Discretising the velocity dist...

dark matt...

From the makers of the smash-hit:

'Who ordered all these operators?'

...agn (IPhT - CEA/Saclay)

...GNUS 2015 - 4th June 2015

Based on arXiv:1502.04224

Discretising the velocity distribution for directional dark matter experiments
or
'Pi in the sky'

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CYGNUS 2015 - 4th June 2015

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NewDark

Based on arXiv:1502.04224

The problem

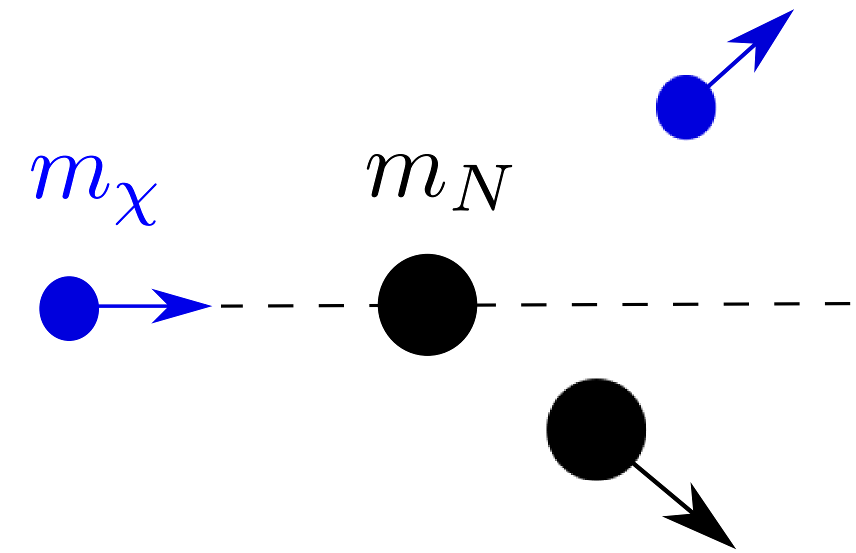
The analysis of direct(ional) detection experiments requires assumptions about the DM velocity distribution $f(\mathbf{v})$.

Poor assumptions about $f(\mathbf{v})$ can lead to biased limits or reconstructions on particle physics parameters such as m_χ and σ^p.

Question:
Can we instead extract $f(\mathbf{v})$ from directional data, without assuming a particular functional form?

Directional event rate

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} = \frac{\rho_0}{4\pi\mu_{\chi p}^2 m_\chi}\sigma^p \mathcal{C}_N F^2(E_R)\hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}})$$



Components:

- Local DM density, $\rho_0 \approx 0.3\,\mathrm{GeV}\,\mathrm{cm}^{-3}$
- DM-proton cross section, σ^p
- 'Enhancement factor', $\mathcal{C}_N$

$$v_{\mathrm{min}} = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}$$

Depends on target nucleus N, and type of interaction (SI/SD)

- Radon transform of velocity distribution, for recoils in dir. $\hat{\mathbf{q}}$:

$$\hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) = \int_{\mathbb{R}^3} f(\mathbf{v})\delta\left(\mathbf{v}\cdot\hat{\mathbf{q}} - v_{\mathrm{min}}\right)\mathrm{d}^3\mathbf{v}$$

Astrophysical uncertainties

Standard Halo Model

Standard Halo Model (SHM) is typically assumed: isotropic, spherically symmetric distribution of particles with $\rho(r) \propto r^{-2}$.

Leads to a Maxwell-Boltzmann distribution in the Galactic frame

$$f_{\mathrm{Gal}}(\mathbf{v}) = (2\pi\sigma_v^2)^{-3/2} \exp\left[-\frac{\mathbf{v}^2}{2\sigma_v^2}\right] \Theta(v - v_{\mathrm{esc}})$$

Perform Galilean transform $\mathbf{v} \rightarrow \mathbf{v} - \mathbf{v}_{\mathrm{lag}}$ to obtain distribution in lab frame:

$$f_{\mathrm{Lab}}(\mathbf{v}) = (2\pi\sigma_v^2)^{-3/2} \exp\left[-\frac{(\mathbf{v} - \mathbf{v}_{\mathrm{lag}})^2}{2\sigma_v^2}\right] \Theta(|\mathbf{v} - \mathbf{v}_{\mathrm{lag}}| - v_{\mathrm{esc}})$$

Standard values:

$$\mathbf{v}_{\mathrm{lag}} = -\mathbf{v}_e(t) \;\sim 180 - 270\,\mathrm{km\,s}^{-1}$$

[astro-ph/9706293,1207.3079, 1209.0759, 1312.1355]

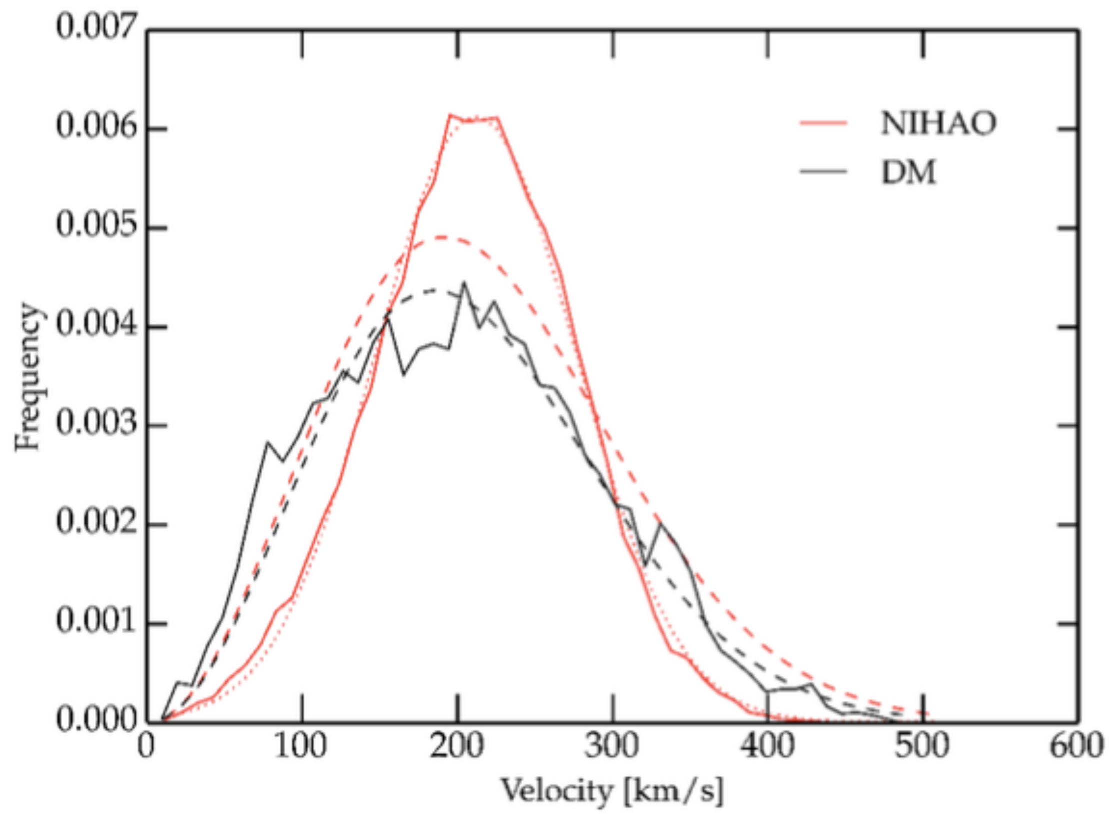
$$\sigma_v \approx v_{\mathrm{lag}}/\sqrt{2}$$

$$v_{\mathrm{esc}} = 533^{+54}_{-41}\,\mathrm{km\,s}^{-1}$$

[1309.4293]

N-body simulations

- Evidence of non-Maxwellian structure from N-body simulations



[0912.2358, 1308.1703, 1503.04814]

- *Streams* may be present due to tidally disrupted satellites [astro-ph/0310334, astro-ph/0309279]
- *Dark disk* may form from sub haloes dragged into the plane of the stellar disk [0901.2938, 1308.1703, 1504.02481]
- *Debris flows*, from sub haloes which are not completely phase-mixed [1105.4166]

Impact on directional detection

- Astrophysical uncertainties have been much studied in *non*-directional experiments [e.g. 1103.5145, 1206.2693, 1207.2039]
- Presence of a dark disk should not affect directional discovery limits, but may bias reconstruction of WIMP mass and cross section [1207.1050]
- May also be able to extract properties of halo, stream, dark disk etc. from directional data - *if* the form of the distribution is known [1202.5035]

Directional detection is the only way to probe the full 3-dimensional velocity distribution $f(\mathbf{v})$ *.*

Attempts at a solution

- Direct inversion of Radon transform $\hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) \rightarrow f(\mathbf{v})$ [hep-ph/0209110]

Mathematically *unstable* - not feasible without huge numbers of events

- Physical parametrisation: assume a particular form for $f(\mathbf{v})$ (e.g. SHM, or SHM with stream) and fit the parameters (e.g. v_{lag} and σ_v). [1012.3960, 1202.5035, 1410.2749]

Fails if $f(\mathbf{v})$ cannot be described by the assumed parametrisation

- Empirical parametrisation…?

Empirical parametrisations

In the analysis of *non-directional* experiments, we have previously looked at general, empirical parametrisations for $f(v)$:

- Binned parametrisation [Peter - 1103.5145, 1207.2039]
- Polynomial parametrisation [1303.6868, 1312.1852]

Allows us to reconstruct both WIMP mass and velocity distribution simultaneously - *without bias*.

But for 3-D, we have an infinite number of 1-D functions to parametrise. Need to define an appropriate basis:

$$f(\mathbf{v}) = f^1(v)A^1(\hat{\mathbf{v}}) + f^2(v)A^2(\hat{\mathbf{v}}) + f^3(v)A^3(\hat{\mathbf{v}}) + \ldots .$$

If we choose the right basis and truncation, we reduce the problem to parametrising a finite number of functions.

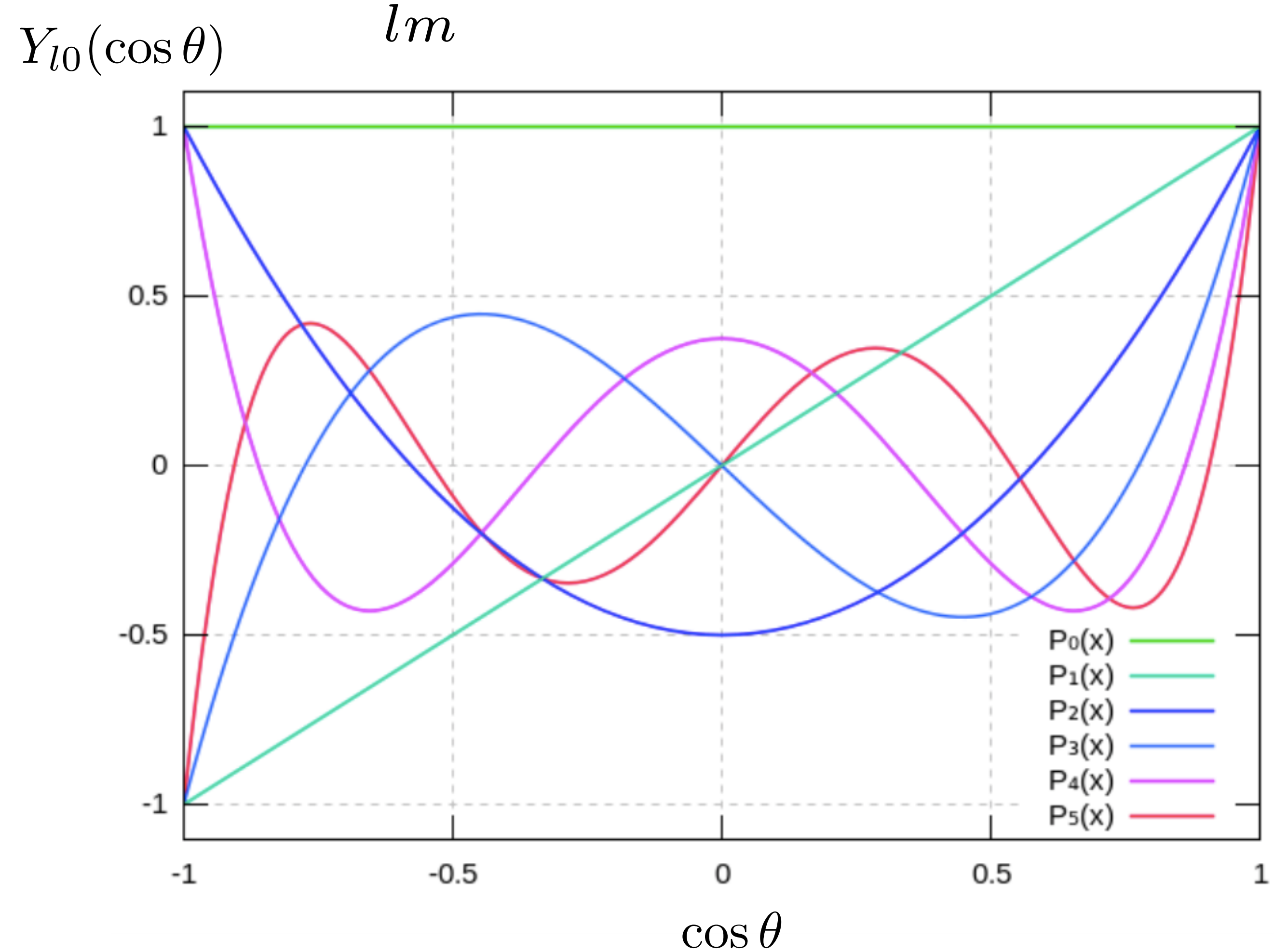
The positivity problem

[Alves et al. - 1204.5487, Lee - 1401.6179]

One possible basis is spherical harmonics. These have nice properties:

$$f(\mathbf{v}) = \sum_{lm} f_{lm}(v) Y_{lm}(\hat{\mathbf{v}})$$

$$\Rightarrow \hat{f}(v_{\min}, \hat{\mathbf{q}}) = \sum_{lm} \hat{f}_{lm}(v_{\min}) Y_{lm}(\hat{\mathbf{q}})$$



However, they are not strictly positive definite!

If we try to fit with spherical harmonics, we cannot guarantee that we get a physical distribution function!

A discretised distribution

Discretising the distribution

Divide the velocity distribution into N angular bins…

$$f(\mathbf{v}) = f(v, \cos\theta', \phi') = \begin{cases} f^1(v) & \text{for } \theta' \in [0, \pi/N] \\ f^2(v) & \text{for } \theta' \in [\pi/N, 2\pi/N] \\ & \vdots \\ f^k(v) & \text{for } \theta' \in [(k-1)\pi/N, k\pi/N] \\ & \vdots \\ f^N(v) & \text{for } \theta' \in [(N-1)\pi/N, \pi] \end{cases}$$

…and then we can parametrise $f^k(v)$ within each angular bin.

In principle, we could also discretise in ϕ', but assuming $f(\mathbf{v})$ is independent of ϕ' does not introduce any error.

Investigating the discretisation error

The idea is to investigate the 'discretisation error' - the difference in rates induced if we use the discretised distribution rather than the full one.

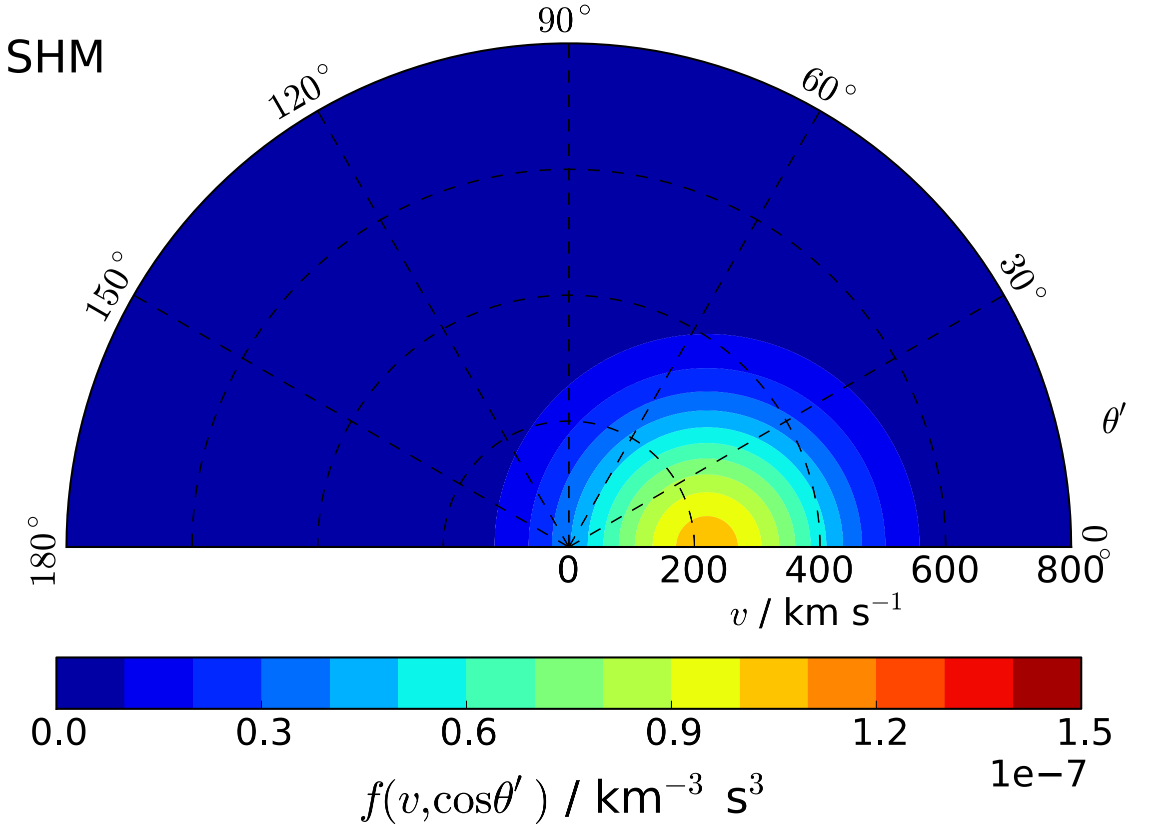
If this error is small enough, we can use the discrete basis to try and reconstruct $f(\mathbf{v})$ reliably.

For now, we will just look at the angular discretisation - we won't look at parametrising the functions $f^k(v)\ldots$

Instead, we fix $f^k(v)$ by setting it equal it is the average over the angular bin:

$$f^k(v) = \frac{1}{\cos((k-1)\pi/N) - \cos(k\pi/N)} \int_{\cos(k\pi/N)}^{\cos((k-1)\pi/N)} f(\mathbf{v})\,\mathrm{d}\cos\theta'\,.$$

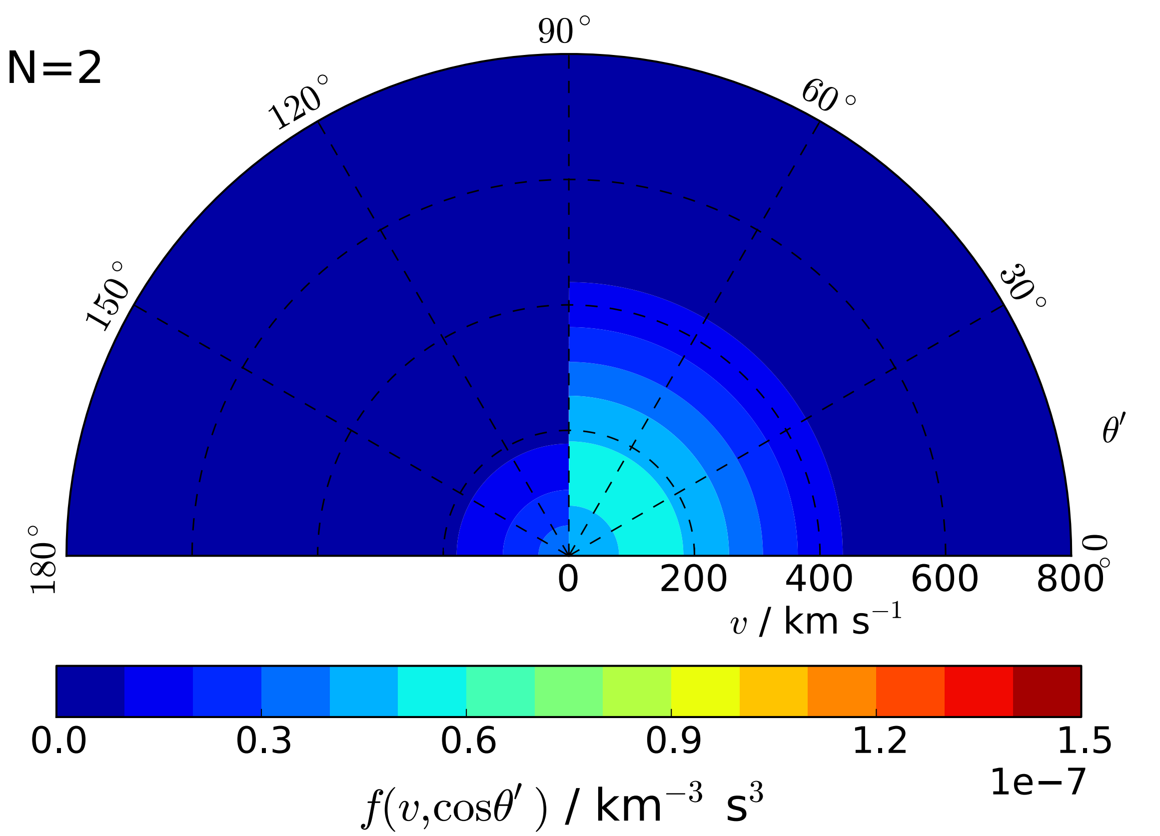
Examples: SHM

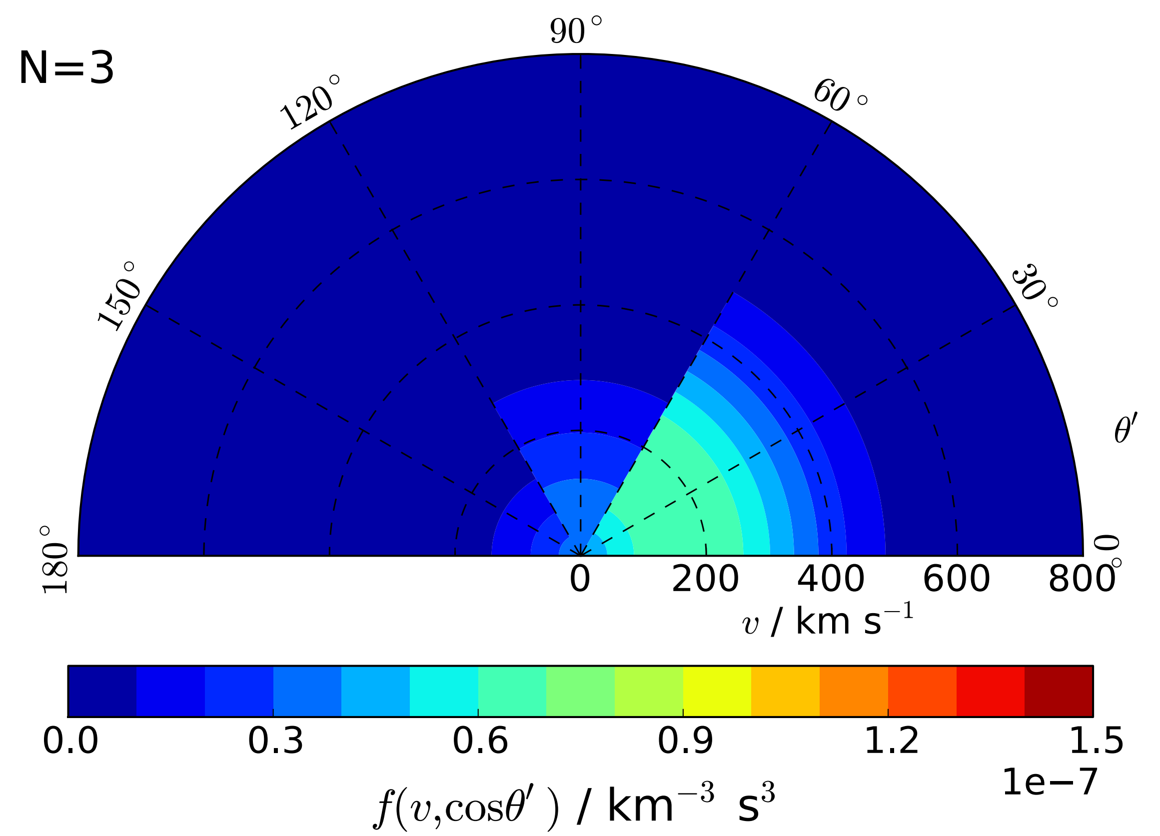


$f(\mathbf{v})$

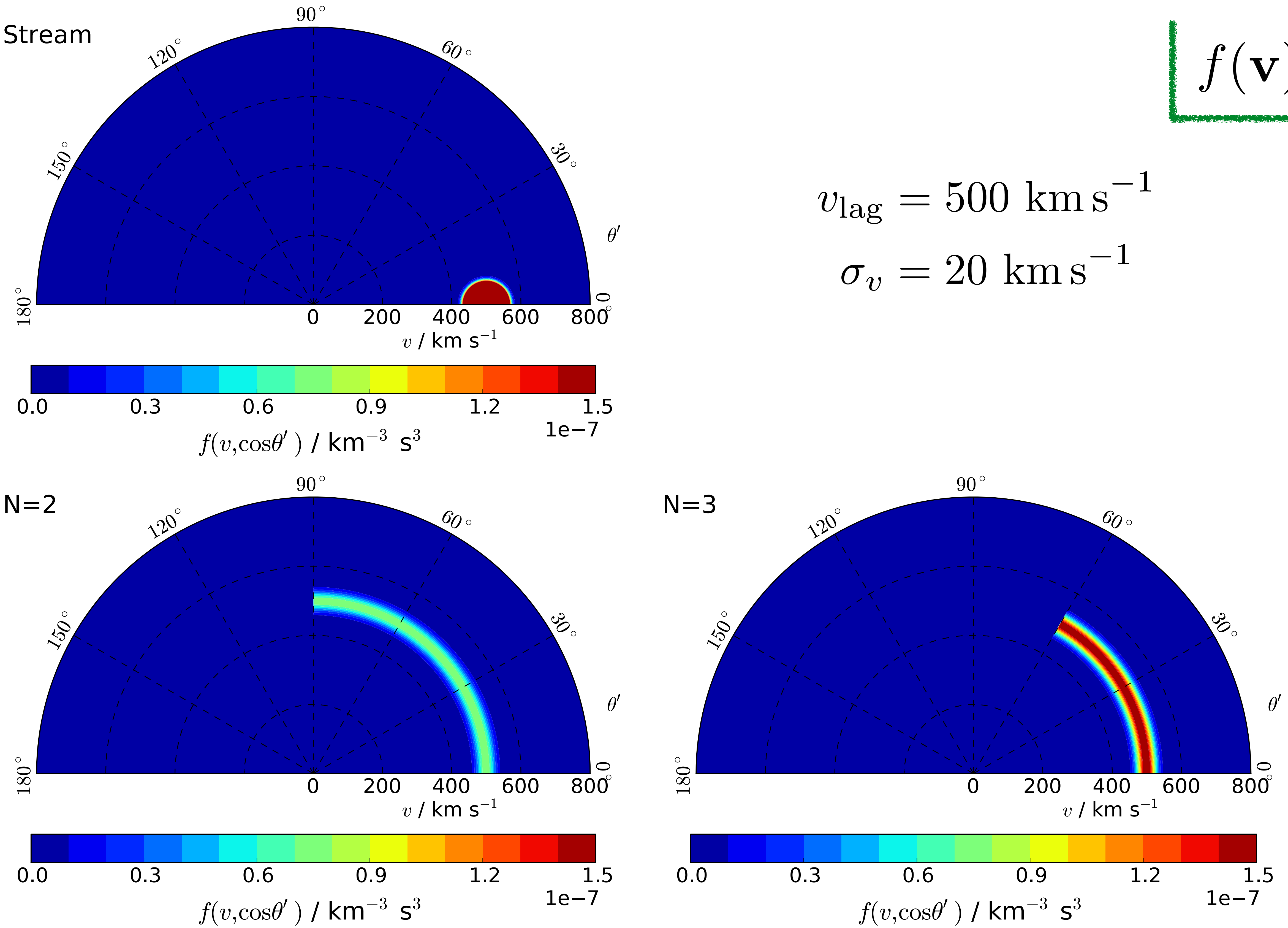
$$v_{\rm lag} = 220\ {\rm km\,s^{-1}}$$

$$\sigma_v = 156\ {\rm km\,s^{-1}}$$





Examples: Stream



$f(\mathbf{v})$

$$v_{\rm lag} = 500\ \mathrm{km\,s^{-1}}$$

$$\sigma_v = 20\ \mathrm{km\,s^{-1}}$$

Integrated Radon Transform (IRT)

We have discarded angular information - we don't expect this discrete distribution to give a good approximation to the full directional event rate.

However, we can consider instead the integrated Radon Transform (IRT):

$$\hat{f}^j(v_{\mathrm{min}}) = \int_{\phi=0}^{2\pi} \int_{\cos(j\pi/N)}^{\cos((j-1)\pi/N)} \hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) \, \mathrm{d}\cos\theta \mathrm{d}\phi \,,$$

We lose information (essentially binning the data) but this should reduce the error involved in using the discretised distribution.

This in turn means that we can use the discretised distribution to parametrise $f(\mathbf{v})$ and extract information from it reliably.

Calculating the Radon Transform

The calculation of the Radon Transform is rather involved, but it can be carried out analytically in the angular variables for an arbitrary number of bins N, and reduced to N integrations over the speed v.

[See 1502.04224 for full expressions - Python code available on request]

For N = 1:

The 'approximation' is exact…

$$\hat{f}^1(v_{\mathrm{min}}) = 8\pi^2 \int_{v_{\mathrm{min}}}^{\infty} f^1(v) v \,\mathrm{d}v = 2\pi \int_{v_{\mathrm{min}}}^{\infty} \frac{f(\mathbf{v})}{v} \,\mathrm{d}^3\mathrm{v}$$

For N = 2:

$$\hat{f}^1(v_{\mathrm{min}}) = 4\pi \int_{v_{\mathrm{min}}}^{\infty} v \left\{ \pi f^1(v) + \tan^{-1}\left(\frac{\sqrt{1-\beta^2}}{\beta}\right) \left[f^2(v) - f^1(v)\right] \right\} \mathrm{d}v$$

$$\hat{f}^2(v_{\mathrm{min}}) = 4\pi \int_{v_{\mathrm{min}}}^{\infty} v \left\{ \pi f^2(v) + \tan^{-1}\left(\frac{\sqrt{1-\beta^2}}{\beta}\right) \left[f^1(v) - f^2(v)\right] \right\} \mathrm{d}v$$

$$\beta = \frac{v_{\mathrm{min}}}{v}$$

Comparison with exact results

Detector set-up

We consider a CF_4 detector with energy threshold 20 keV, with perfect angular and energy resolution. Assume 100 GeV WIMP.
Déjà vu?

We compare:

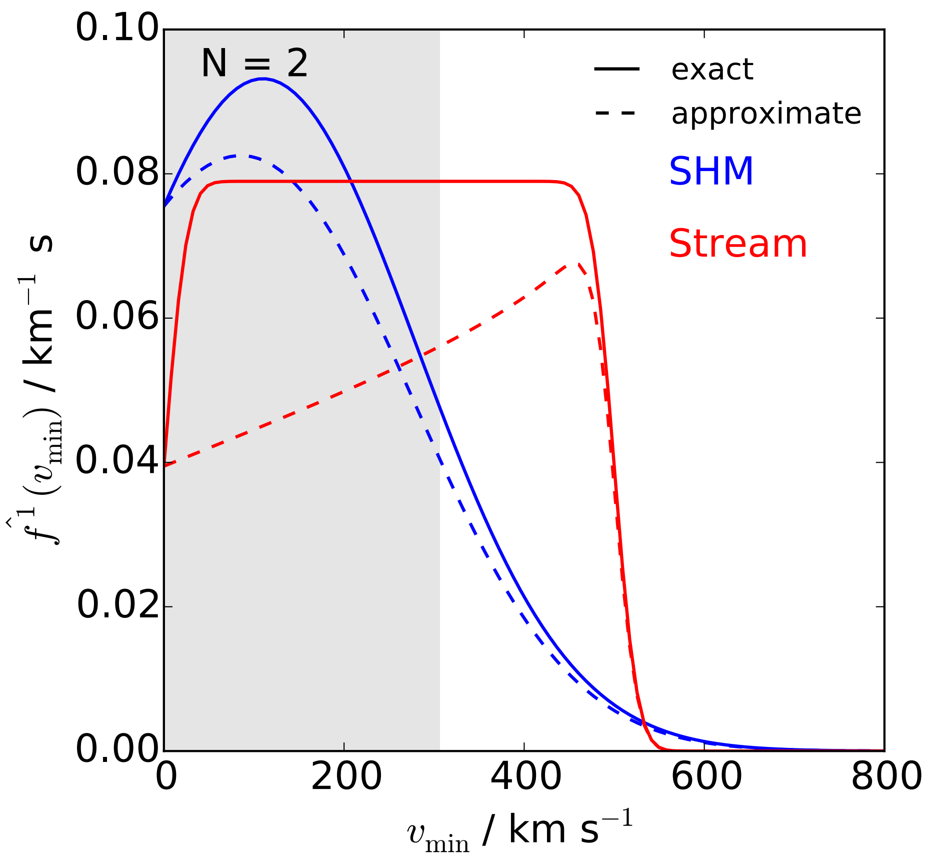
——— *Exact IRT* - calculated from the true, full distribution
- - - - - *Approx. IRT* - calculated from discretised distribution

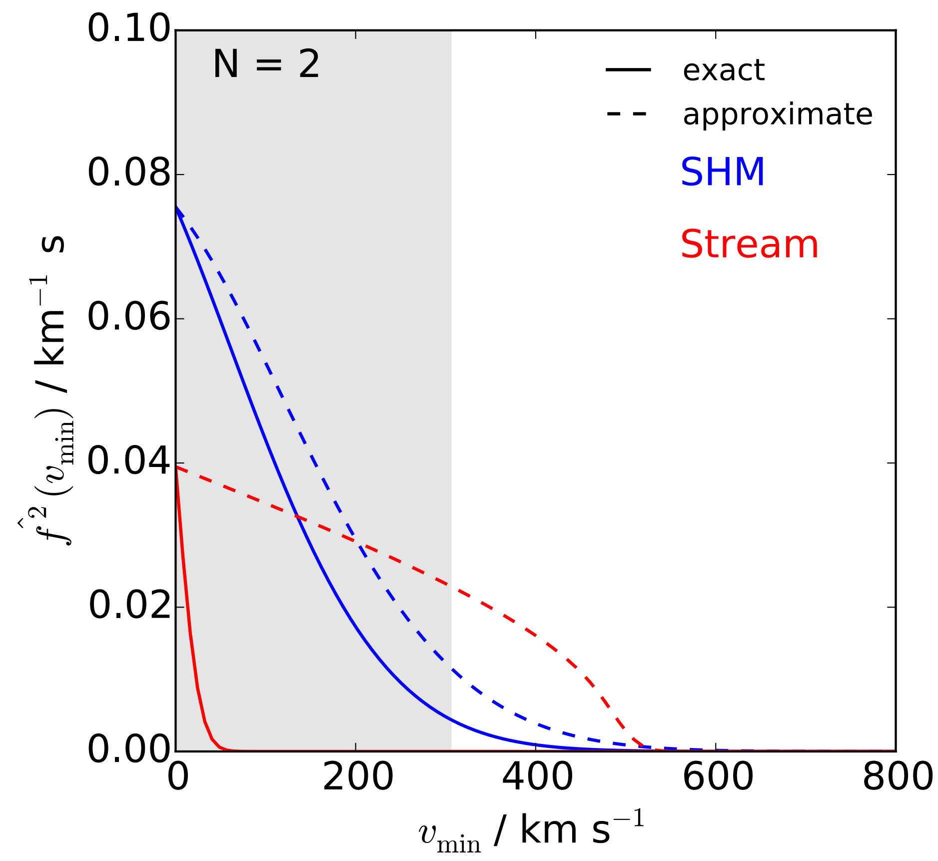
We also compare the total number of events in each angular bin assuming 50 signal events and 1 isotropic background event:

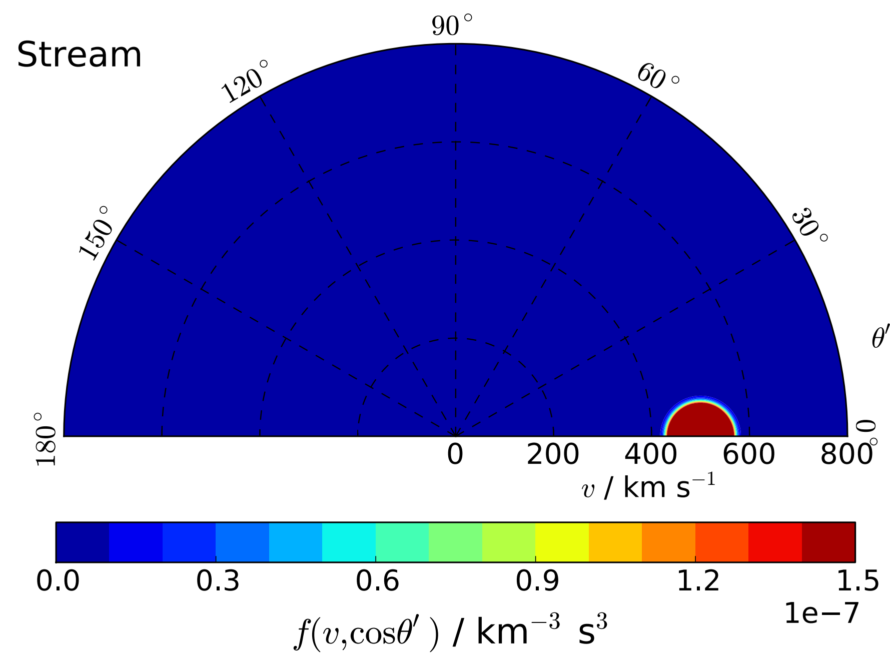
$$N_j \propto \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} \hat{f}^j(v_{\mathrm{min}}(E_R)) F^2(E_R) \, \mathrm{d}E_R$$

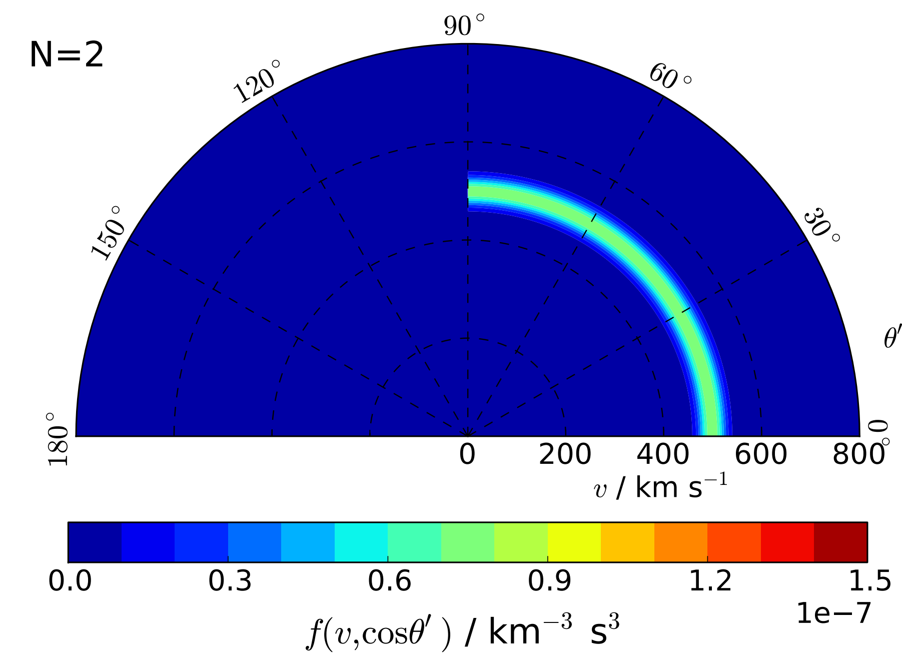
$$+ \mathrm{BG}$$

N = 2 discretisation

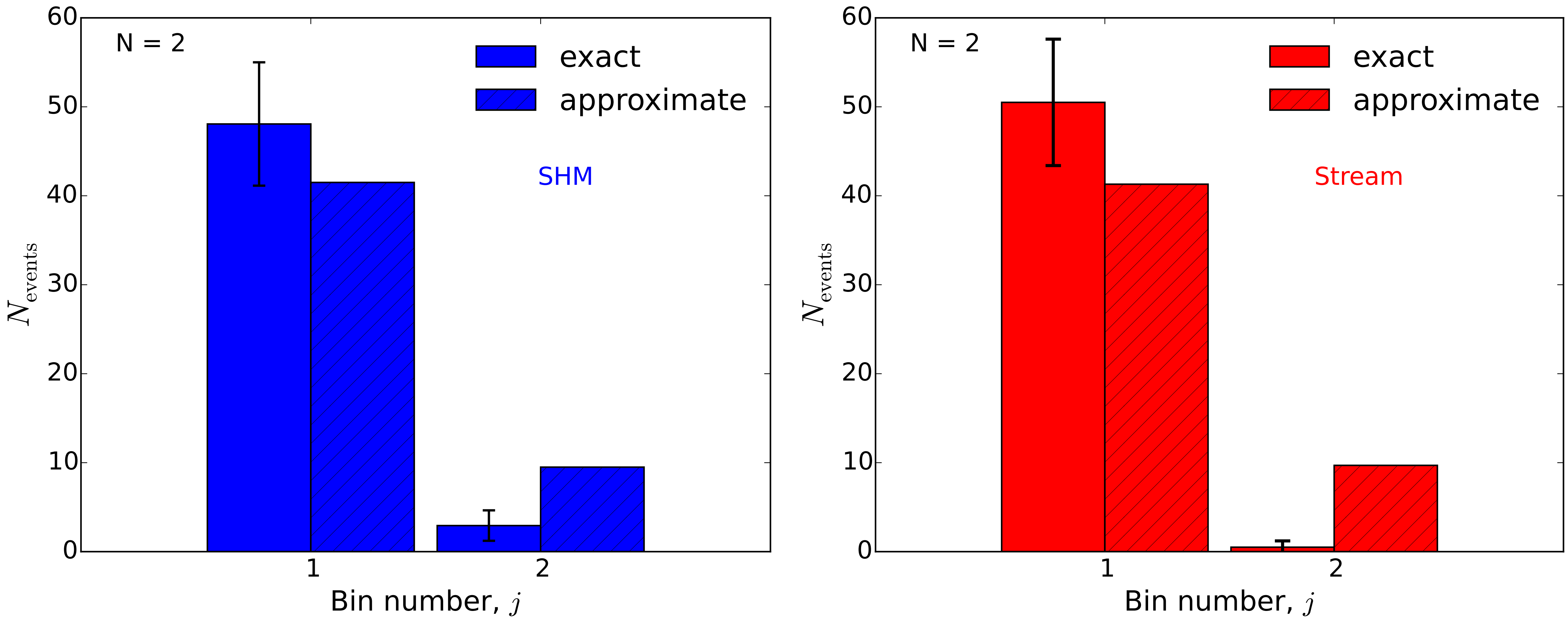




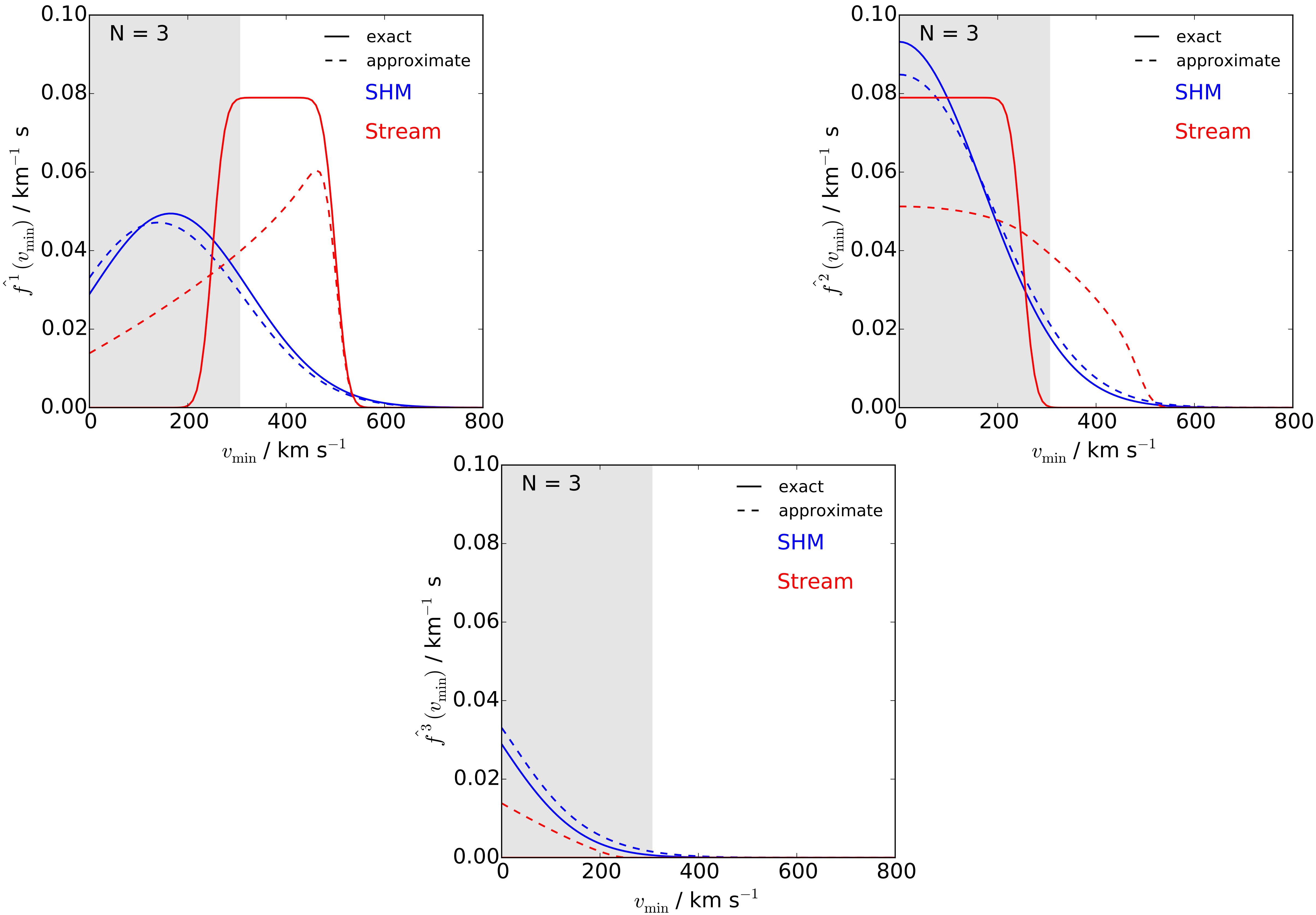




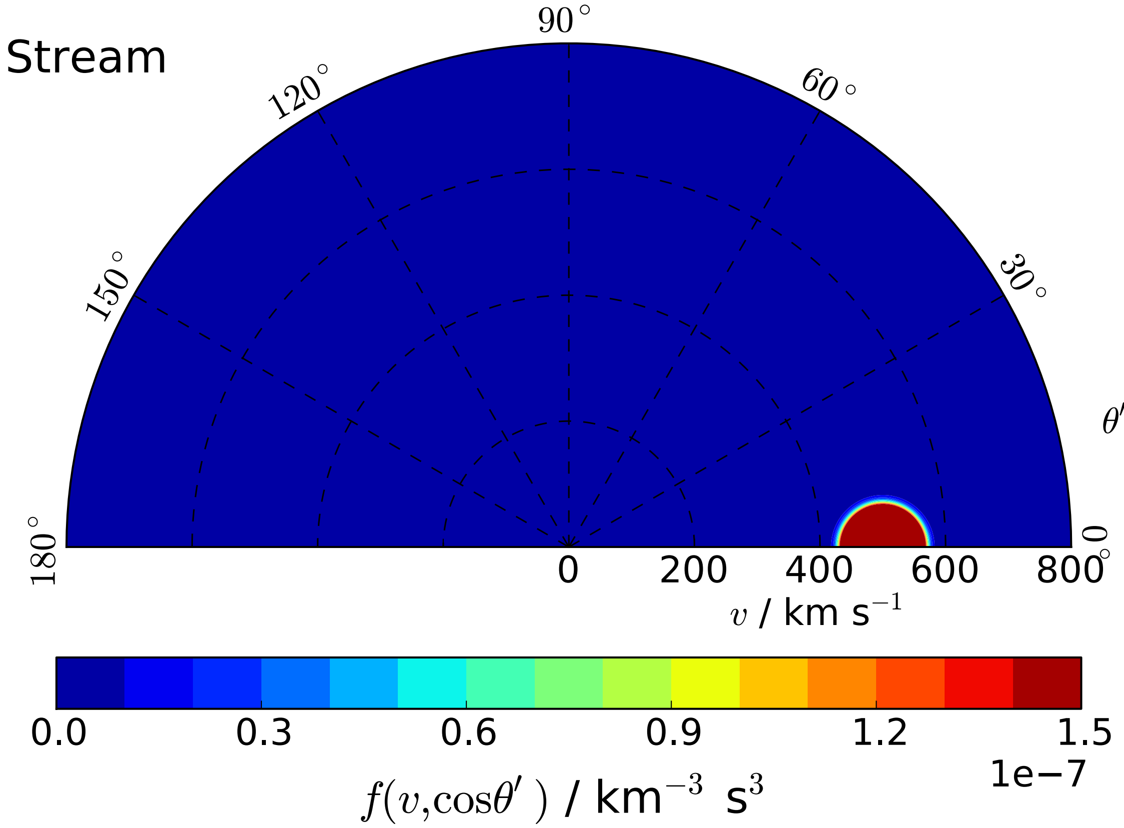
N = 2 discretisation - event numbers



N = 3 discretisation



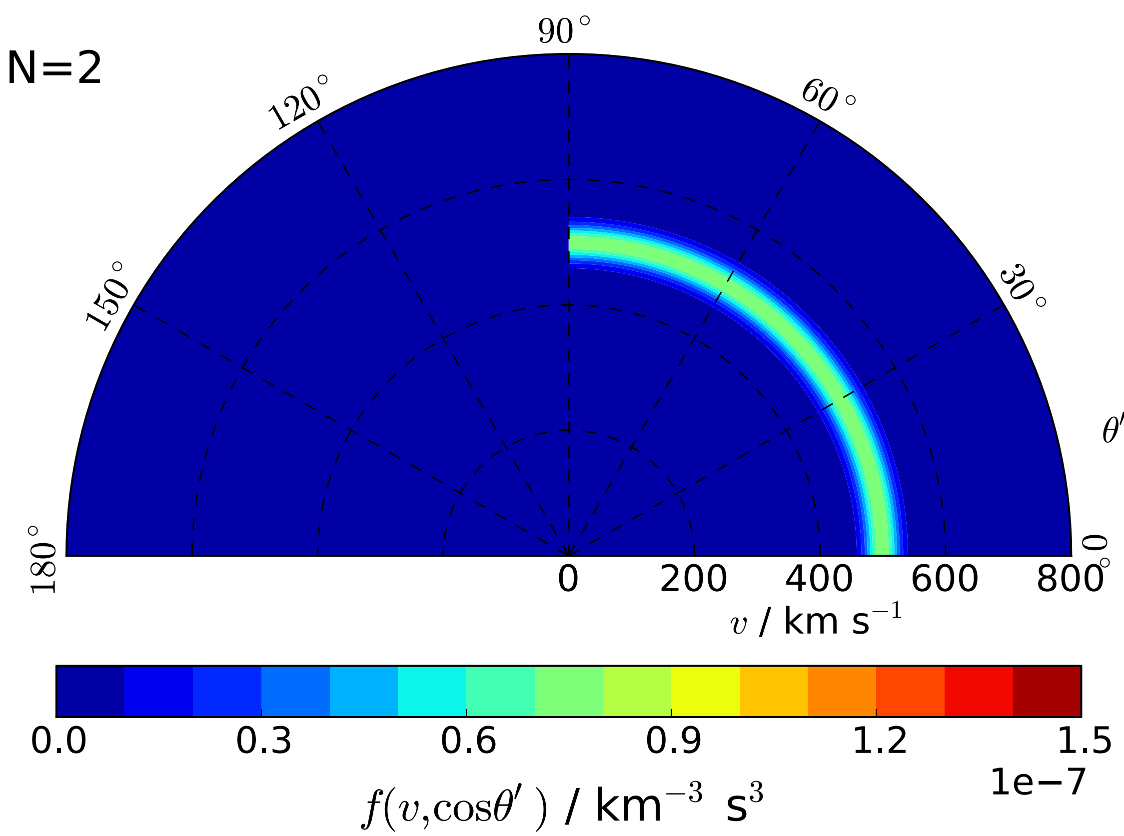
Examples: Stream (revisited)

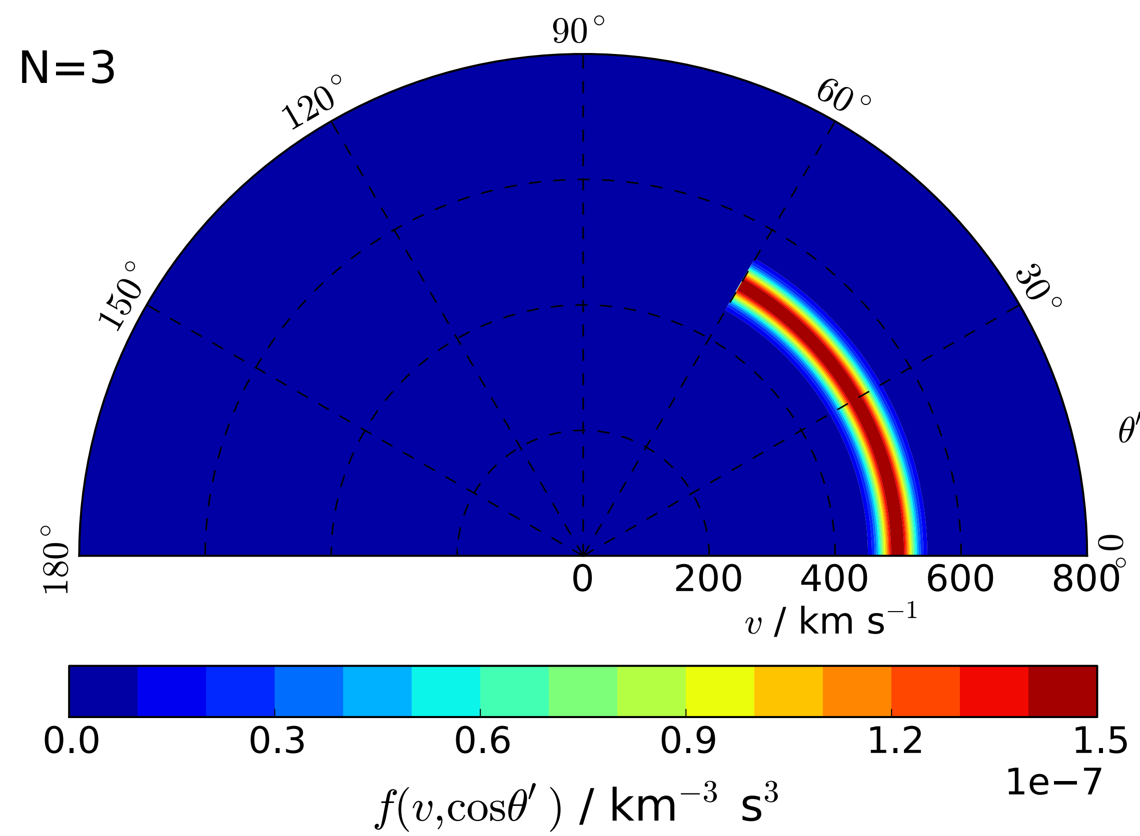


$f(\mathbf{v})$

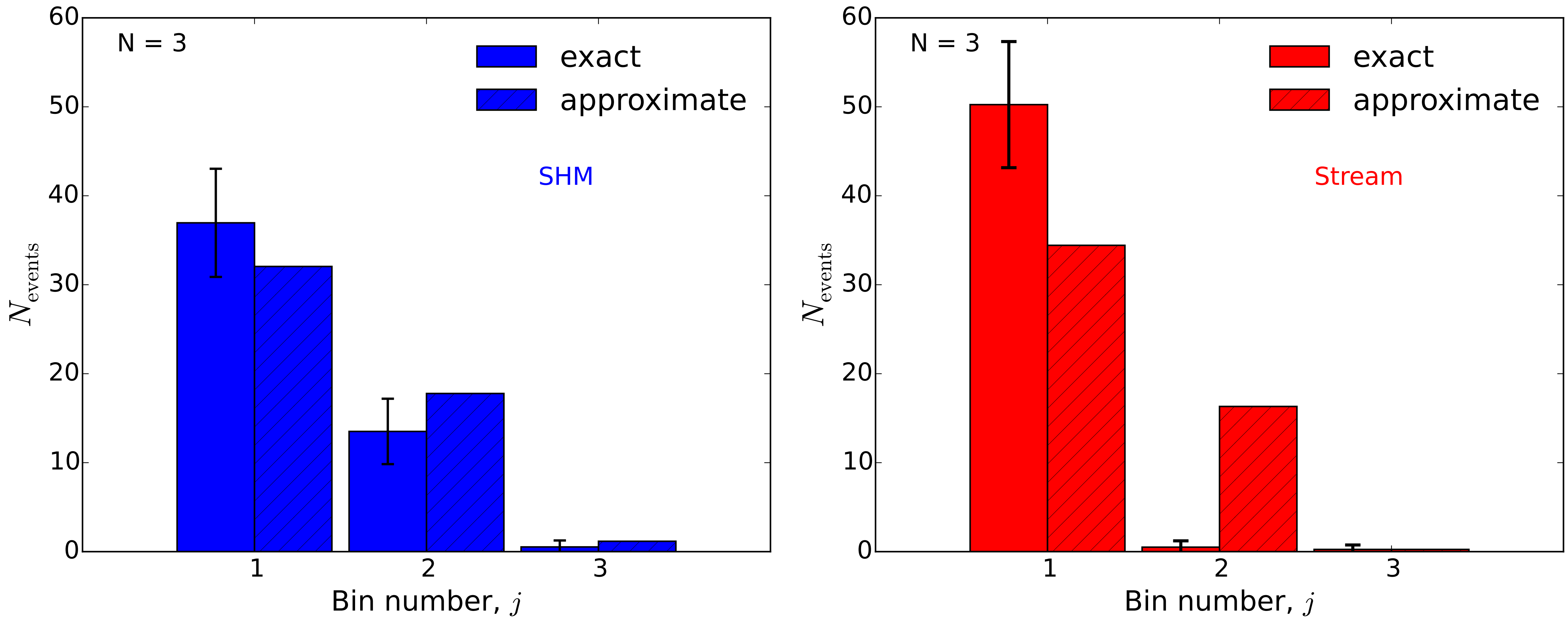
$$v_{\rm lag} = 500\ {\rm km\,s^{-1}}$$

$$\sigma_v = 20\ {\rm km\,s^{-1}}$$



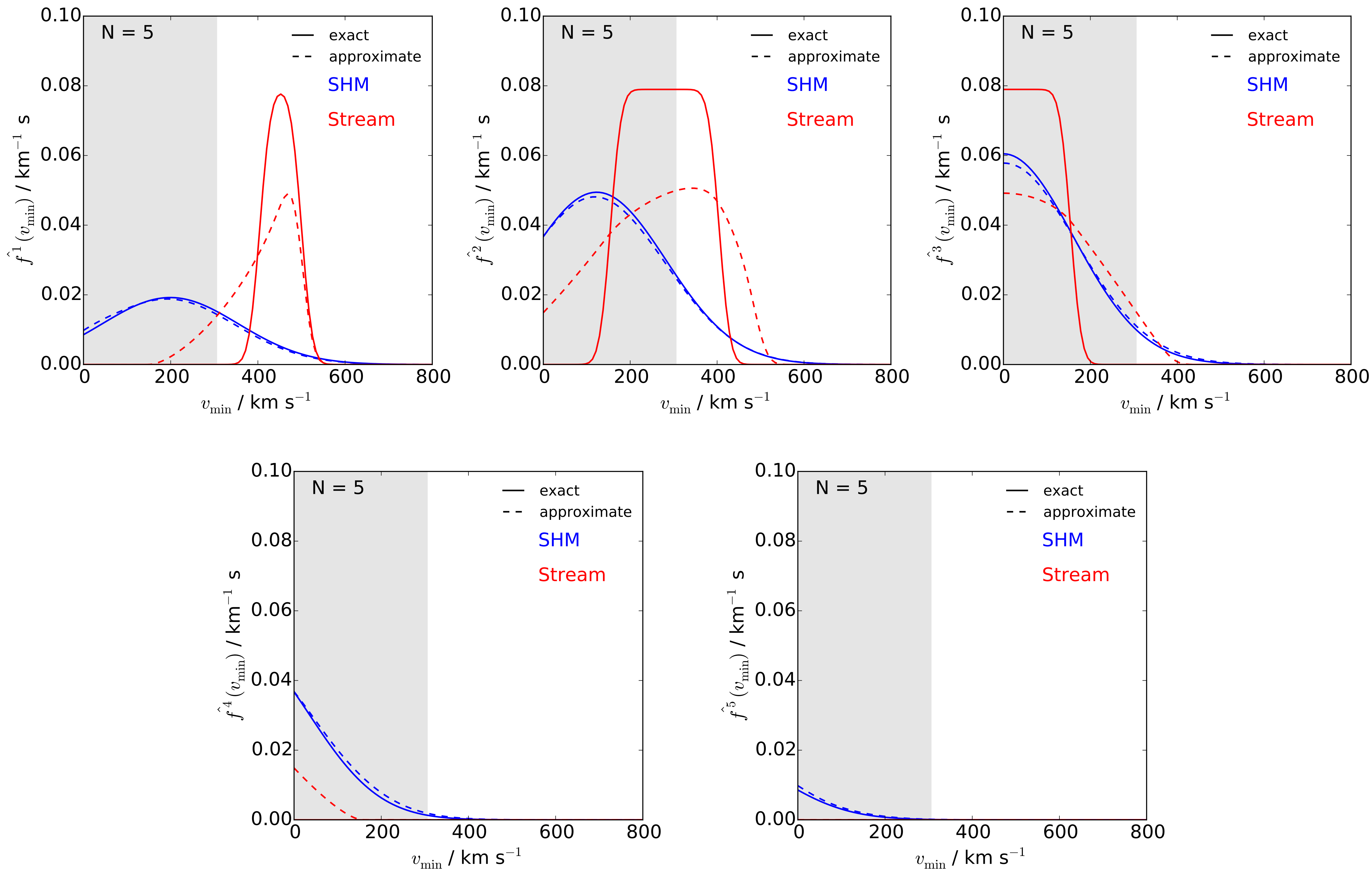


N = 3 discretisation - event numbers

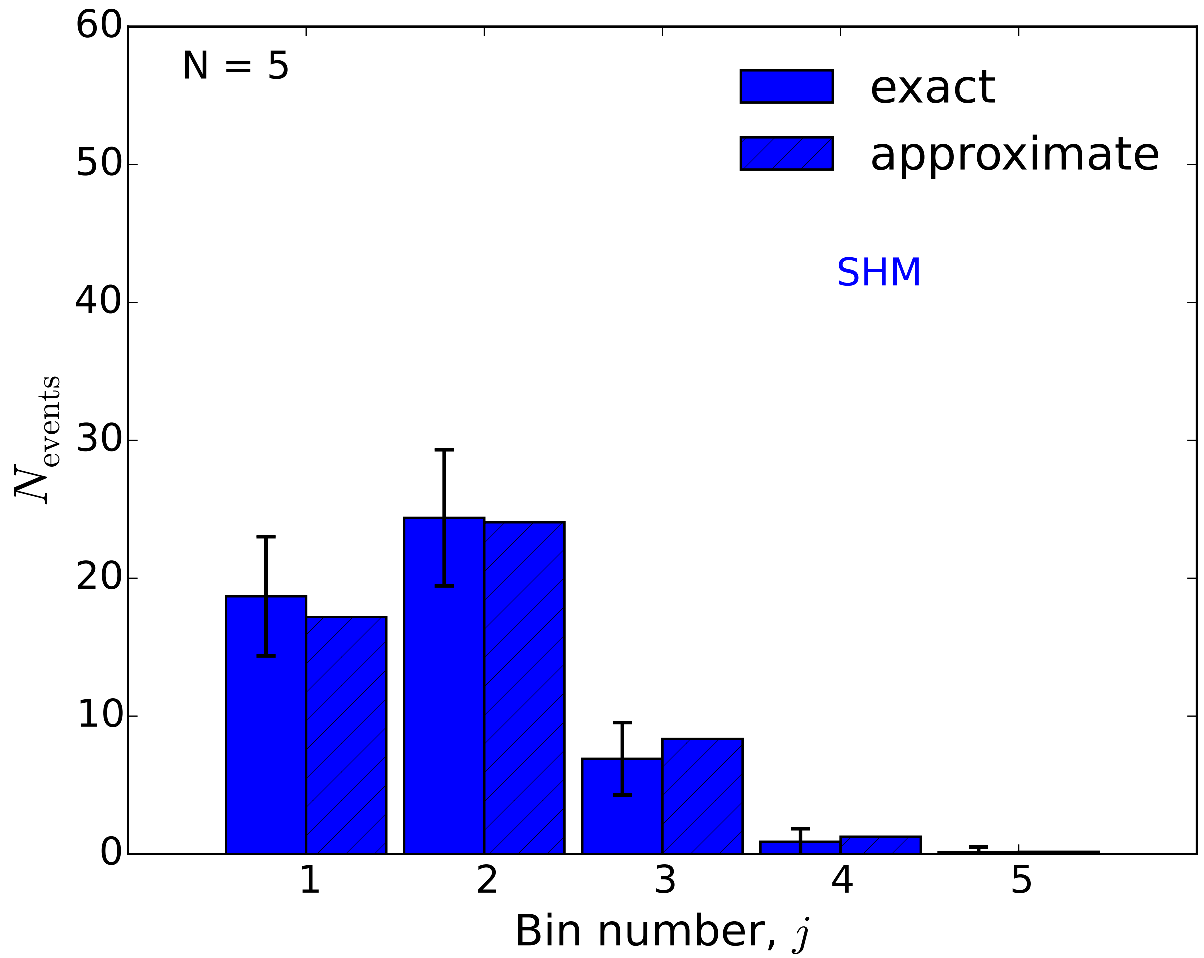


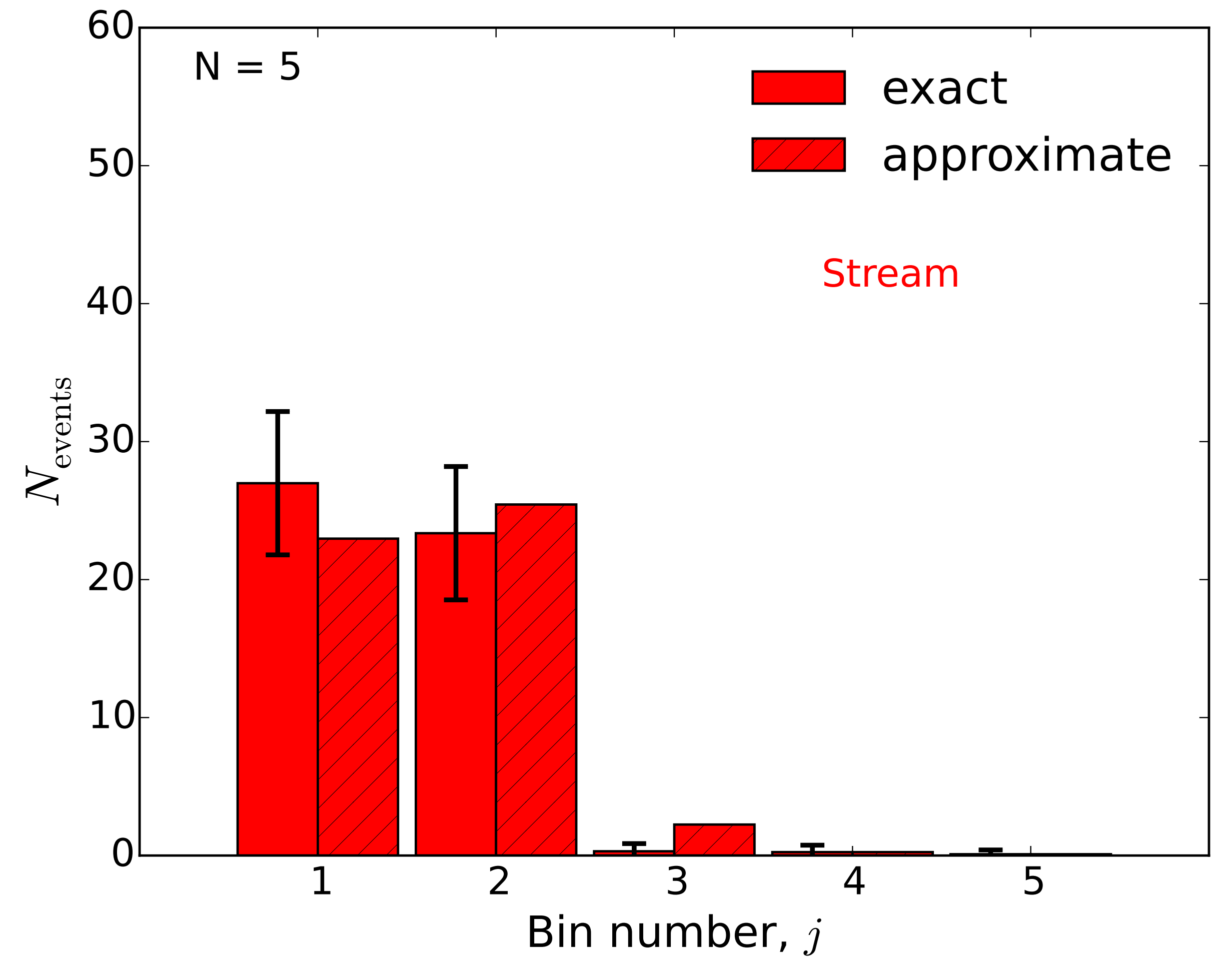
This approach can also be applied if sense recognition is not possible - simply 'fold' bins together…

N = 5 discretisation



N = 5 discretisation - event numbers





In principle, we can carry on increasing N indefinitely…

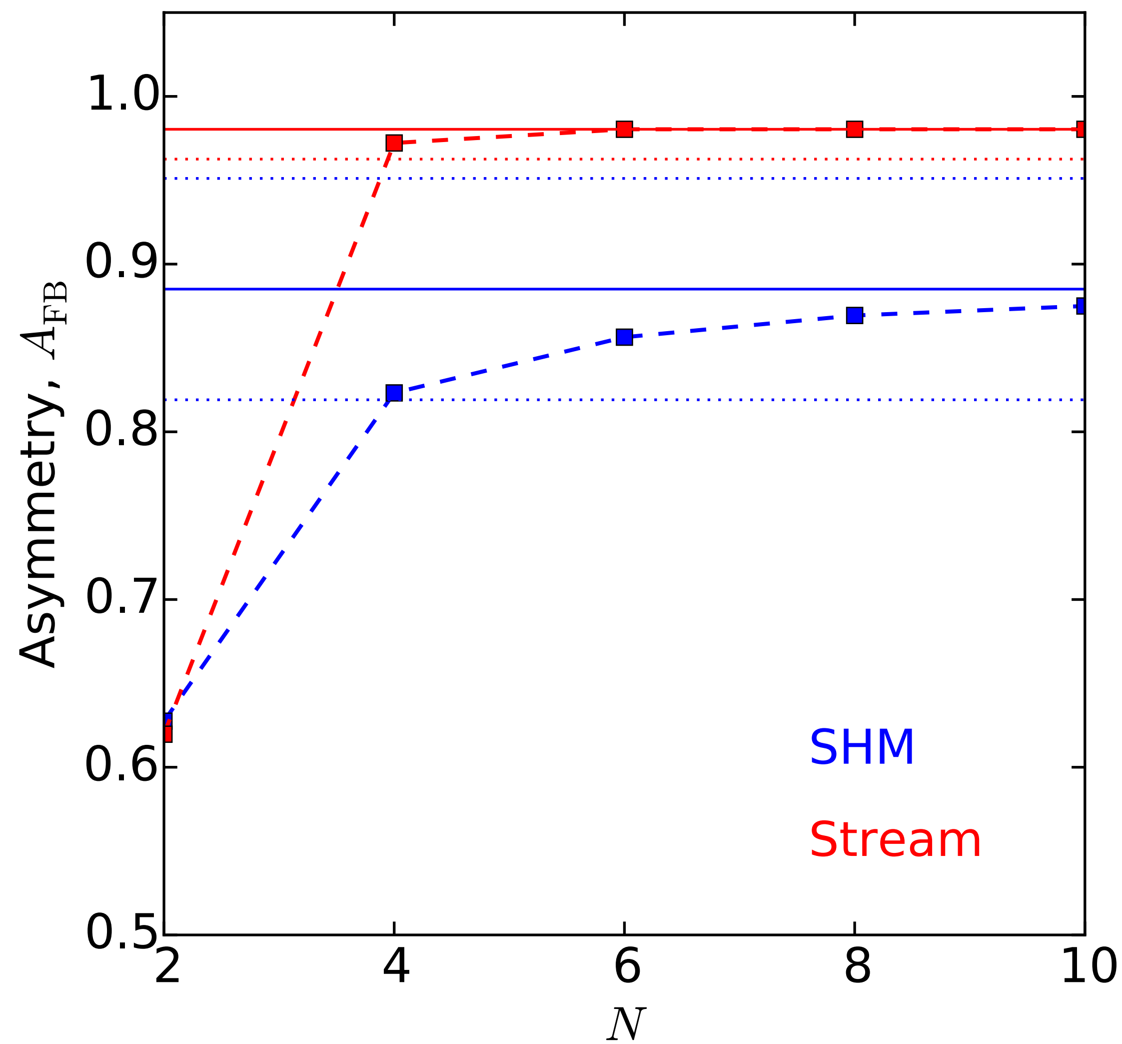
…but the conference is nearly over…

Forward-backward asymmetry

$$N_{\mathrm{F}} = \int_{E_{\mathrm{th}}}^{E_{\mathrm{max}}} \int_{\phi=0}^{2\pi} \int_{0}^{1} \frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} \,\mathrm{d}\cos\theta \,\mathrm{d}\phi \,\mathrm{d}E_R$$

$$N_{\mathrm{B}} = \int_{E_{\mathrm{th}}}^{E_{\mathrm{max}}} \int_{\phi=0}^{2\pi} \int_{-1}^{0} \frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} \,\mathrm{d}\cos\theta \,\mathrm{d}\phi \,\mathrm{d}E_R$$

$$A_{\mathrm{FB}} = \frac{N_{\mathrm{F}} - N_{\mathrm{B}}}{N_{\mathrm{F}} + N_{\mathrm{B}}}$$



Issues and future directions

- Need to include finite angular resolution - $\Delta\theta \sim 20^\circ - 80^\circ$ [1202.3372]
- Need to determine how to align the basis
  - perhaps using median recoil direction…
- Need to find an optimal method for choosing the number of bins
- Other distributions (e.g. dark disk) could be fit even better…

Going forwards, we now need to combine this discrete basis with a parametrisation for each of the $f^k(v)$ and bring it to bear on mock data.

Is the error induced smaller than the potential bias due to astrophysical uncertainties?

Conclusions

- Presented a new angular basis for the DM velocity distribution
- The integrated Radon Transform (IRT) can be calculated for arbitrary numbers of angular bins N
- Compared the shape of the IRT and event numbers in each bin
  - N = 2 is a poor approx. for all distributions
  - N = 3 and above works well for smooth distributions
  - Directional stream distribution requires a much larger number of bins - but this is an *extreme* example
- Next step is to perform a full analysis of mock data - what information can we extract from the discretised velocity distribution in future directional detectors?

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Thank you