $\vec{v}_{\mathrm{lag}}$:

$$\hat{f}^T(v_{\min}, \hat{q}) = \frac{\left(2\sigma_v^2 + v_{\mathrm{lag}}^2 \sin^2\theta\right)}{\sqrt{2\pi}\sigma_v c^2} \exp\left[-\frac{(v_{\min} - v_{\mathrm{lag}}\cos\theta)^2}{2\sigma_v^2}\right]$$

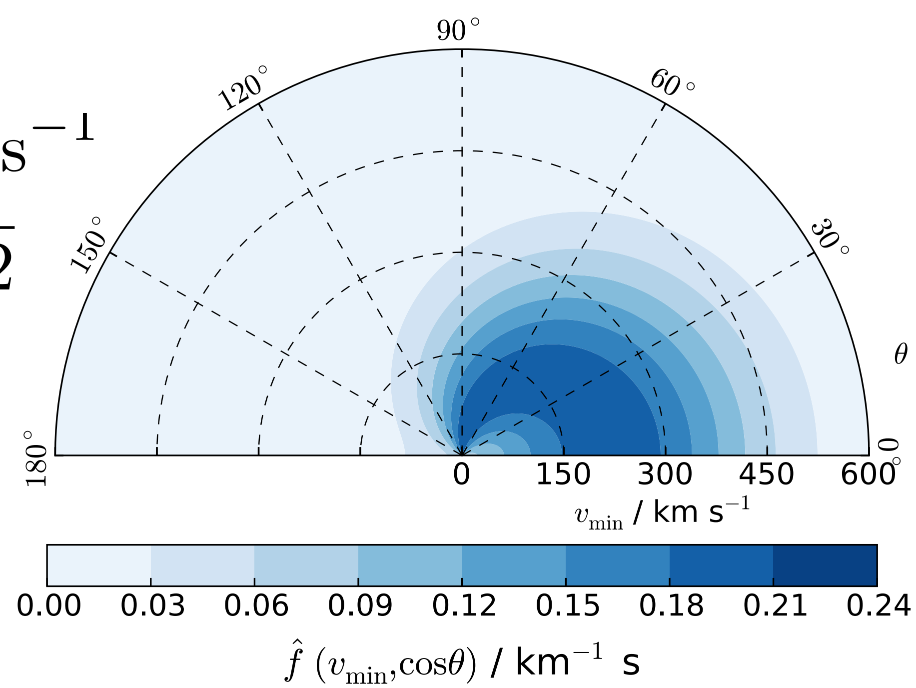
Transverse Radon Transform (examples)

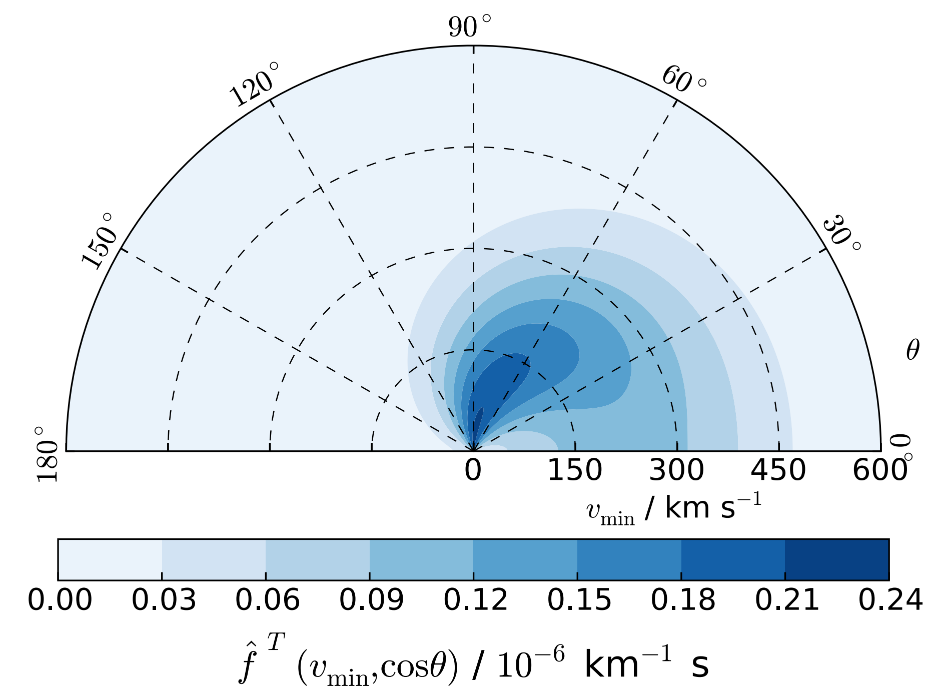
$\hat{f}(v_{\min}, \hat{q})$ $\hat{f}^T(v_{\min}, \hat{q})$

SHM:

$v_{\text{lag}} = 220\,\text{km}\,\text{s}^{-1}$

$\sigma_v = v_{\text{lag}}/\sqrt{2}$

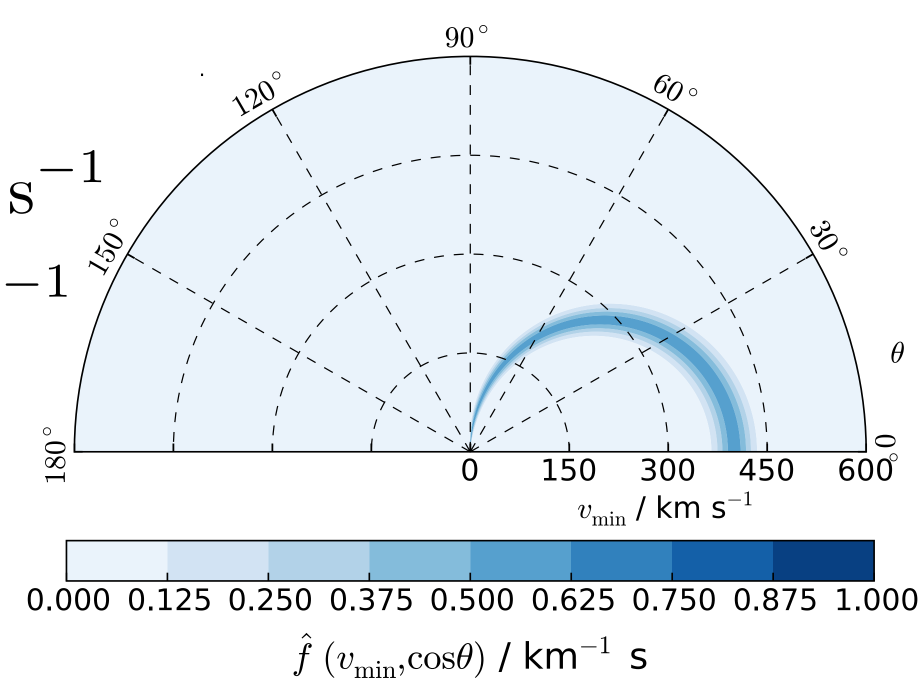


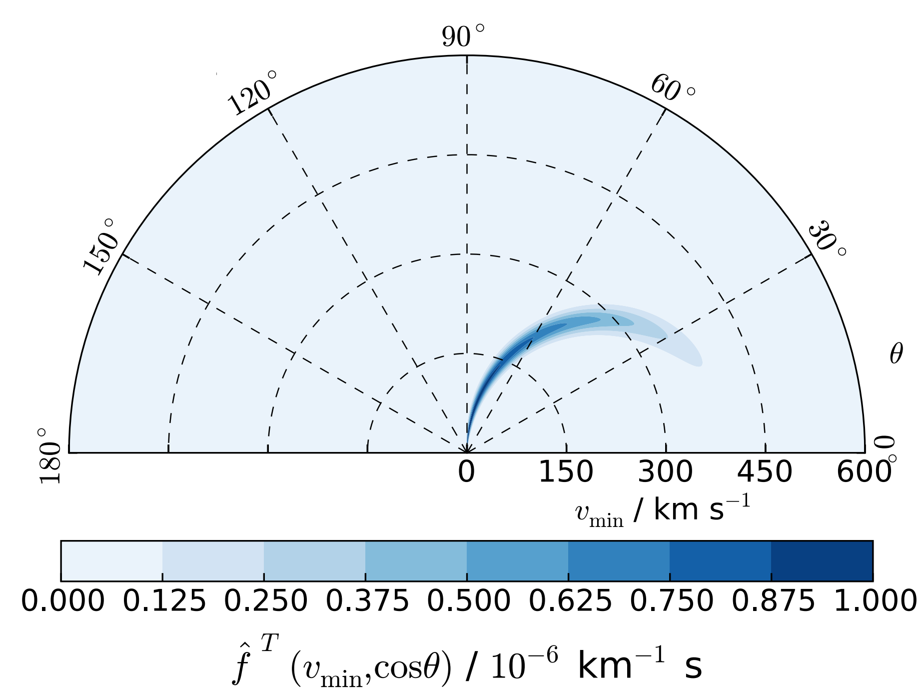


Stream:

$v_{\text{lag}} = 400\,\text{km}\,\text{s}^{-1}$

$\sigma_v = 20\,\text{km}\,\text{s}^{-1}$





Directionality of NREFT operators

Assumptions

Need to specify a detector before we can compare the directional spectra…

Assume a *very ideal* detector:

- CF_4 target, with sense recognition
- Perfect energy and angular resolution
- Zero backgrounds
- Energy range: $E_R \in [20, 50]\,\mathrm{keV}$ [Drift-IId,arXiv:1010.3027]

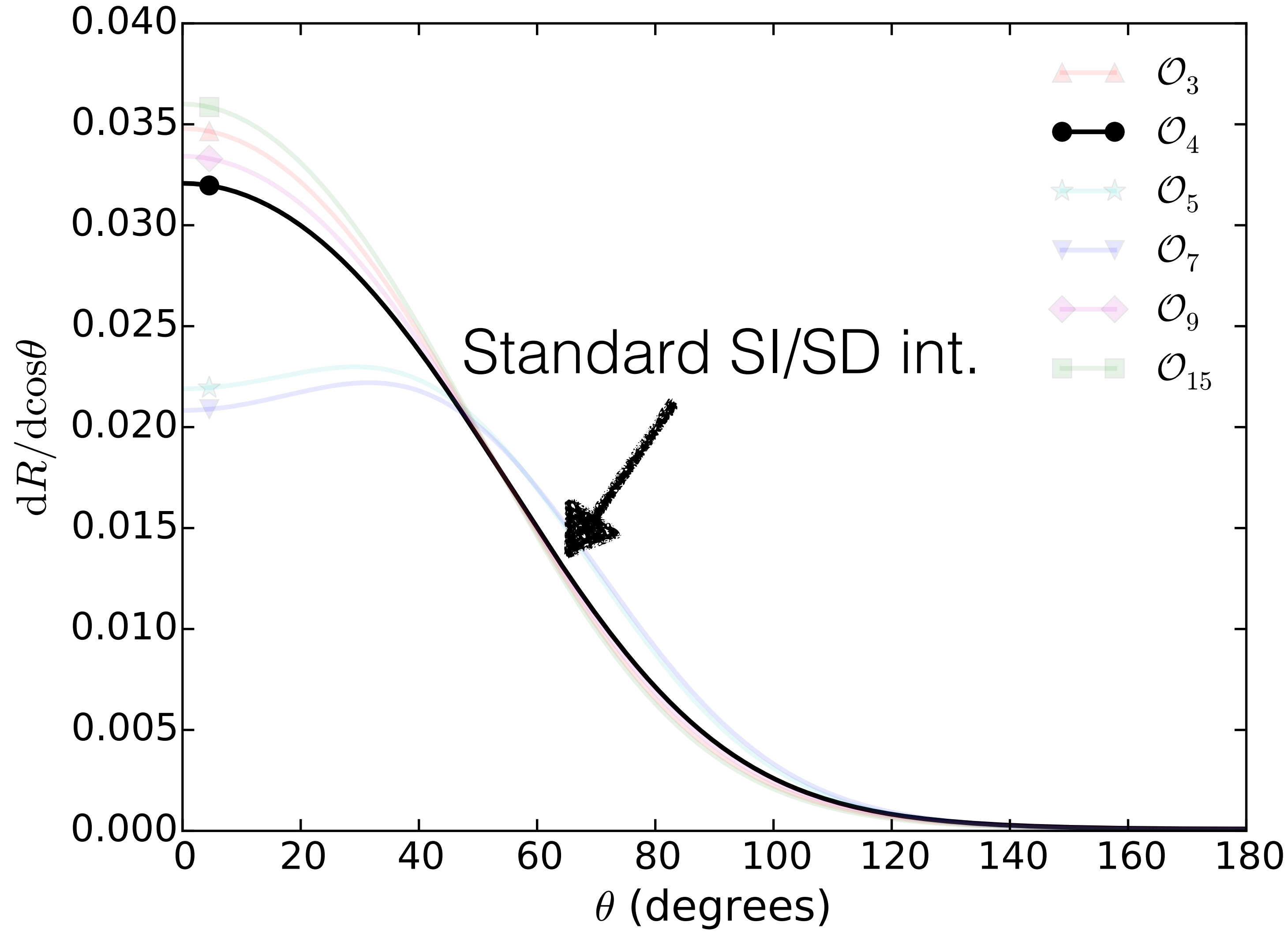
Assume $m_\chi = 100\ \mathrm{GeV}$ (unless otherwise stated) with SHM distribution.

Calculate the directional rate (integrated over energy):

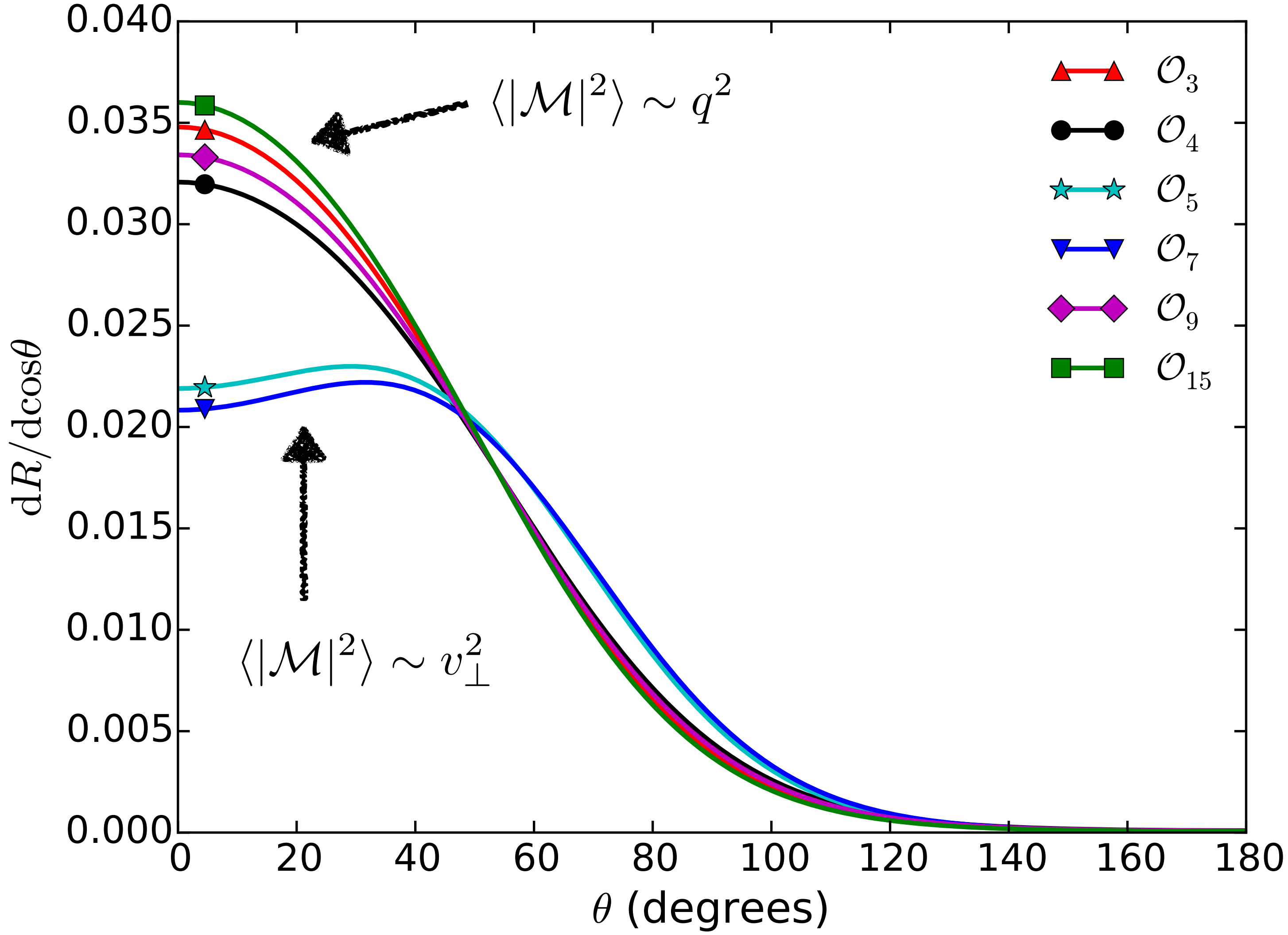
$$\frac{\mathrm{d}R}{\mathrm{d}\Omega_q} = \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} \frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q}\,\mathrm{d}E_R$$

Normalise operators to give the same number of events…

Directional Spectra



Directional Spectra



Note: $q = 2\mu_{\chi N}\vec{v}\cdot\hat{q}$
$= 2\mu_{\chi N} v\cos\theta$

$$\langle|\mathcal{M}|^2\rangle \sim \begin{cases} 1 & : \mathcal{O}_1, \mathcal{O}_4\,, \\ v_\perp^2 & : \mathcal{O}_7, \mathcal{O}_8\,, \\ q^2 & : \mathcal{O}_9, \mathcal{O}_{10}, \mathcal{O}_{11}, \mathcal{O}_{12}\,, \\ v_\perp^2 q^2 & : \mathcal{O}_5, \mathcal{O}_{13}, \mathcal{O}_{14}\,, \\ q^4 & : \mathcal{O}_3, \mathcal{O}_6\,, \\ q^4(q^2 + v_\perp^2) & : \mathcal{O}_{15}\,. \end{cases}$$

Most isotropic:

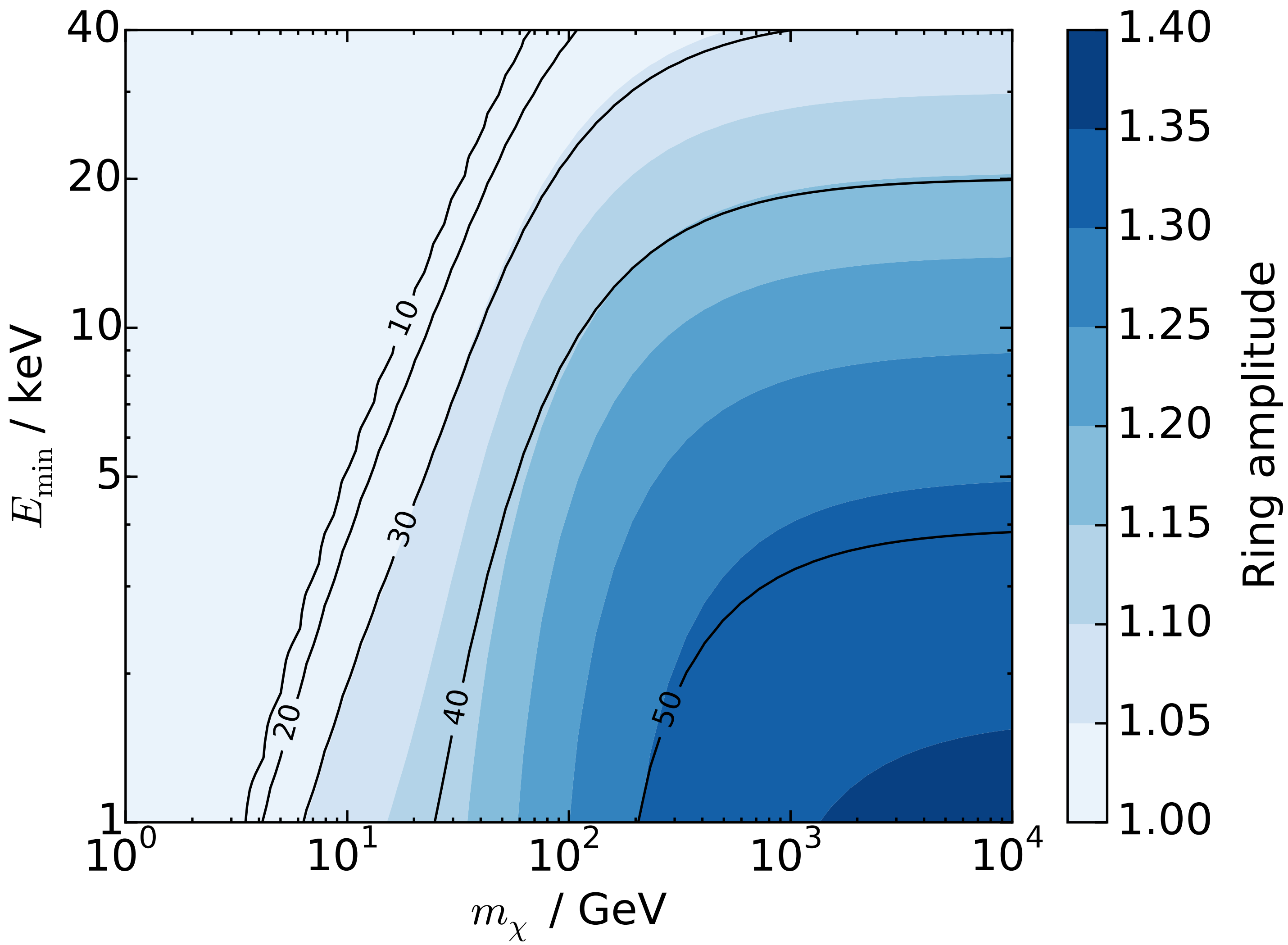
$\mathcal{O}_7 = \vec{S}_n \cdot \vec{v}_\perp$

Least isotropic:

$\mathcal{O}_{15} = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_n})((\vec{S}_n \times \vec{v}_\perp)\cdot\frac{\vec{q}}{m_n})$

A (new) ring-like feature

Operators with $\langle |\mathcal{M}|^2 \rangle \sim (v_\perp)^2$ lead to a 'ring' in the directional rate.



Contours:
ring opening angle
in degrees

Shading:
ring amplitude (ratio of
ring to centre)

A ring in the standard rate has been previously studied [Bozorgnia et al. - 1111.6361], but *this* ring occurs for lower WIMP masses and higher threshold energies.

Statistical tests

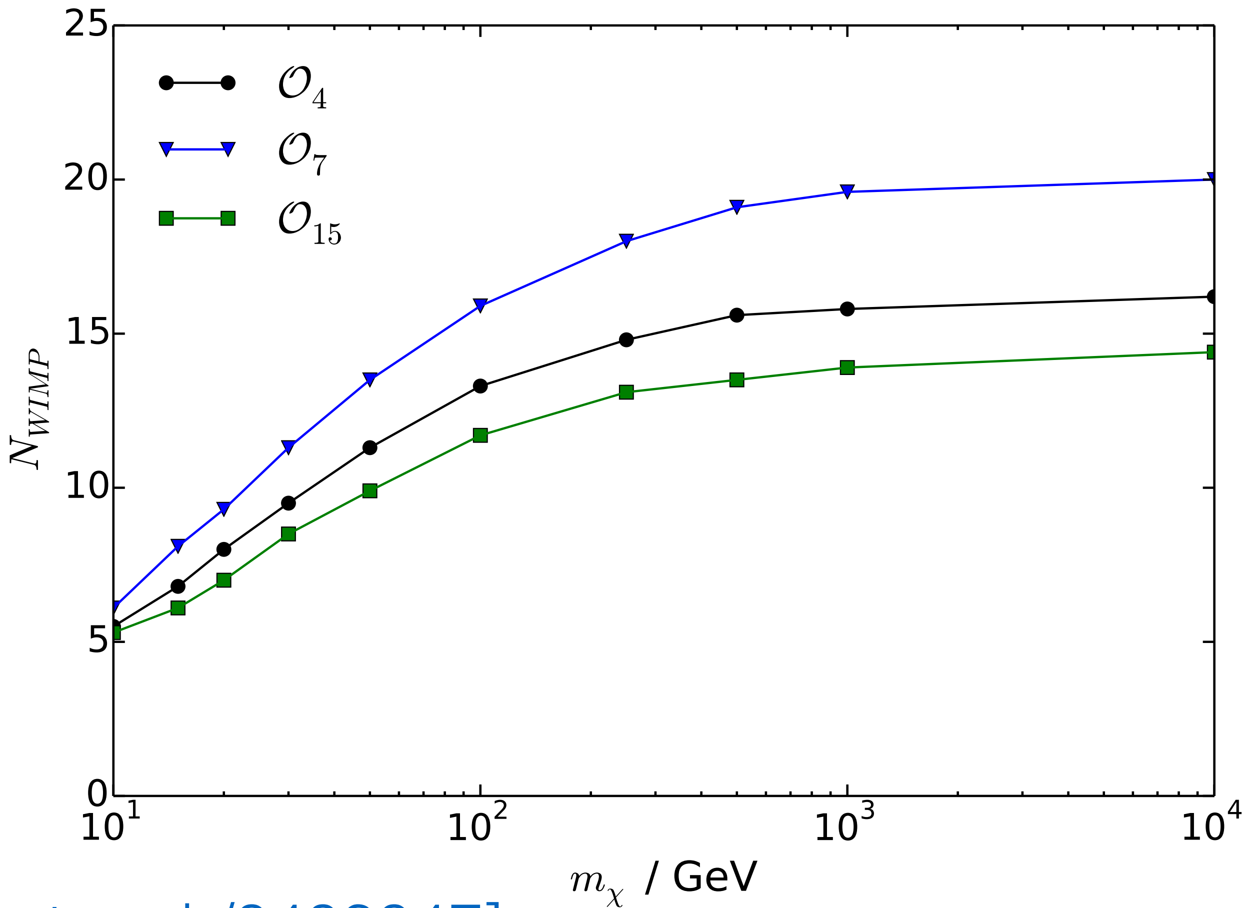
$F_{4,4} \sim 1$

$F_{7,7} \sim v_\perp^2$

$F_{15,15} \sim q^4(q^2 + v_\perp^2)$

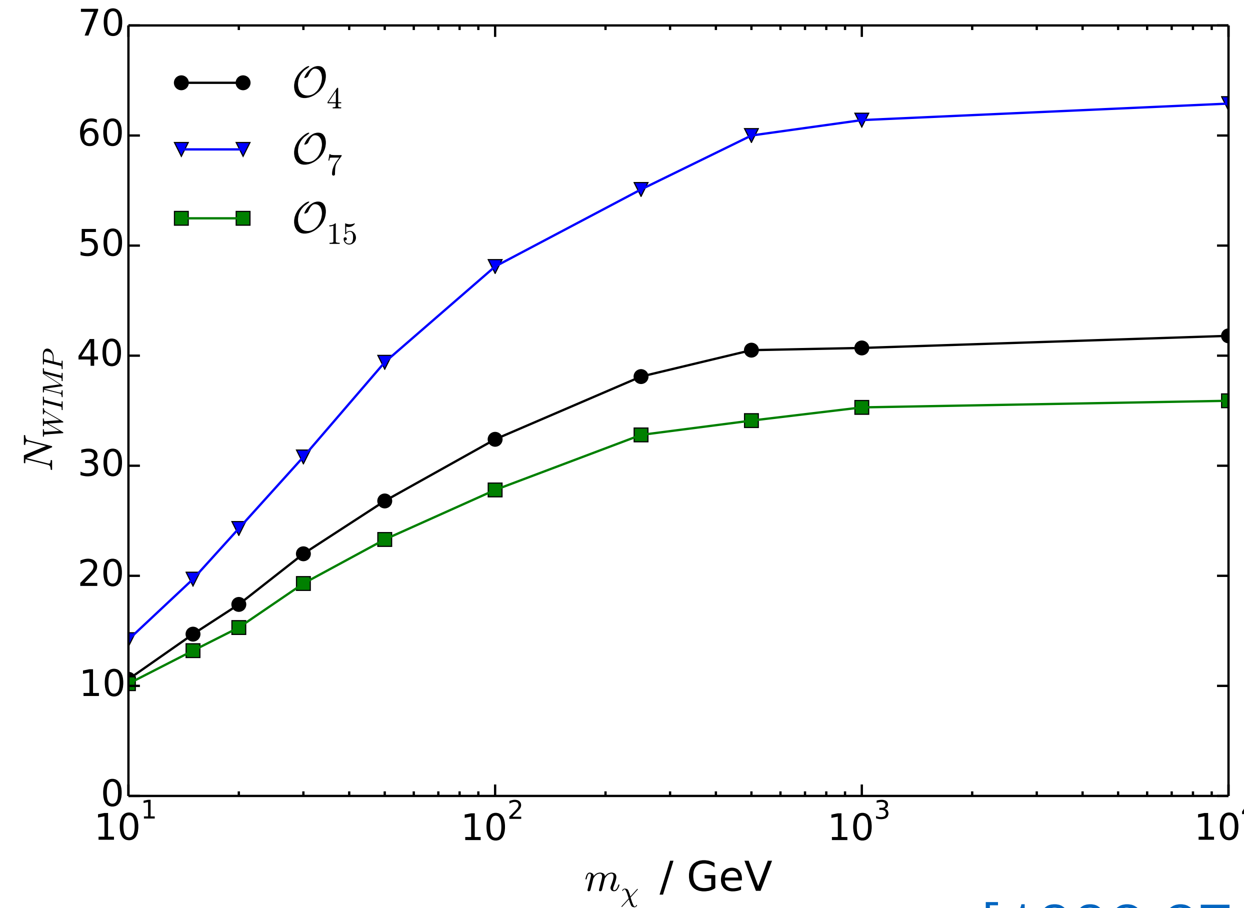
Calculate the number of signal events required to…

…reject isotropy…



[astro-ph/0408047]

…confirm the median recoil dir…



[1002.2717]

…at the 2σ level in 95% of experiment.

Distinguishing NREFT operators

Distinguishing operators

Generate data assuming an NREFT operator ($\mathcal{O}_7$ or $\mathcal{O}_{15}$).

Assume data is a combination of standard SI/SD interaction and non-standard NREFT operator. Fit to data with two free parameters m_χ and A .

A : fraction of events which are due to non-standard NREFT interaction.

Perform likelihood ratio test (in 10000 pseudo-experiments) to determine the significance with which we can reject SD-only interactions:

Null hypothesis, $\mathbf{H_0}$: all events are due to SD interactions, A = 0

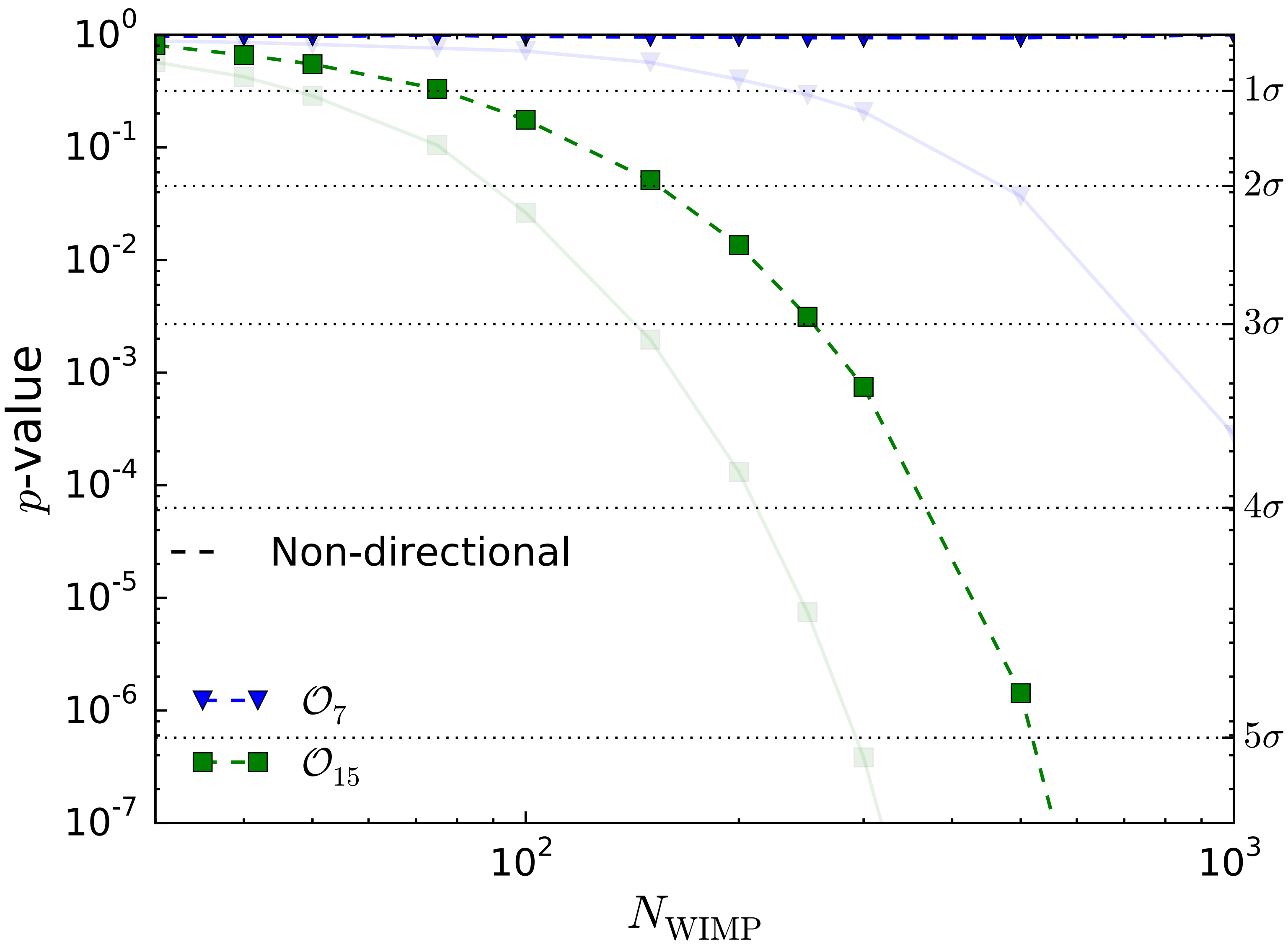
Alt. hypothesis, $\mathbf{H_1}$: there is some contribution from NREFT ops, $A \neq 0$

Distinguishing operators: Energy-only

$F_{4,4} \sim 1$

$F_{7,7} \sim v_\perp^2$

$F_{15,15} \sim q^4(q^2 + v_\perp^2)$

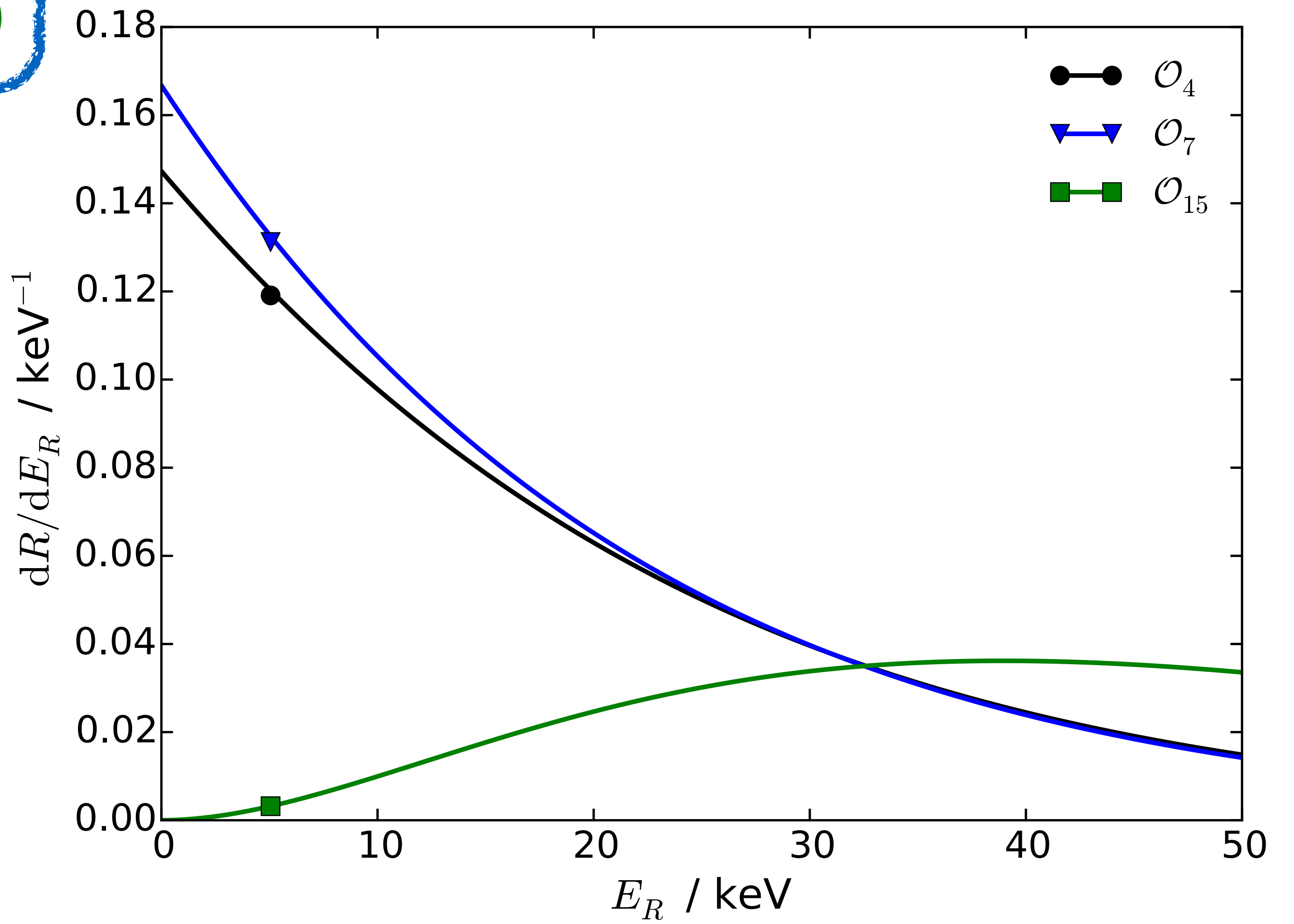


Comparing energy spectra

$F_{4,4} \sim 1$

$F_{7,7} \sim v_\perp^2$

$F_{15,15} \sim q^4(q^2 + v_\perp^2)$



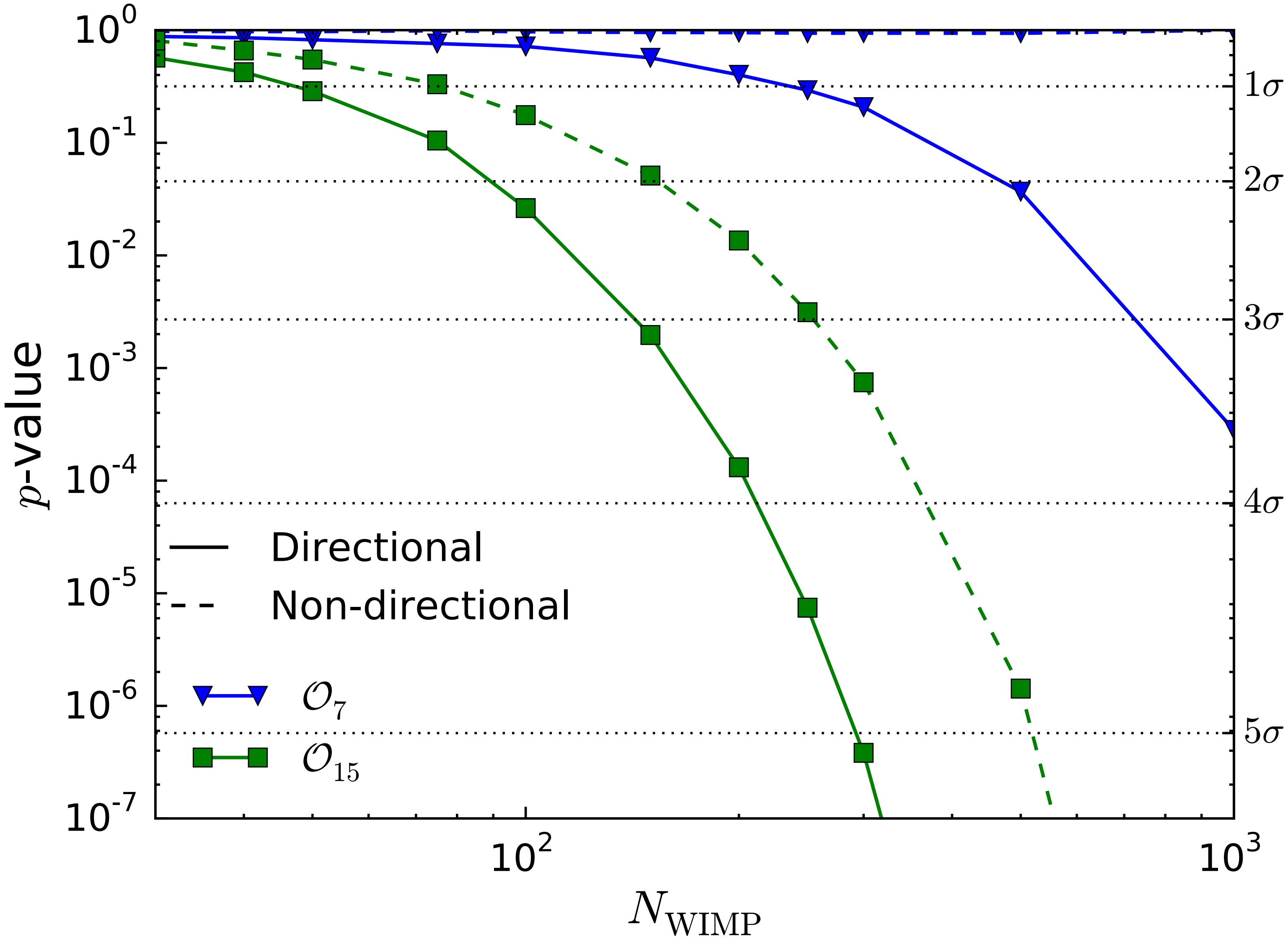
Energy spectra for $\mathcal{O}_4$ and $\mathcal{O}_7$ are indistinguishable:
Forward recoils are suppressed, but transverse recoils are enhanced.
For slowly varying distributions, these two effects roughly cancel.

Distinguishing operators: Energy + Directionality

$F_{4,4} \sim 1$

$F_{7,7} \sim v_\perp^2$

$F_{15,15} \sim q^4(q^2 + v_\perp^2)$



Consequences for relativistic theories

Many 'dictionaries' are available which allow us to translate from relativistic interactions to NREFT interactions
[e.g. 1211.2818, 1307.5955, 1505.03117]

$$\mathcal{L}_1 = \bar{\chi}\chi\bar{n}n \quad \Rightarrow \quad \langle \mathcal{L}_1 \rangle = 4 m_\chi m_n \mathcal{O}_1$$

$$\rightarrow \quad \langle |\mathcal{M}|^2 \rangle \sim F_M(q^2)$$

$$\mathcal{L}_6 = \bar{\chi}\gamma^\mu\gamma^5\chi\bar{n}\gamma_\mu n \quad \Rightarrow \quad \langle \mathcal{L}_6 \rangle = 8 m_\chi (m_n \mathcal{O}_8 + \mathcal{O}_9)$$

$$\rightarrow \quad \langle |\mathcal{M}|^2 \rangle \sim v_\perp^2 F_M(q^2)$$

These two relativistic operators cannot be distinguished without directional detection.

Open issues

- We have assumed an *ideal* detector - lower limits on the event numbers (need to be convolved with detector effects…)
- Different signatures possible for different target materials - see (very) recent paper by Catena [1505.06441]
- Astrophysical uncertainties are expected to be comparable with particle physics uncertainties
    - inability to distinguish different operators depends on SHM-type distribution (may be different for sharp stream-like distributions)
- May be possible to distinguish operators using other methods - measuring annual modulation [1504.06772]

In the future, it would be interesting to examine astrophysical uncertainties in detail, and to compare different approaches to distinguishing NREFT operators.

Conclusions

- NREFT operators lead to new directional signatures:
  - Operators coupling to q^2 lead to more directional rates
  - Operators coupling to $v_\perp^2$ lead to more isotropic rates

- Possible ring-like feature, even for high thresholds and light DM
- Factor of ~2 uncertainty in number of events needed to confirm WIMP signal

- With $\mathcal{O}(100-500)$ events it should be possible to distinguish NREFT operators from standard operators at the 2σ level
- Directional sensitivity allows us to discriminate between operators which otherwise would be indistinguishable using energy-only experiments

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Thank you