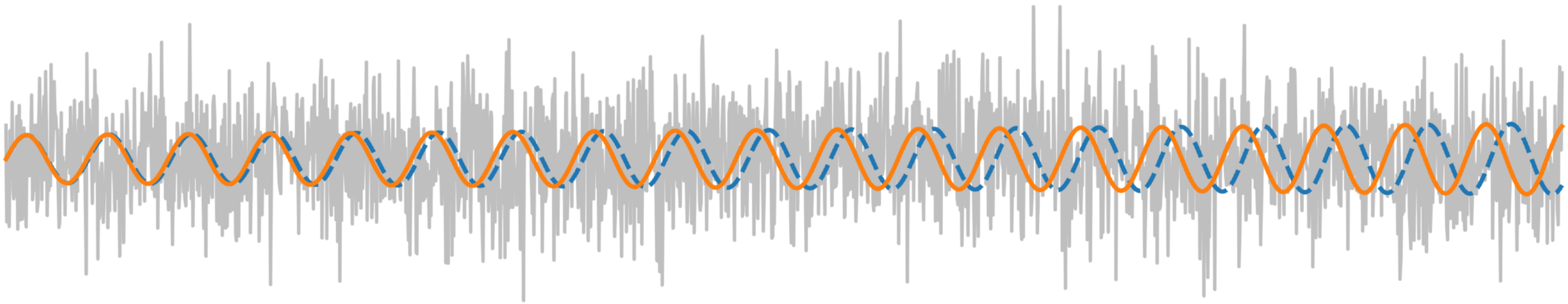
Detecting Dark Matter around Black Holes with Gravitational Waves



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UPV/EHU Seminar, 27th May 2020

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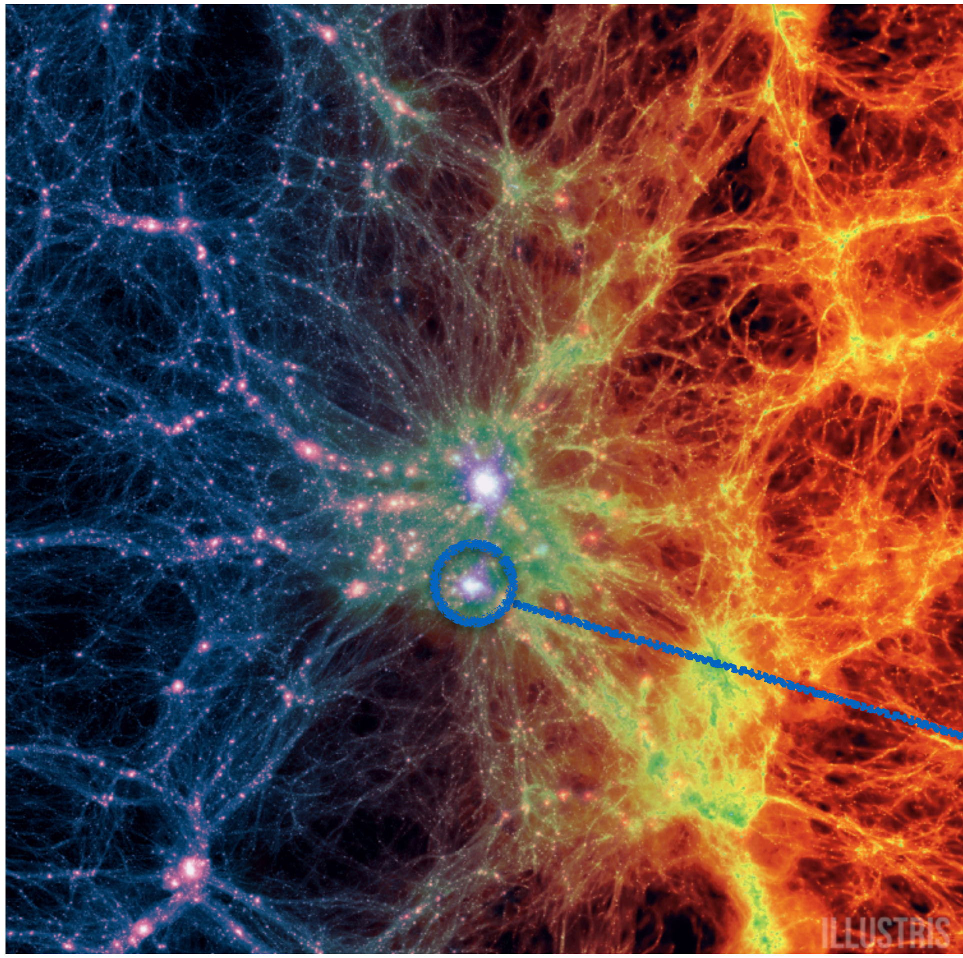
Abstract. With growing agony of not finding a dark matter (DM) particle in direct search experiments so far (for example in XENON1T), frameworks where the freeze-out of DM is driven by number changing processes within the dark sector itself and do not contribute to direct search, like Strongly Interacting Massive Particle (SIMP) are gaining more attention. In this analysis, we ideate a simple scalar DM framework stabilised by $\mathcal{Z}_3$ symmetry to serve

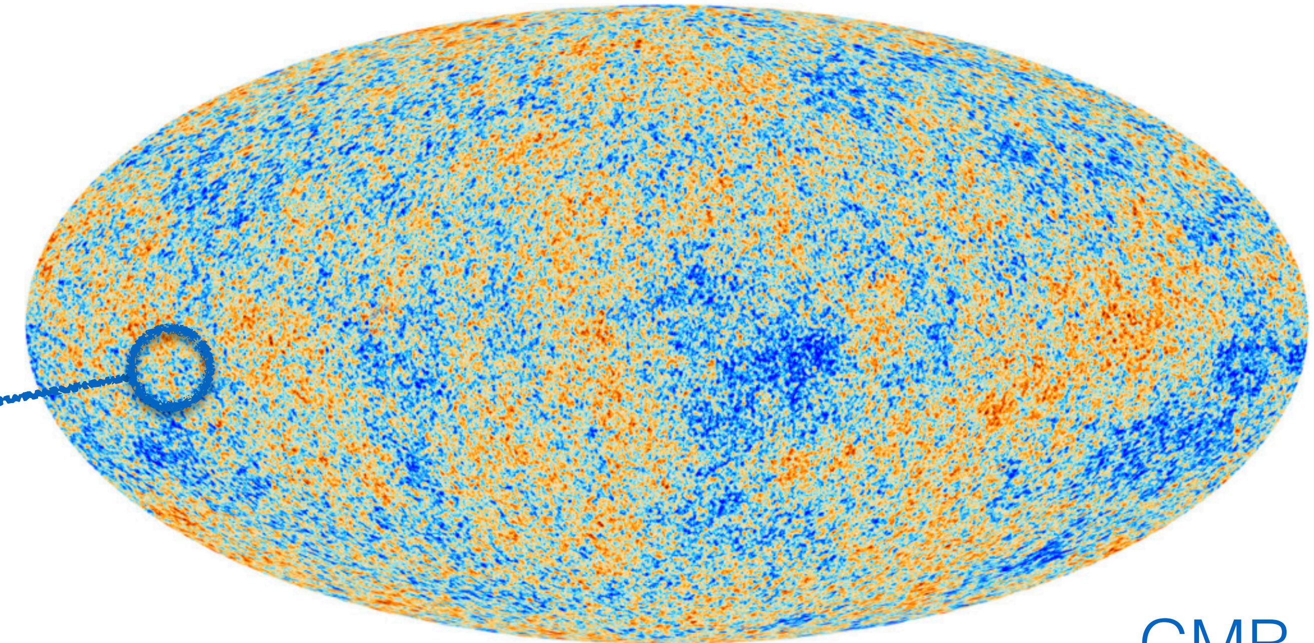
From 'SIMPler realisation of Scalar Dark Matter' [1904.07562]

[With thanks to @TimonEmken]

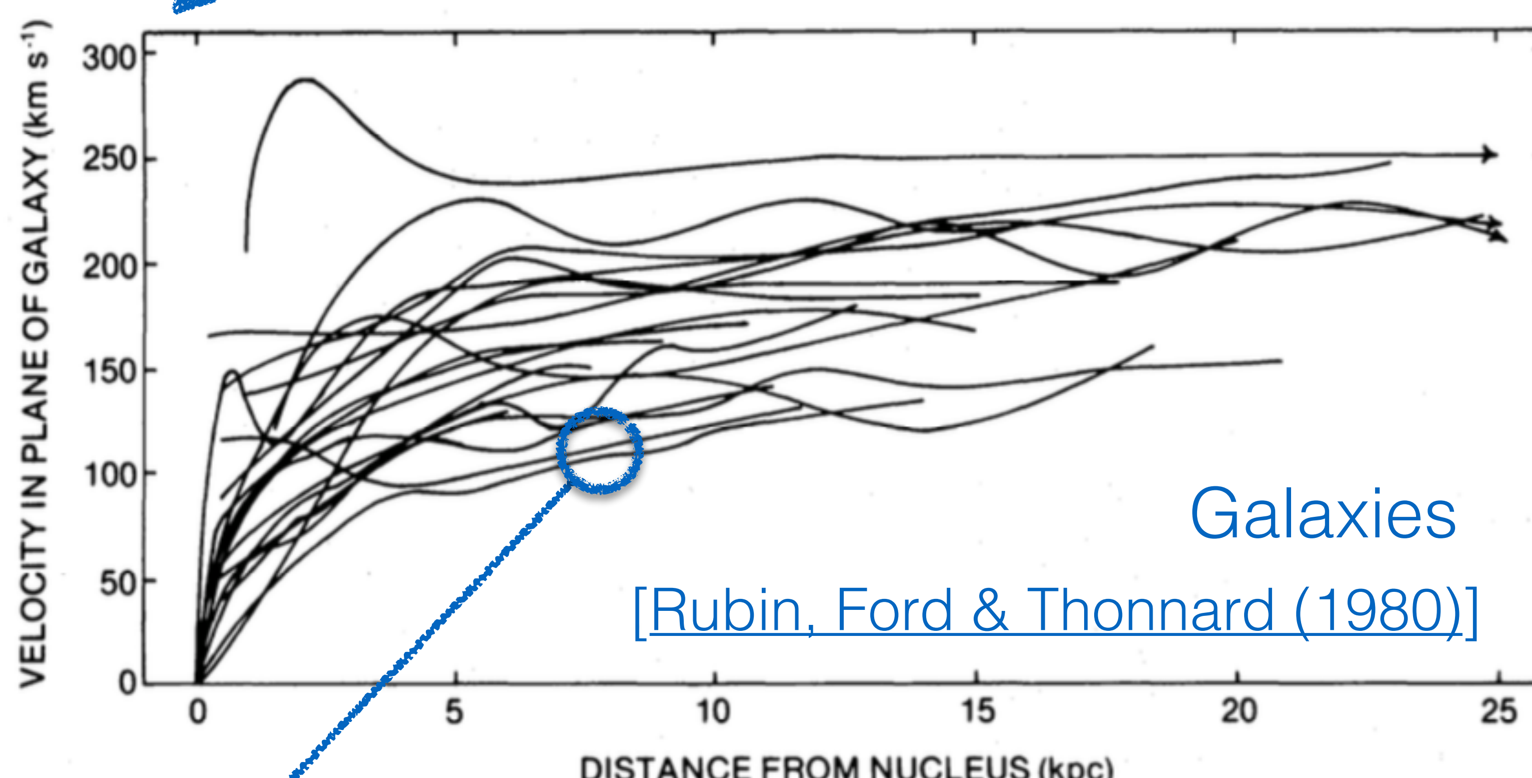
Dark Matter on all scales



Galaxy clusters
[Illustris, 1405.2921]
[astro-ph/0006397]



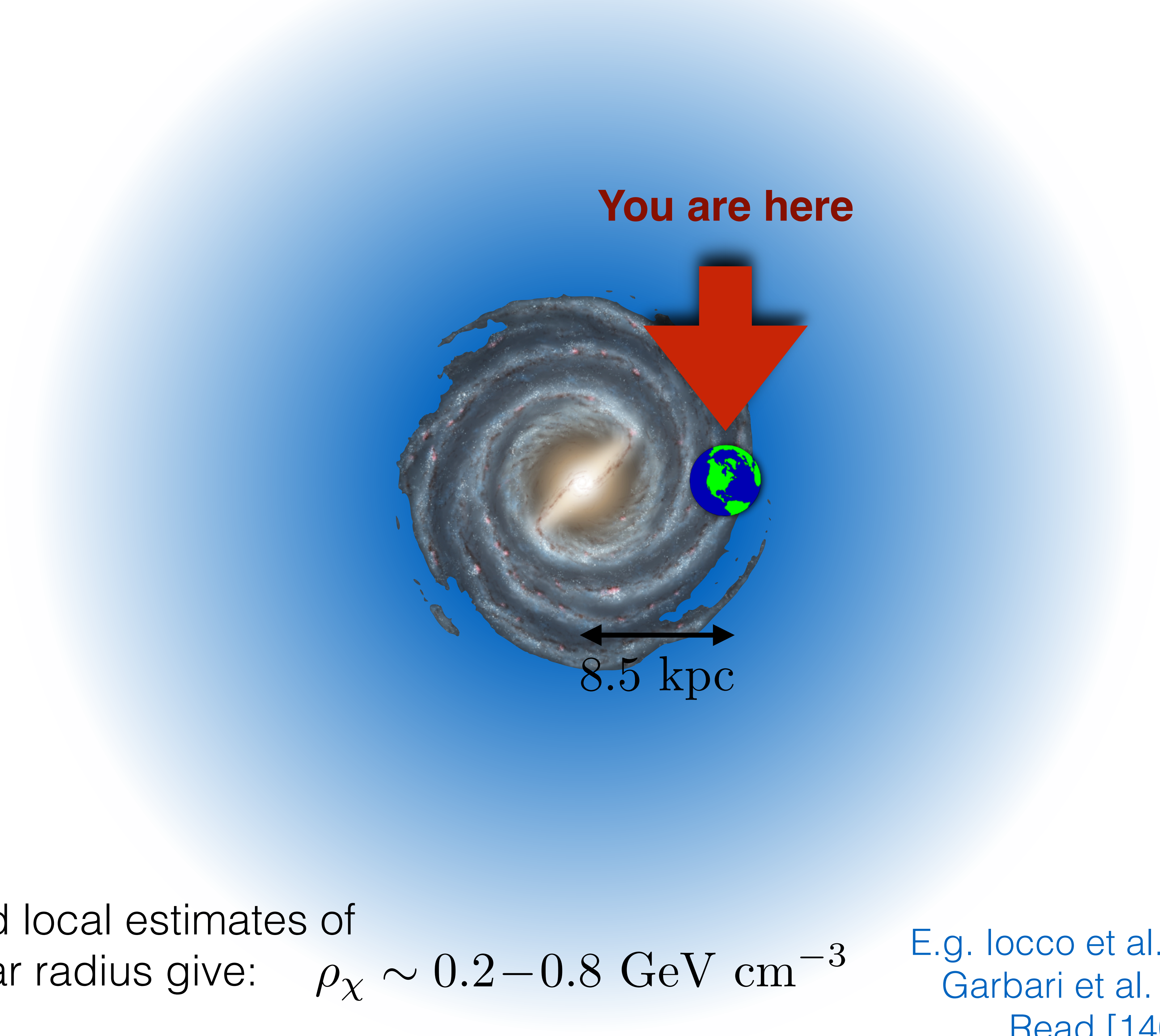
CMB
Planck [1502.01589]



Galaxies
[Rubin, Ford & Thonnard (1980)]

Dark Matter at Earth

NOT TO SCALE



Global and local estimates of DM at Solar radius give: $\rho_\chi \sim 0.2 - 0.8~\mathrm{GeV~cm^{-3}}$

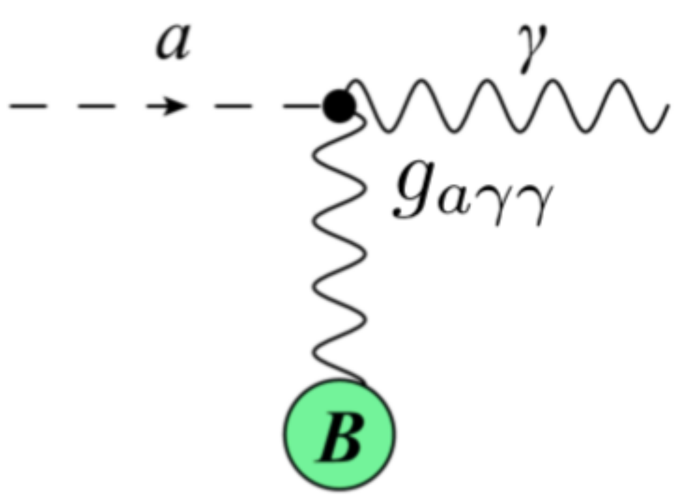
E.g. Iocco et al. [1502.03821], Garbari et al. [1206.0015], Read [1404.1938]

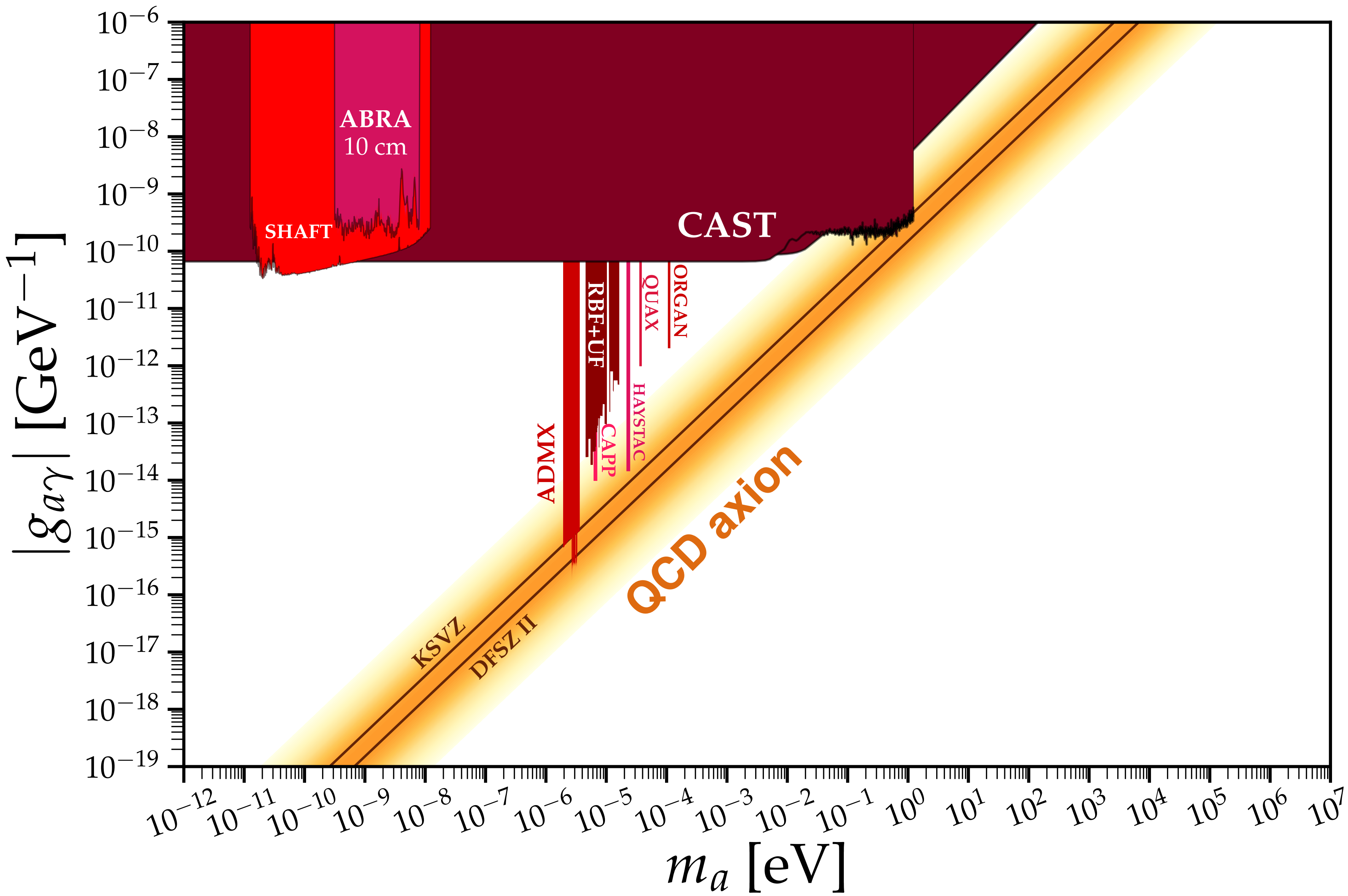
Axion Dark Matter

[Slide credit: Marco Chianese]

Dark Matter could be in the form of light pseudo scalar ‘axions',
which may convert to photons (and vice versa) in an external magnetic field:

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$
$$= -\frac{1}{4} g_{a\gamma\gamma} a \boldsymbol{E} \cdot \boldsymbol{B}$$





[O'Hare, https://cajohare.github.io/AxionLimits/]

Axion Dark Matter

[Slide credit: Marco Chianese]

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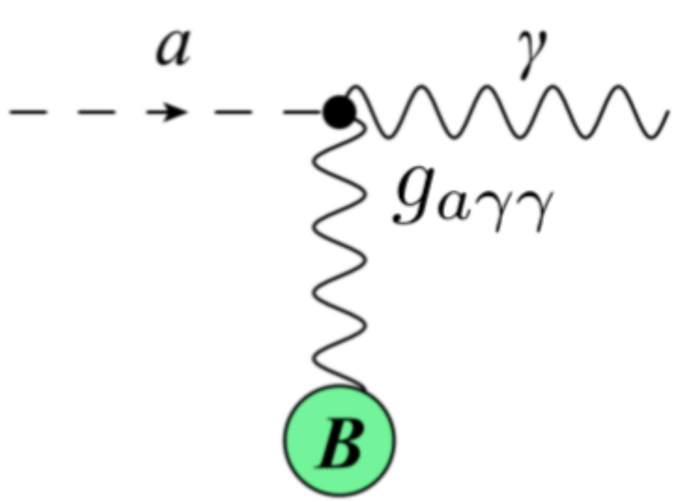
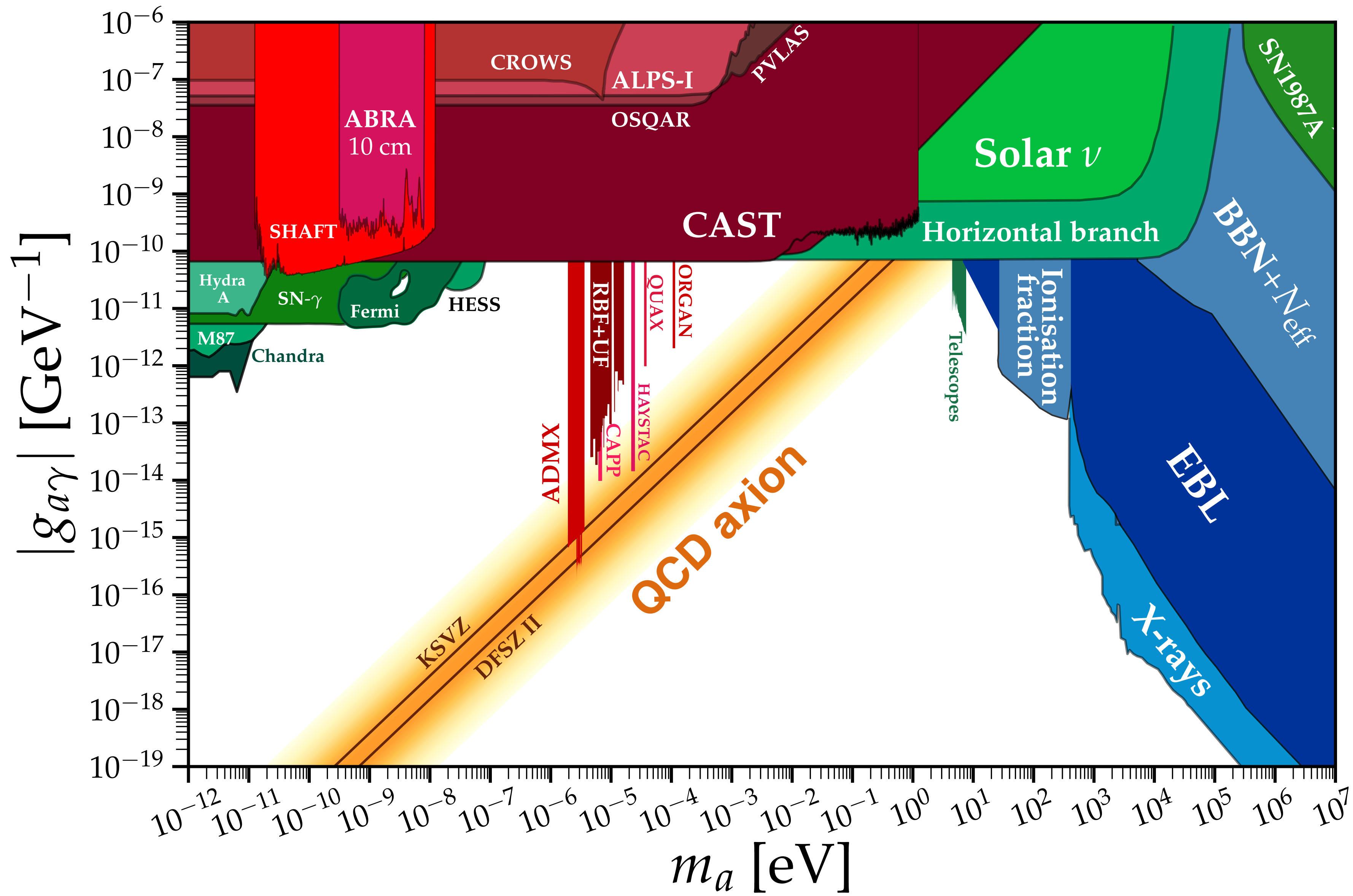
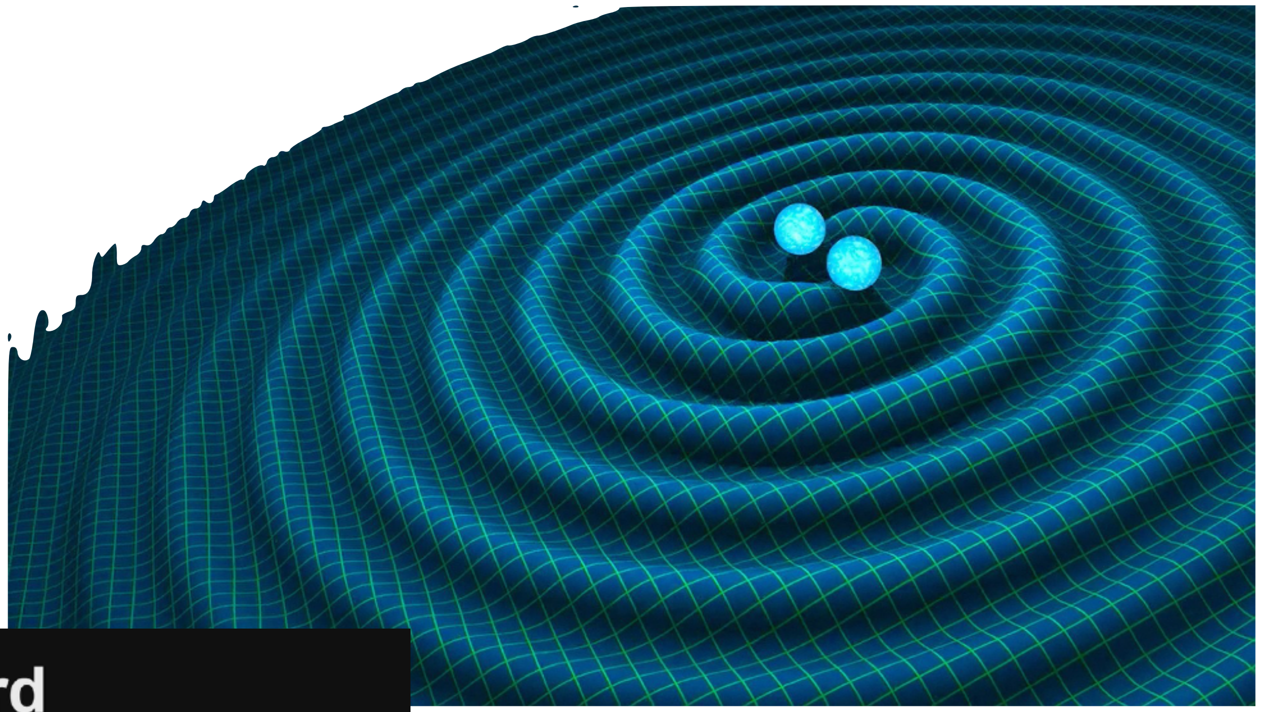


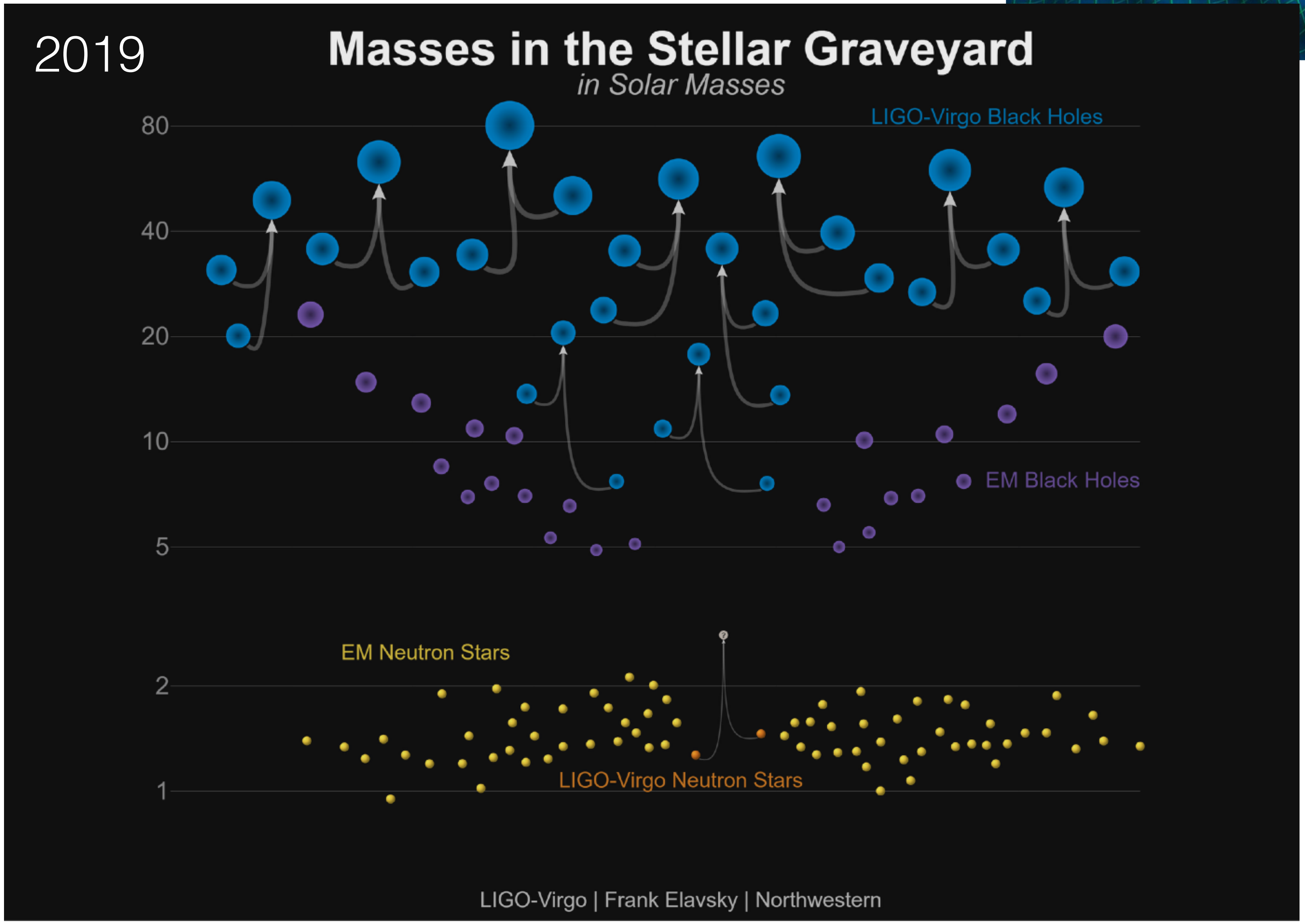


[O’Hare, https://cajohare.github.io/AxionLimits/]

Gravitational Waves (GWs)

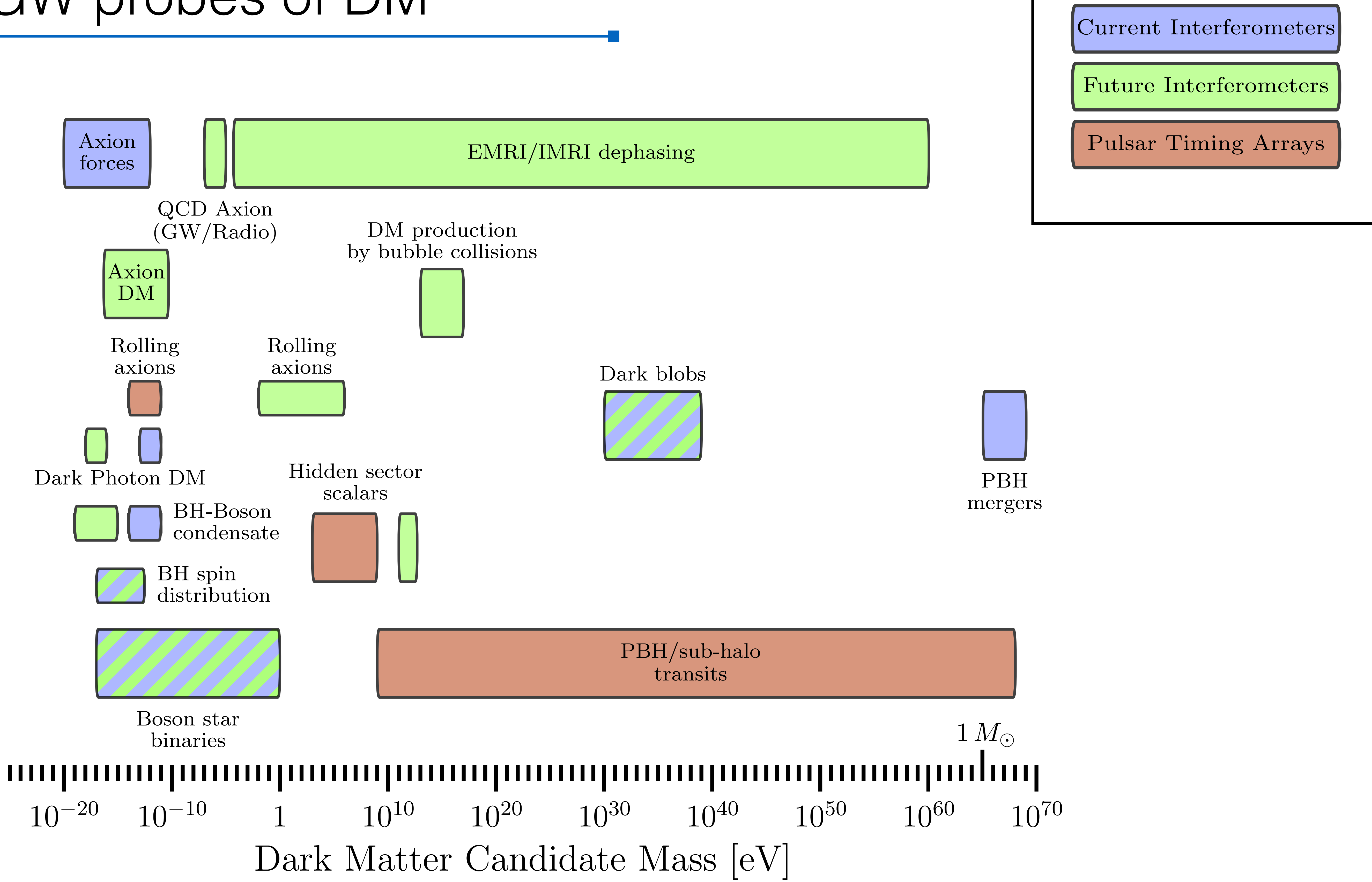


LIGO/Virgo/Northwestern Univ. (Frank Elavsky)



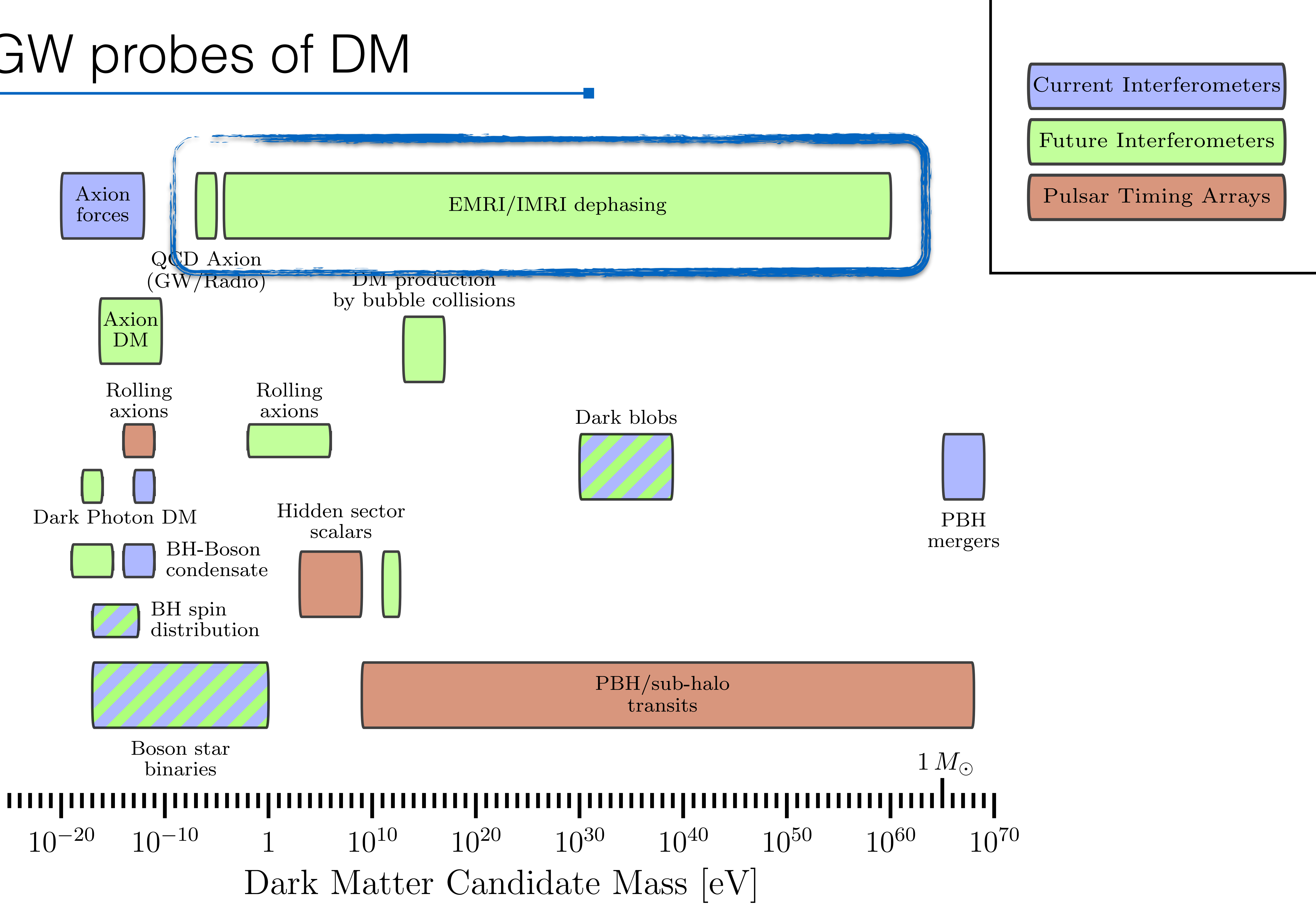
R. HURT / CALTECH-JPL / HANDOUT/ ESA

GW probes of DM



[1907.10610]

GW probes of DM

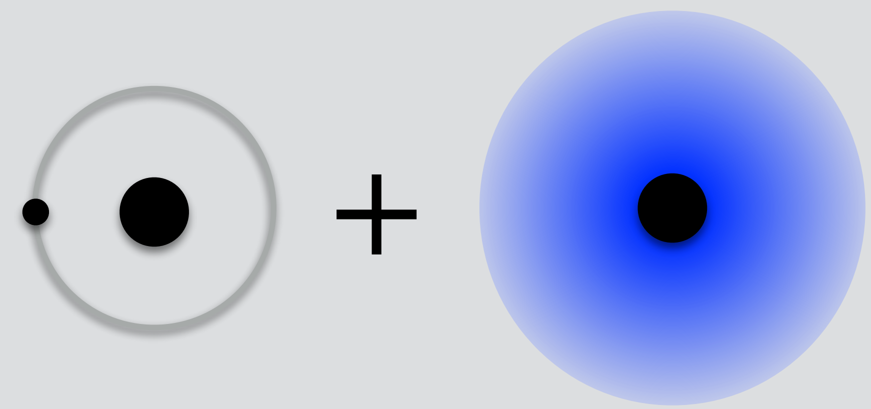


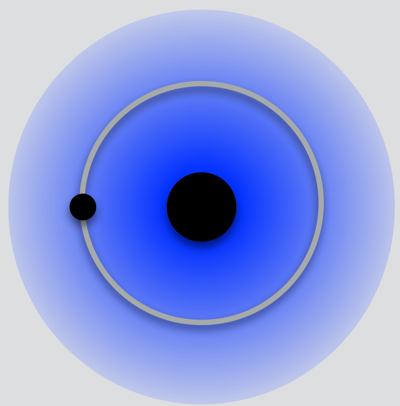
[1907.10610]

Overview

Intermediate Mass-Ratio Inspirals (IMRIs) and Dark Matter spikes

[Eda et al, 1301.5971, and others]



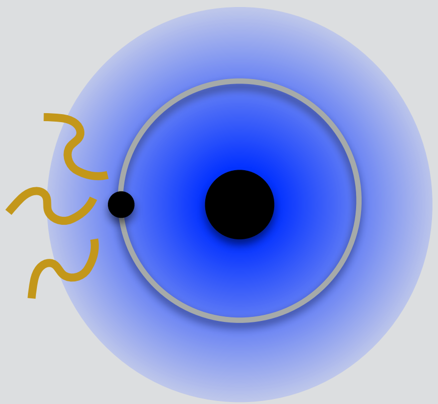


Dark Matter 'de-phasing' revisited

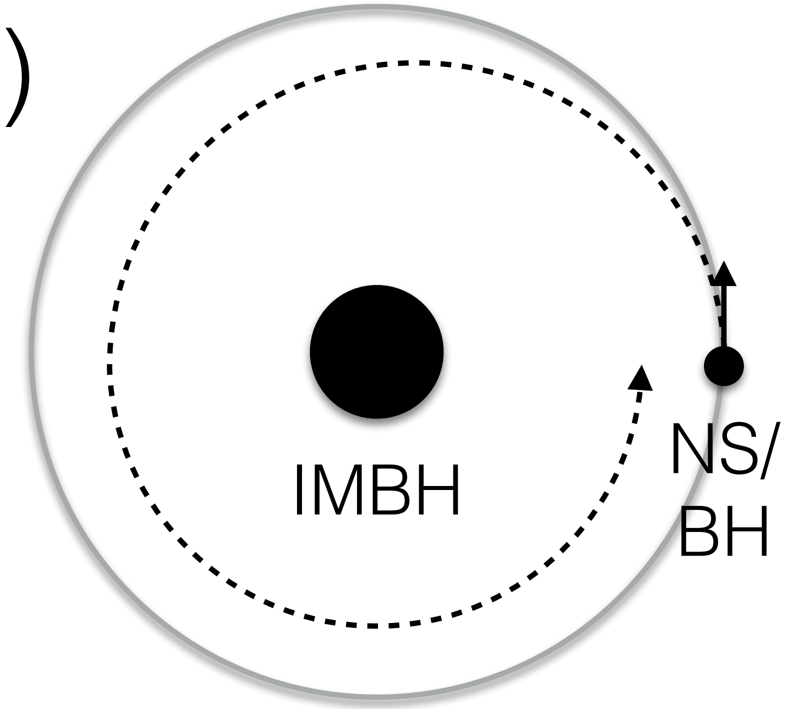
[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]

GW + EM signals of QCD axion Dark Matter

[Edwards, Chianese, **BJK**, Nissanke & Weniger, Phys. Rev. Lett. 124, 161101, 1905.04686]



Intermediate Mass Ratio Inspiral (IMRI)

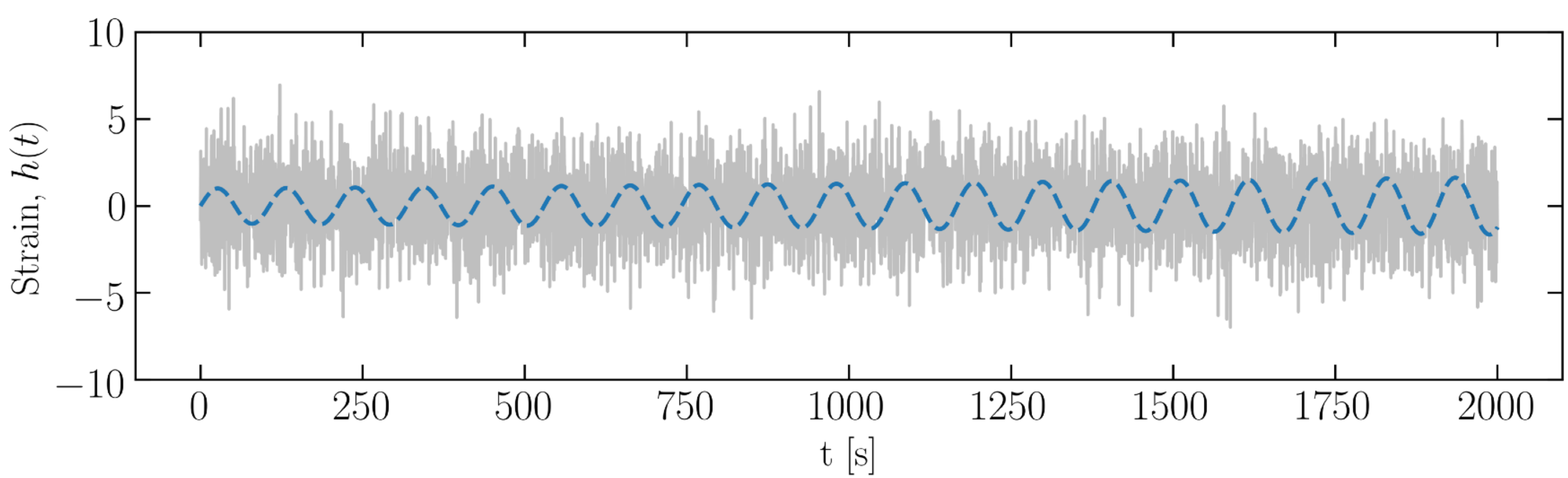


Stellar mass compact object (NS/BH) inspirals towards intermediate mass black hole (IMBH)

$$M_{\mathrm{IMBH}} \sim 10^3 - 10^5\, M_\odot$$

GW emission causes long, slow inspiral:

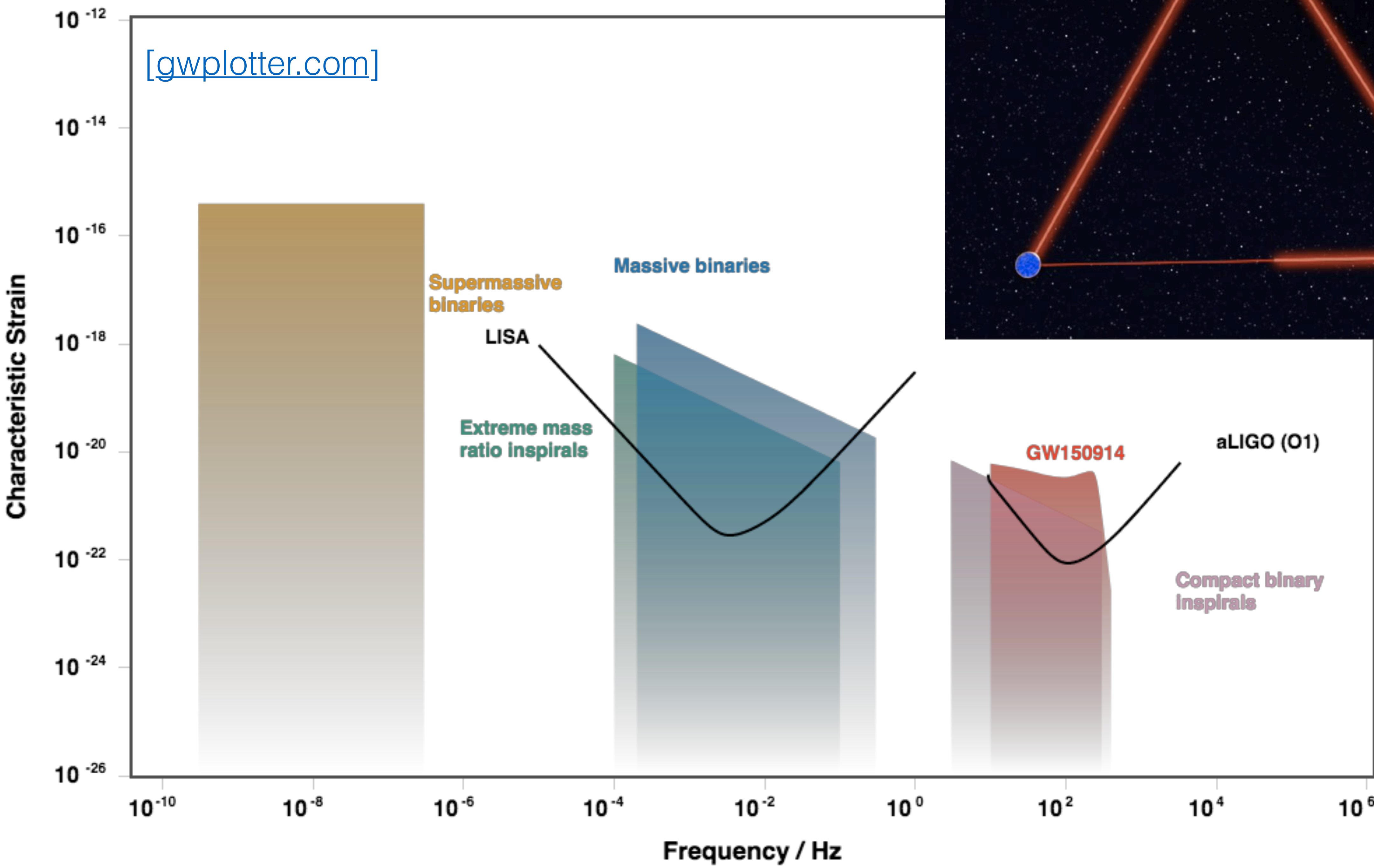
$$\dot{E}_{\mathrm{GW}} \approx \frac{32G^4}{5c^5}\frac{{M_{\mathrm{IMBH}}}^3 {M_{\mathrm{NS}}}^2}{r^5} \propto (f_{\mathrm{GW}})^{10/3}$$

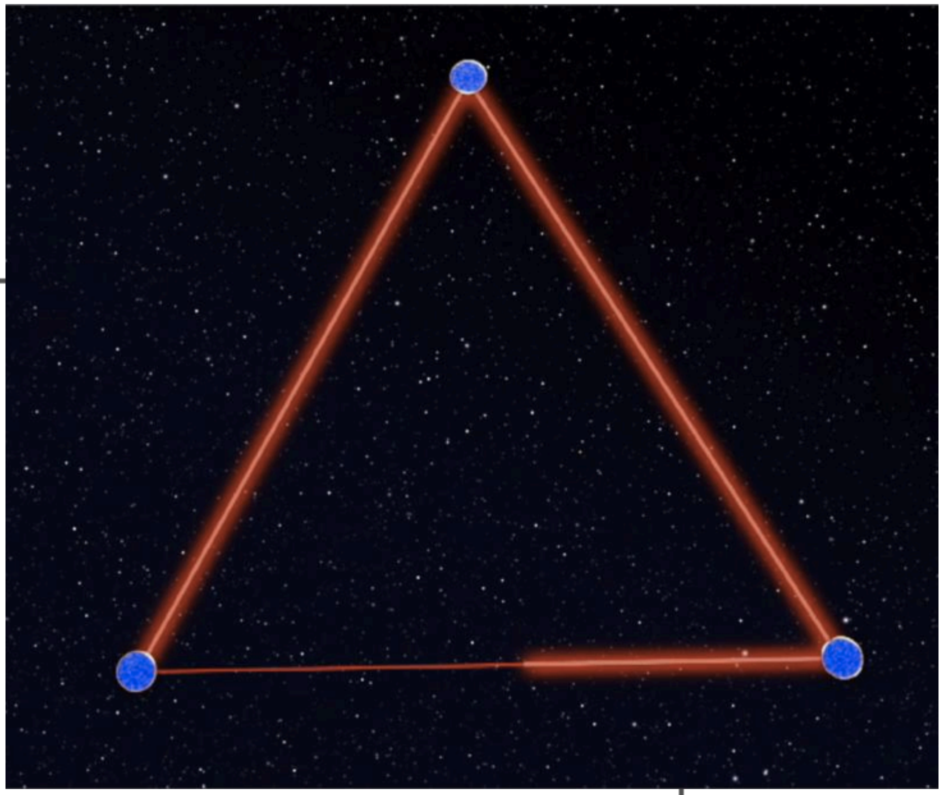


LISA: GWs in Space

© AEI / MM / exozet

Laser **I**nterferometer **S**pace **A**ntenna
(planned for the 2030s) [1702.00786]





LISA should detect ~ 3 - 10 IMRIs per year [1711.00483]

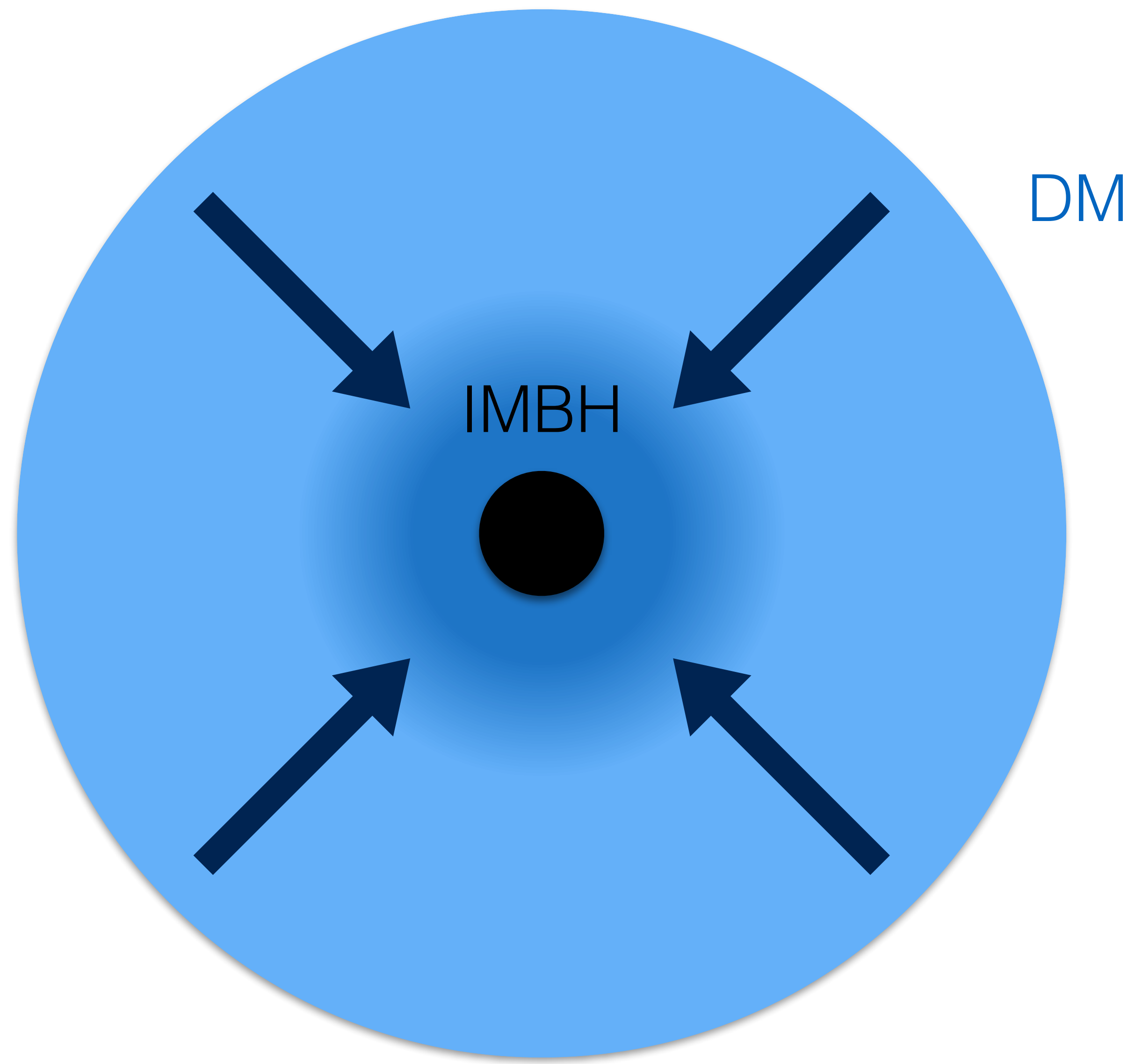
Dark Matter ‘Mini-spikes’

Depending on the formation mechanism of the IMBH, expect an over-density of DM:

$$\rho_{\mathrm{DM}}(r) = \rho_{\mathrm{sp}} \left(\frac{r_{\mathrm{sp}}}{r}\right)^{\gamma_{\mathrm{sp}}}$$

For BH forming in an NFW halo, from adiabatic growth expect:

$$\gamma_{\mathrm{sp}} = 7/3 \approx 2.333$$



For 1000 Solar mass IMBH, forming at z ~ 20, get typical values:

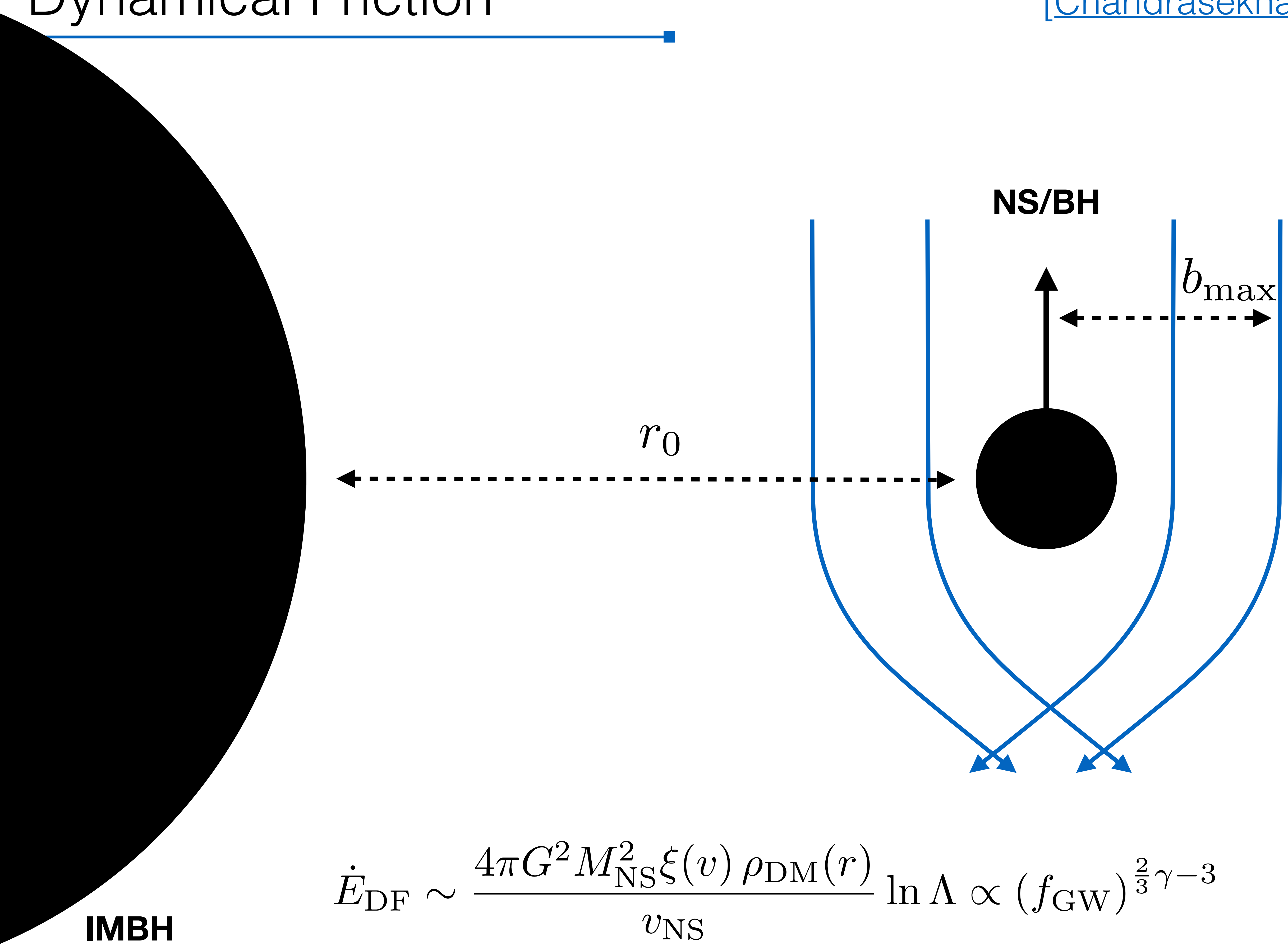
$$\rho_{\mathrm{sp}} = 200\, M_\odot\, \mathrm{pc}^{-3}$$
$$r_{\mathrm{sp}} = 0.5\, \mathrm{pc}$$

Density can reach $\rho \sim 10^{24}\, M_\odot\, \mathrm{pc}^{-3}$ (~10^{24} times larger than local density)

[astro-ph/9906391, astro-ph/0501555, astro-ph/0501625, astro-ph/0509565, 0902.3665, 1305.2619]

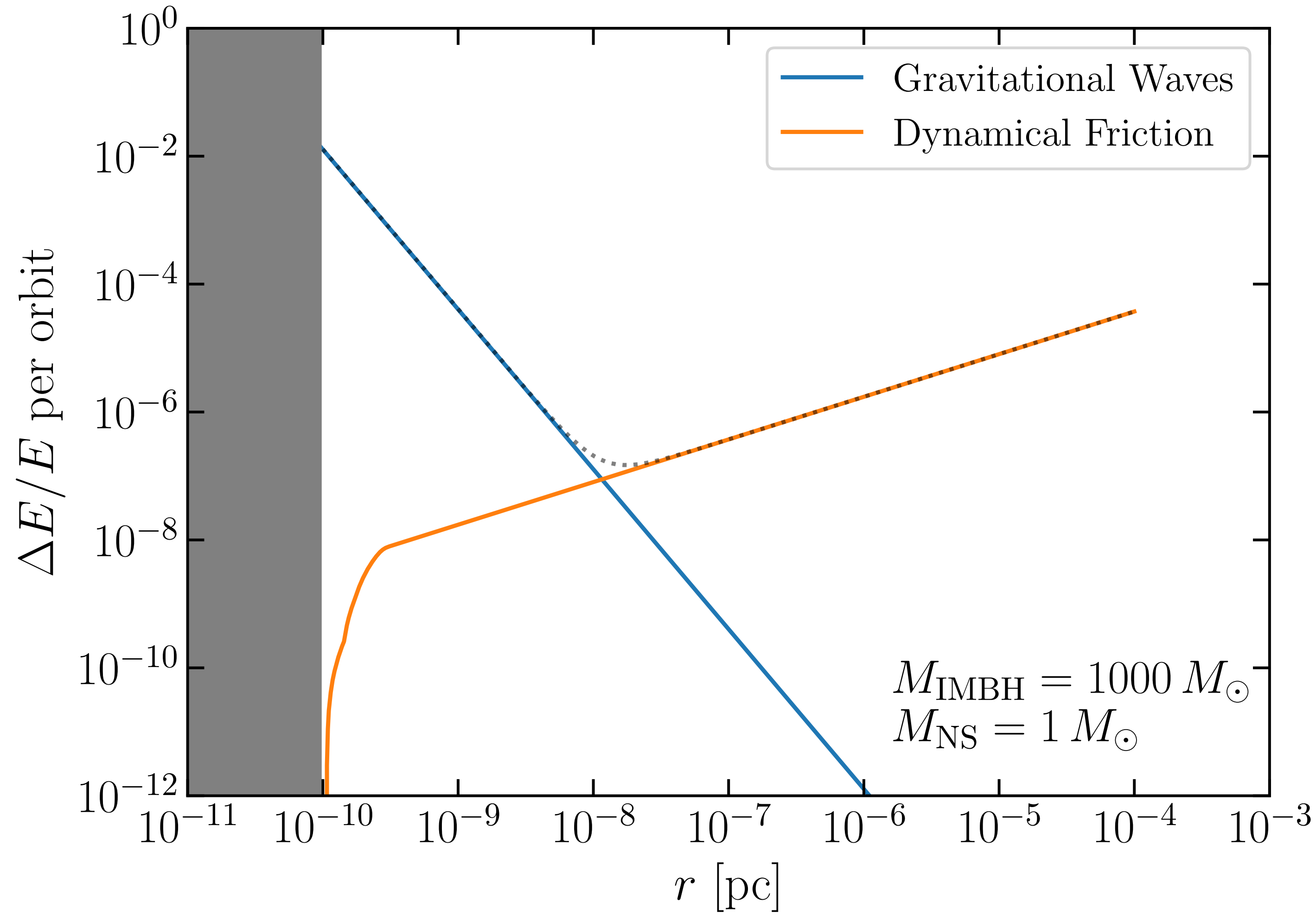
Dynamical Friction

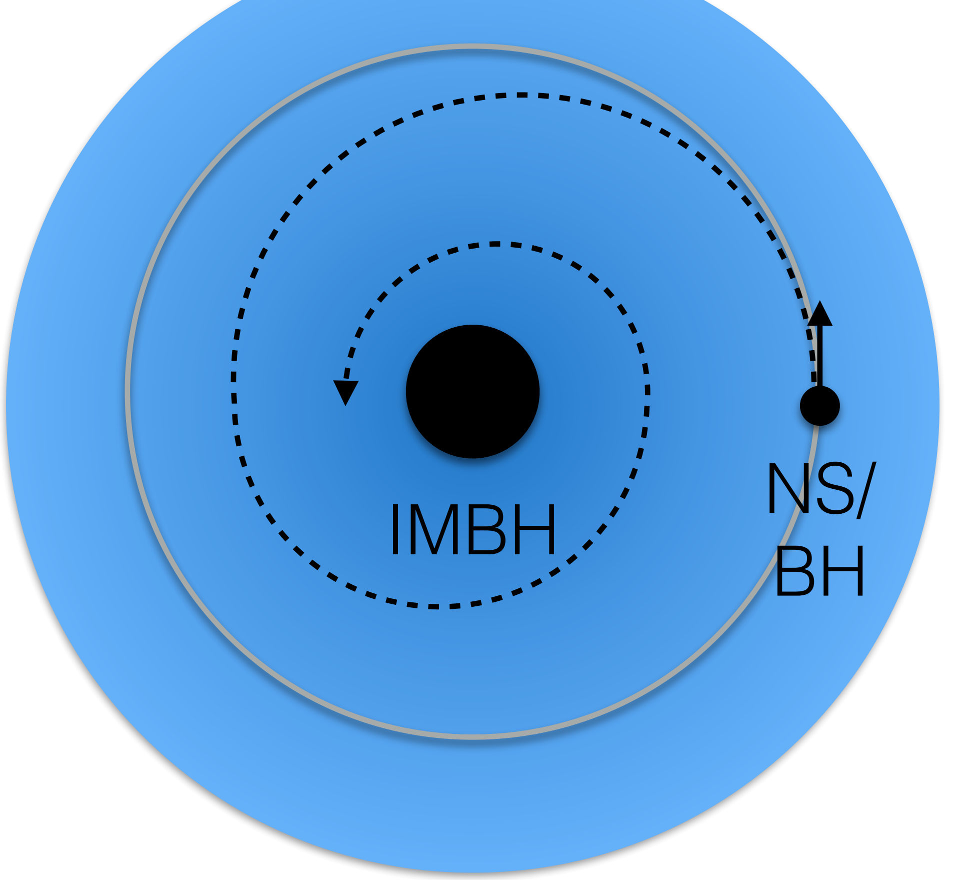
[Chandrasekhar, 1943]



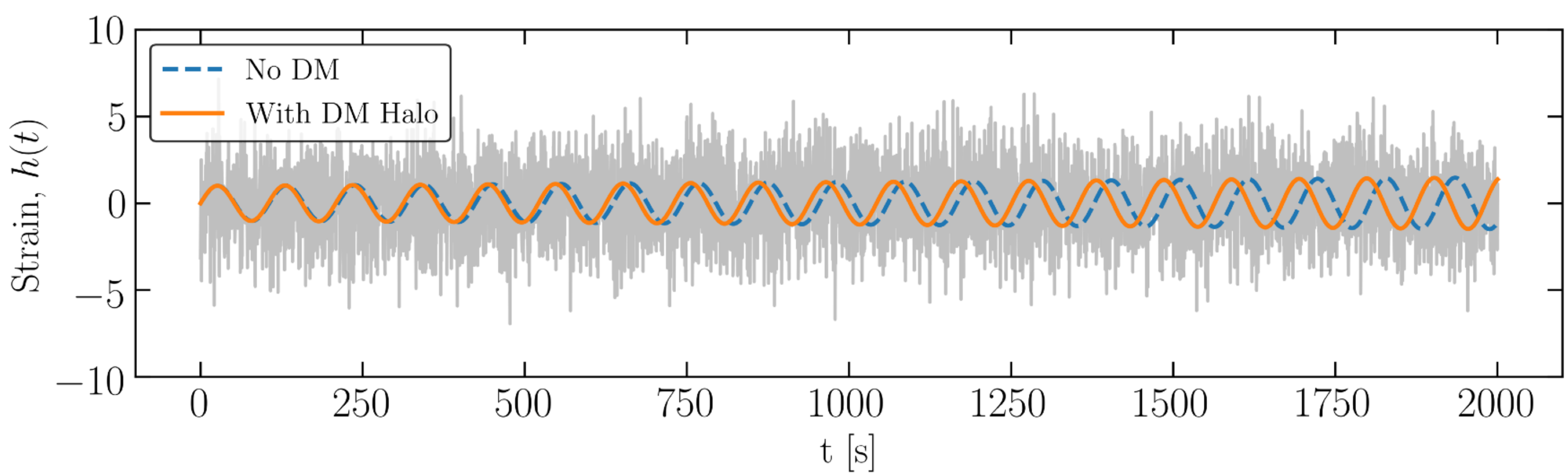
$$\dot{E}_{\mathrm{DF}} \sim \frac{4\pi G^2 M_{\mathrm{NS}}^2 \xi(v)\, \rho_{\mathrm{DM}}(r)}{v_{\mathrm{NS}}} \ln \Lambda \propto (f_{\mathrm{GW}})^{\frac{2}{3}\gamma - 3}$$

IMRI + Dark Matter





$$-\dot{E}_{\rm orb} = \dot{E}_{\rm GW} + \dot{E}_{\rm DF}$$



'De-phasing' signal

How does DM affect the number of cycles?

Benchmark:

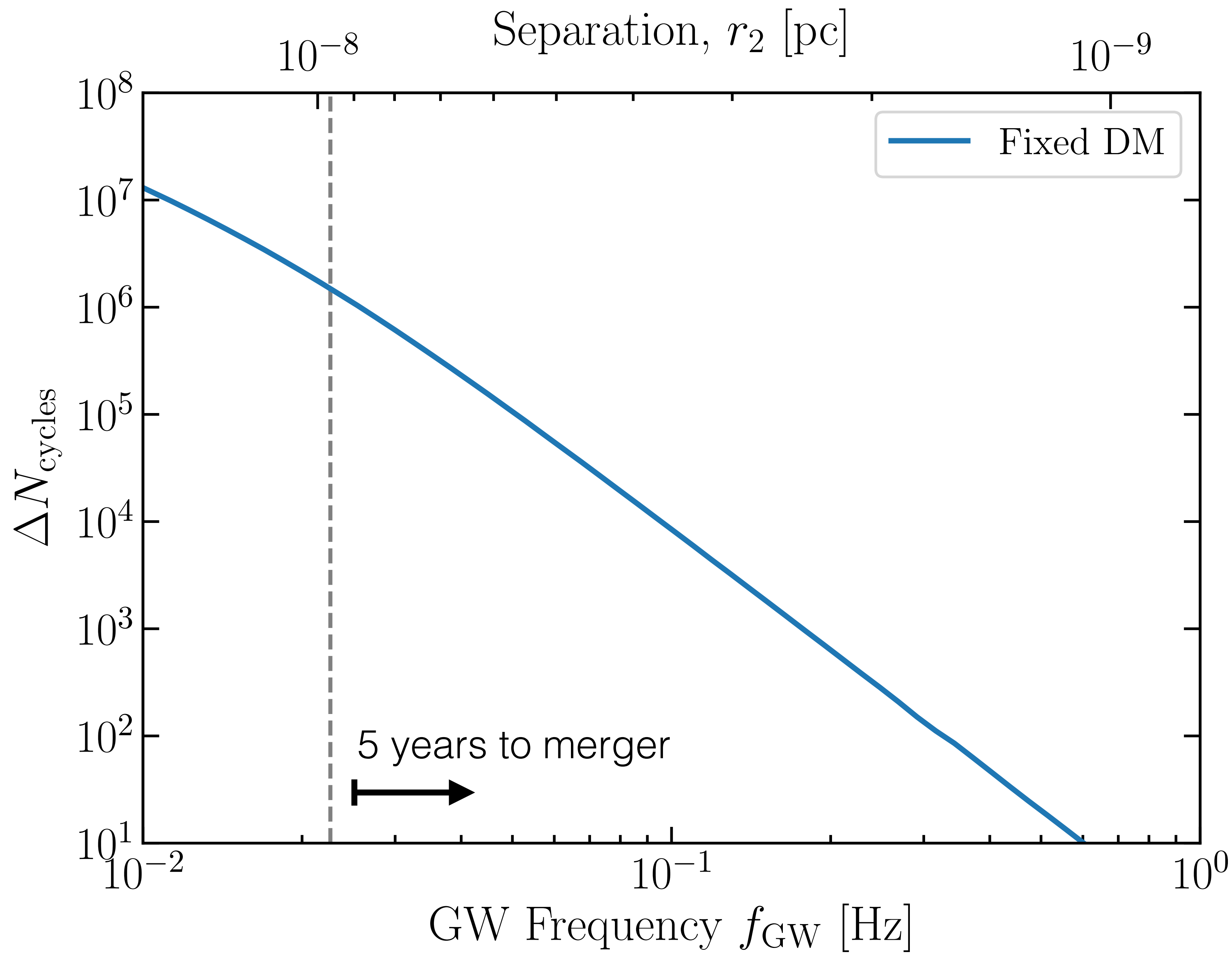
$$M_{\rm IMBH} = 10^3\,M_\odot$$

$$M_{\rm NS} = 1\,M_\odot$$

$$r_{\rm ini} \sim 10^{-8}\,\mathrm{pc}$$

$$t_{\rm merge}^{\rm vacuum} \sim 5\,\mathrm{yr}$$

$$N_{\rm cycles}^{\rm vacuum} \sim 6\times 10^6$$



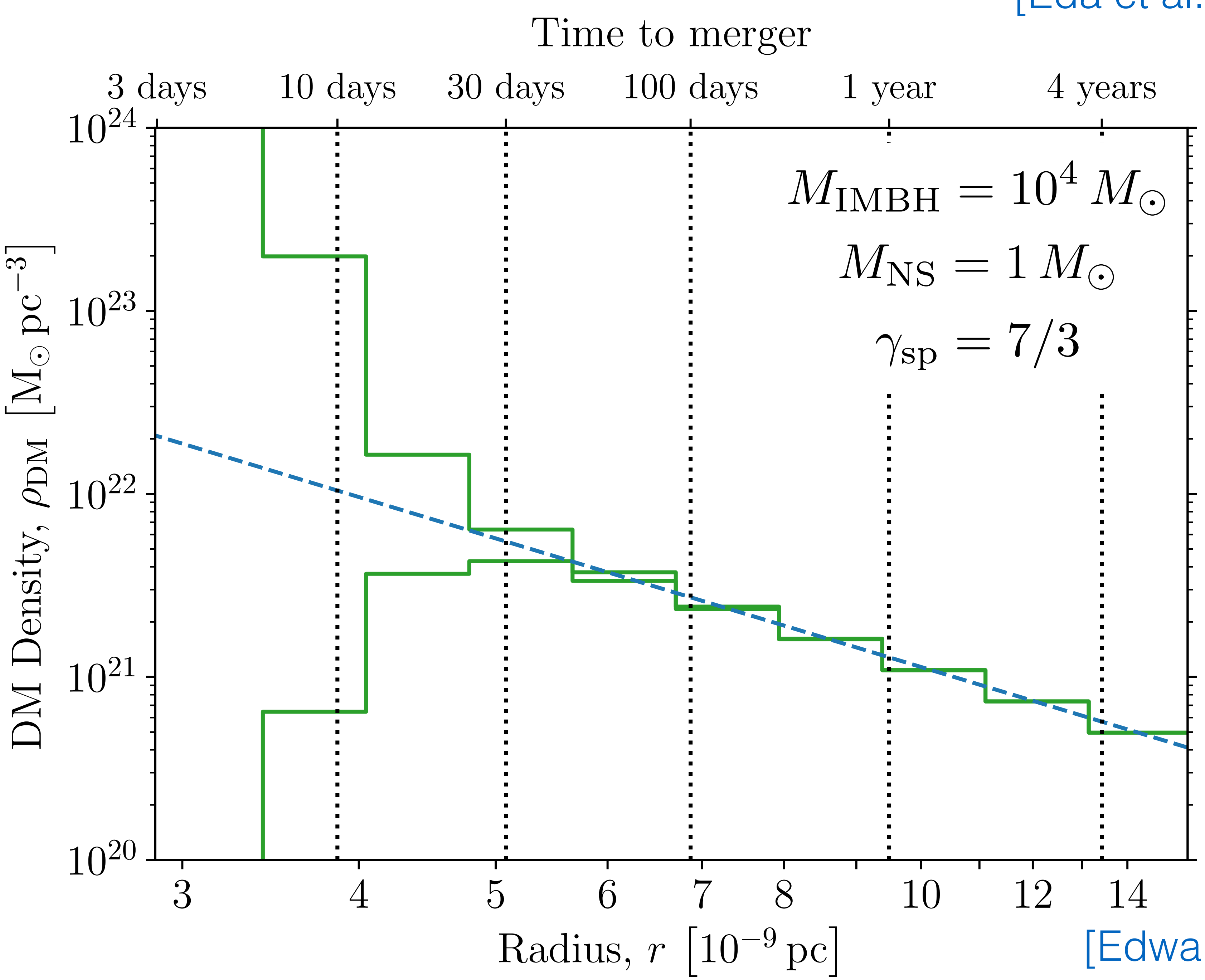
Need to know the signal to better than ~1 part in 10^6!

[Eda et al. 1301.5971, 1408.3534]
[See also 1302.2646, 1404.7140, 1404.7149]

Extracting DM Properties

With LISA, should be able to detect this 'de-phasing' and reconstruct chirp mass and DM spike properties to high precision

[Eda et al. 1301.5971, 1408.3534]

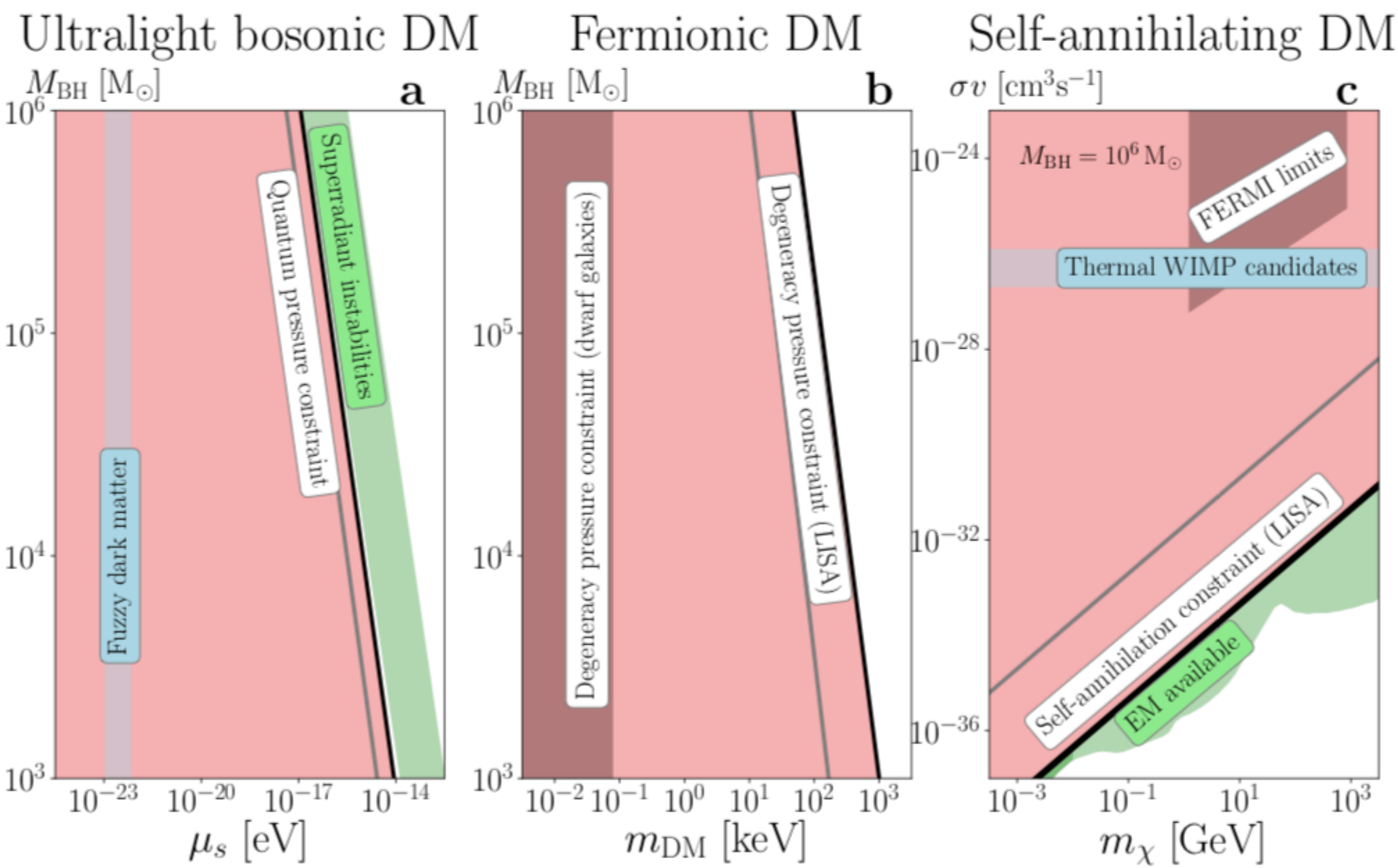


[Edwards, Chianese, **BJK**, Nissanke & Weniger, 1905.04686]

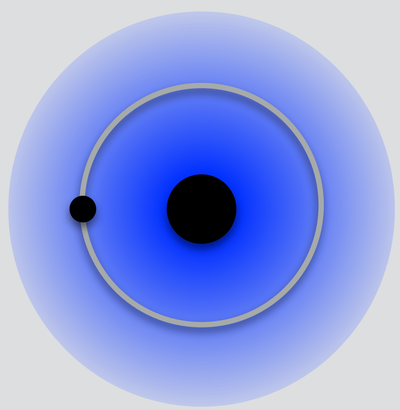
Nature of Dark Matter

Red regions would be ruled out by observation of a DM spike!

[1906.11845]

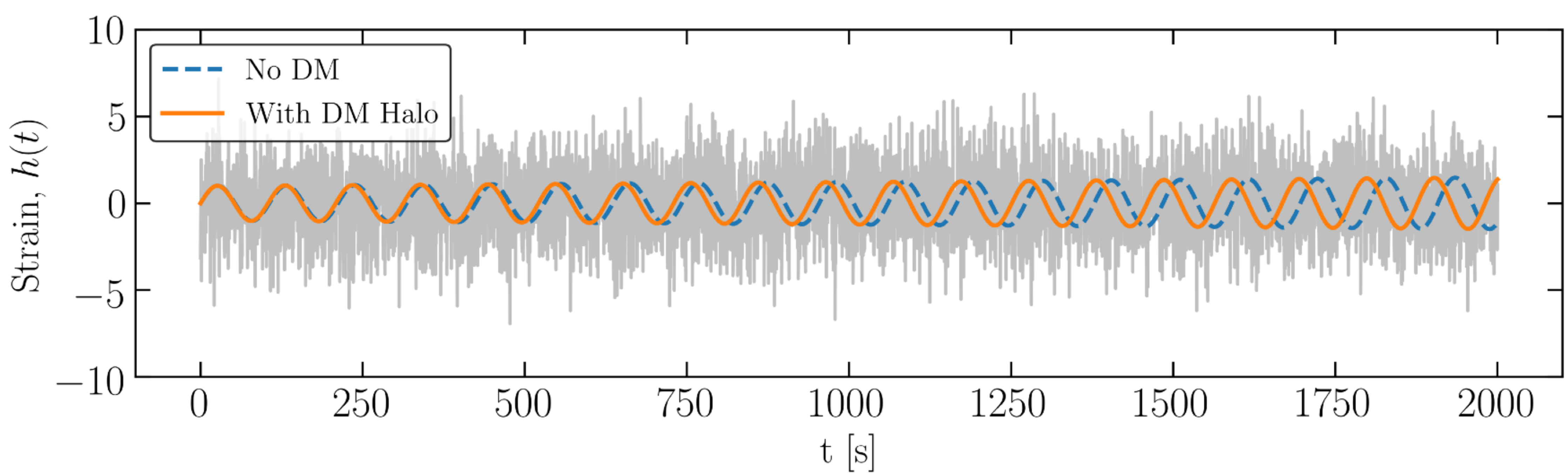


[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, 1905.01238]



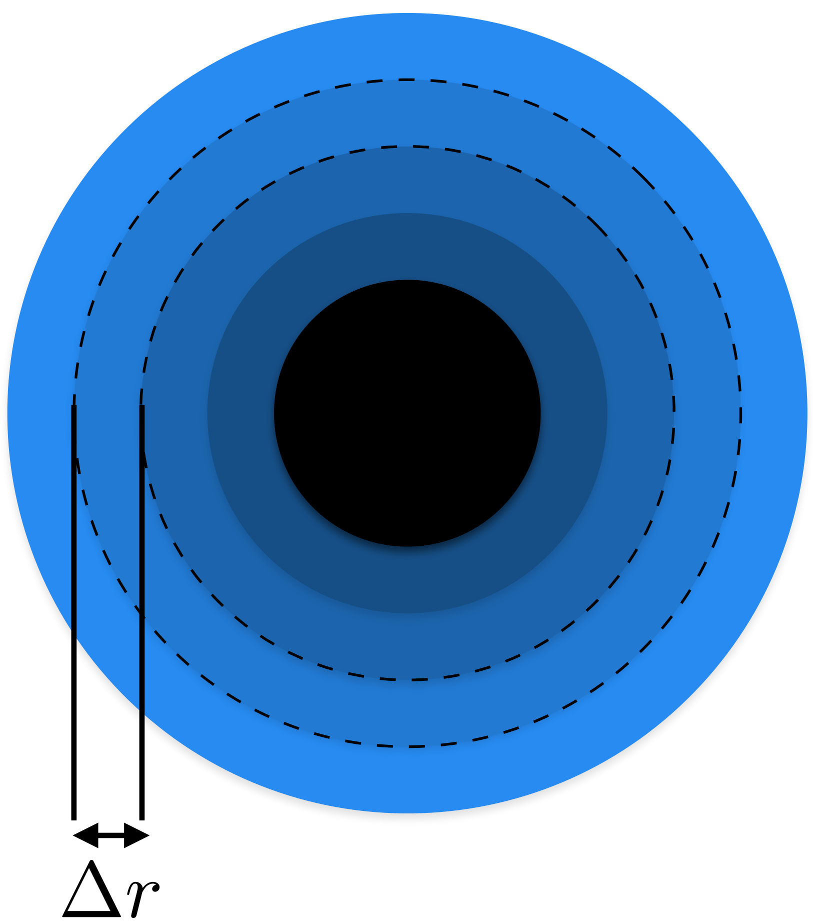
Dark Matter ‘de-phasing’ revisited

[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]

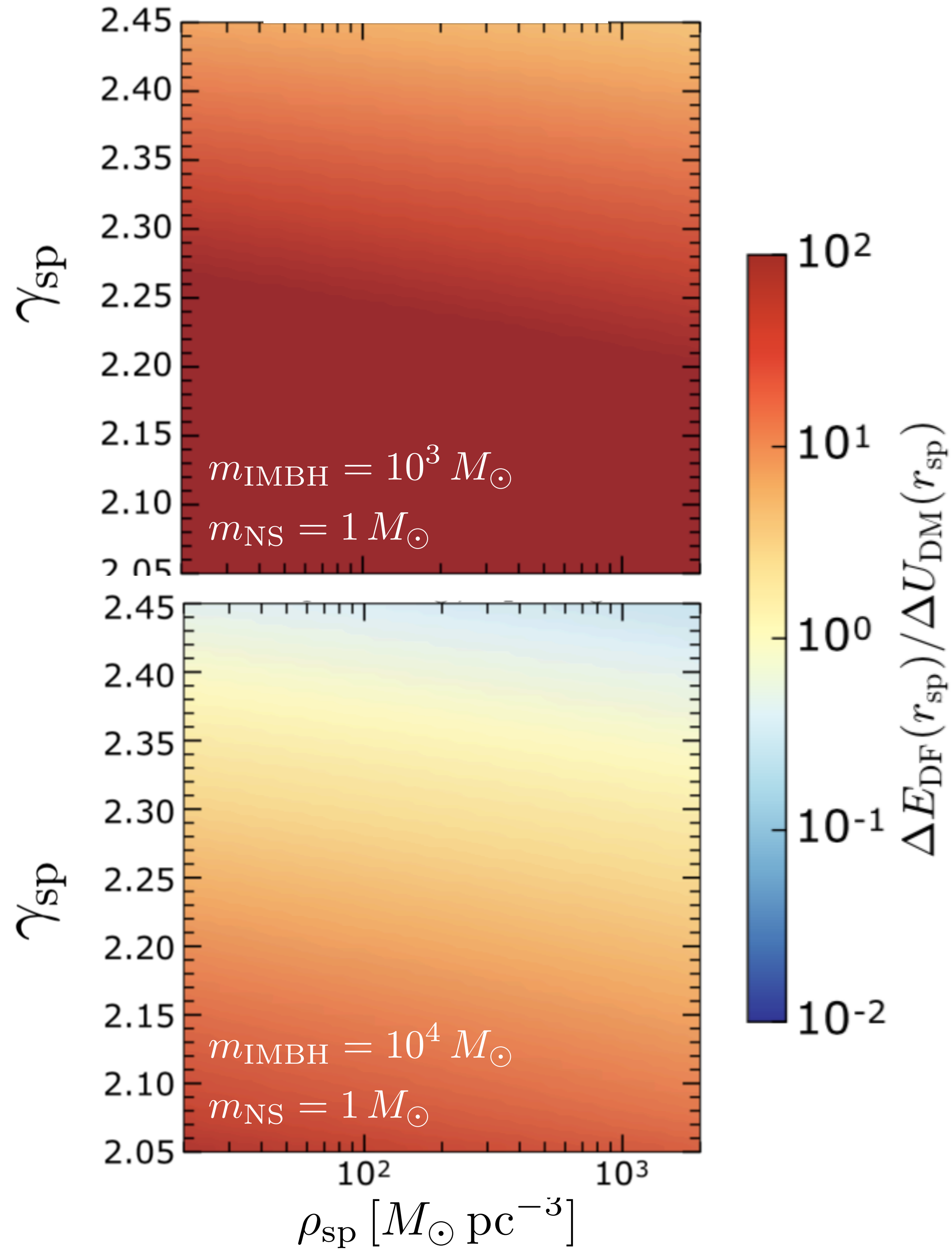


Energy Budget

Q: How much energy is *available* for dynamical friction?



A: Binding energy of DM $\Delta U_{\rm DM}$ over radius Δr



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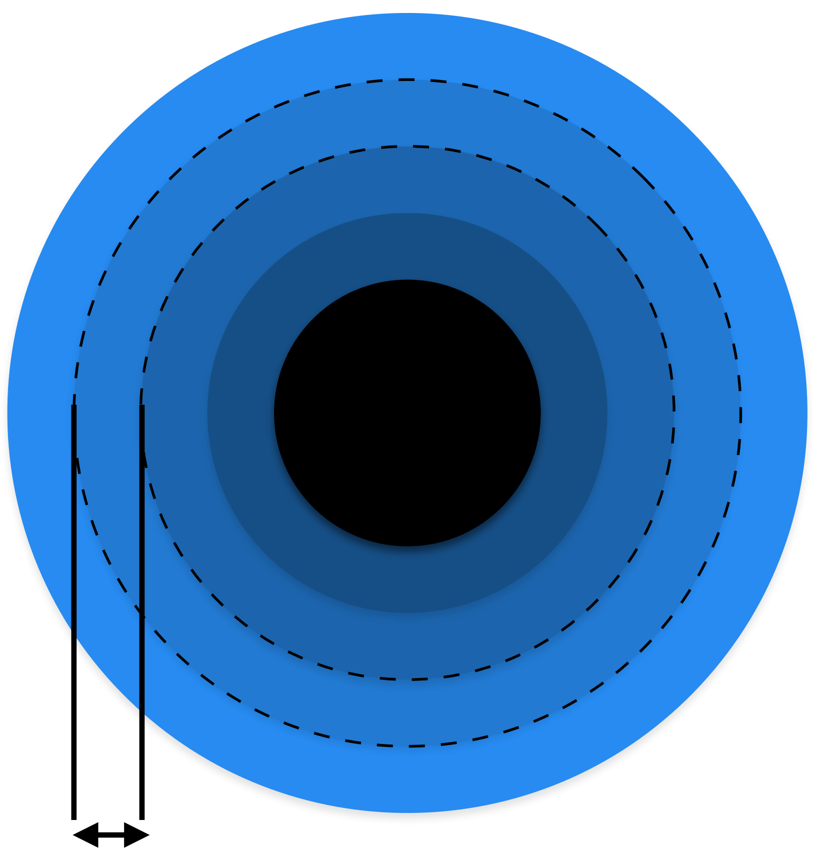
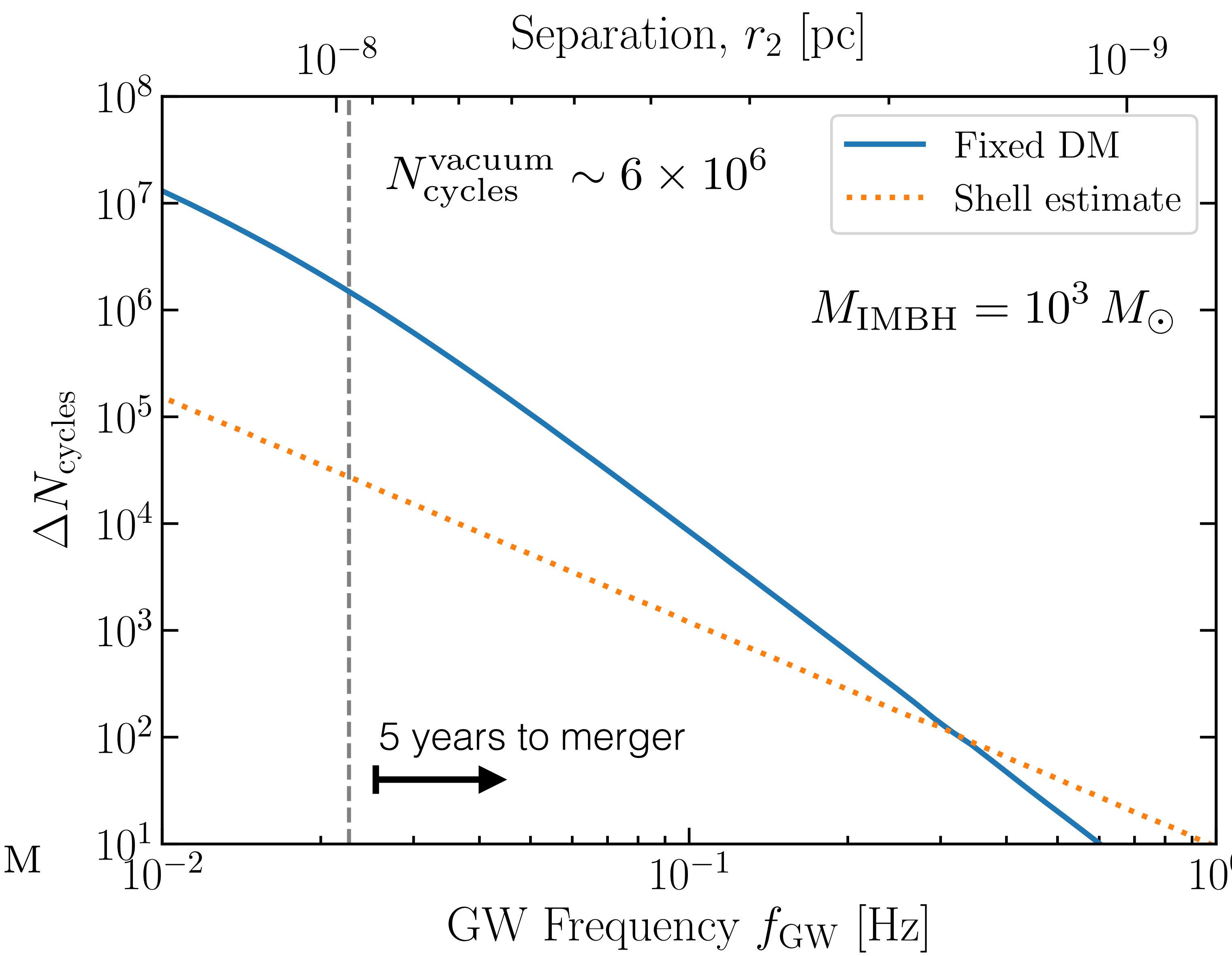


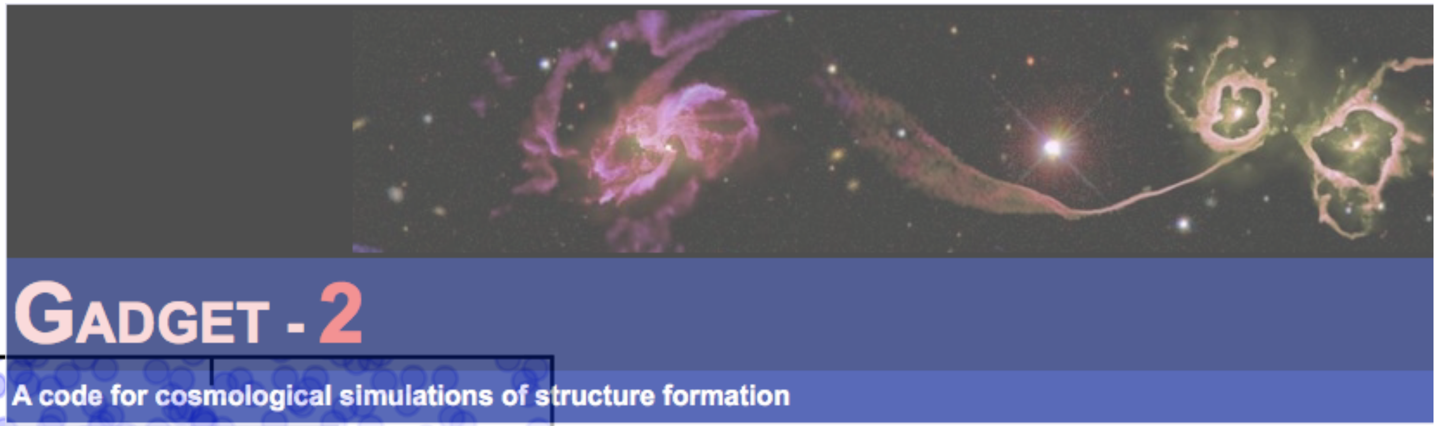
A: Binding energy of DM ΔU_{DM} over radius Δr

Evolve the system by fixing the dynamical friction force to extract *all* binding energy from a shell at a given radius:

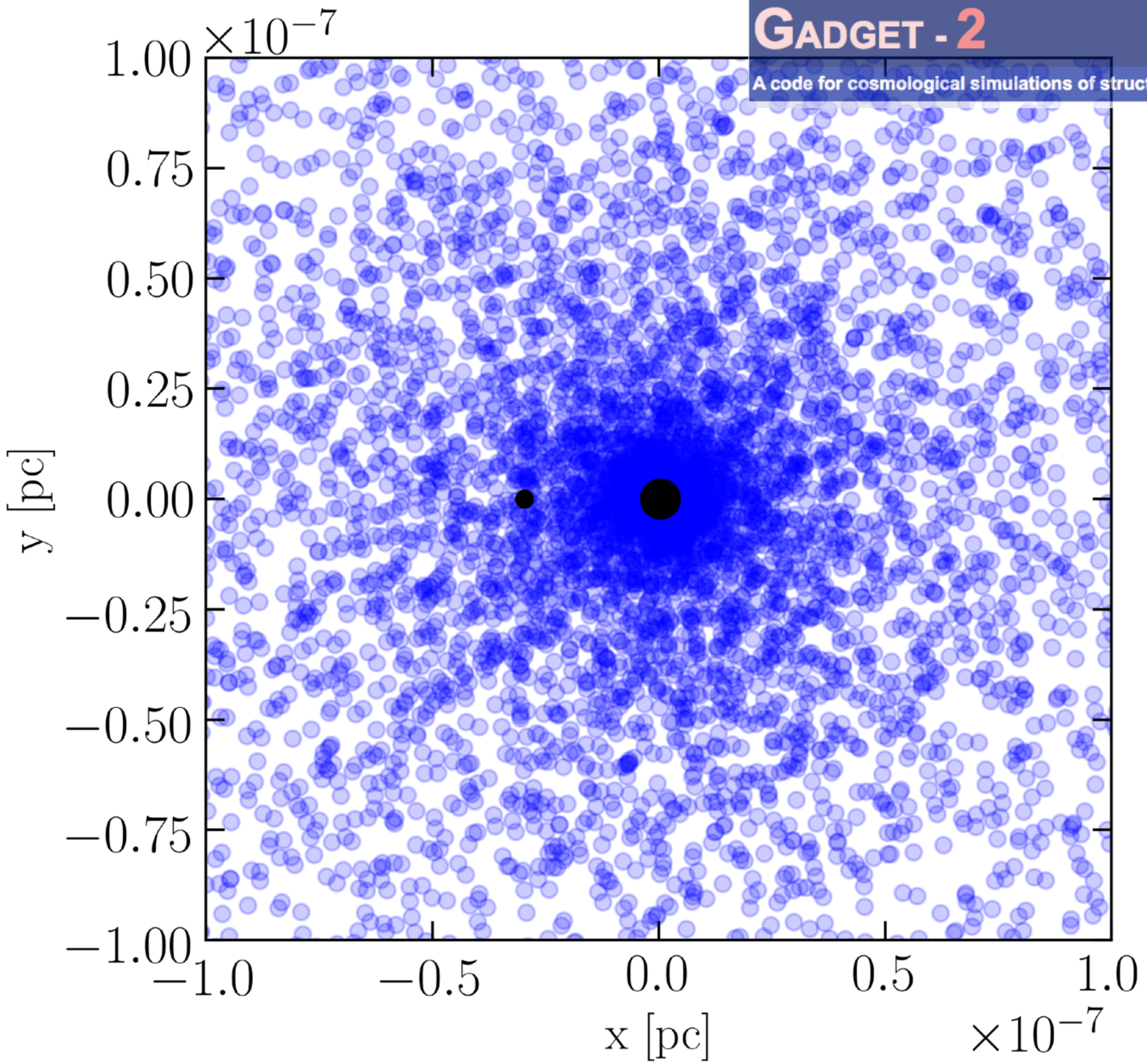
$$\dot{E}_{\mathrm{DF}} = \dot{r}\,\frac{\mathrm{d}U_{\mathrm{DM}}}{\mathrm{d}r}$$



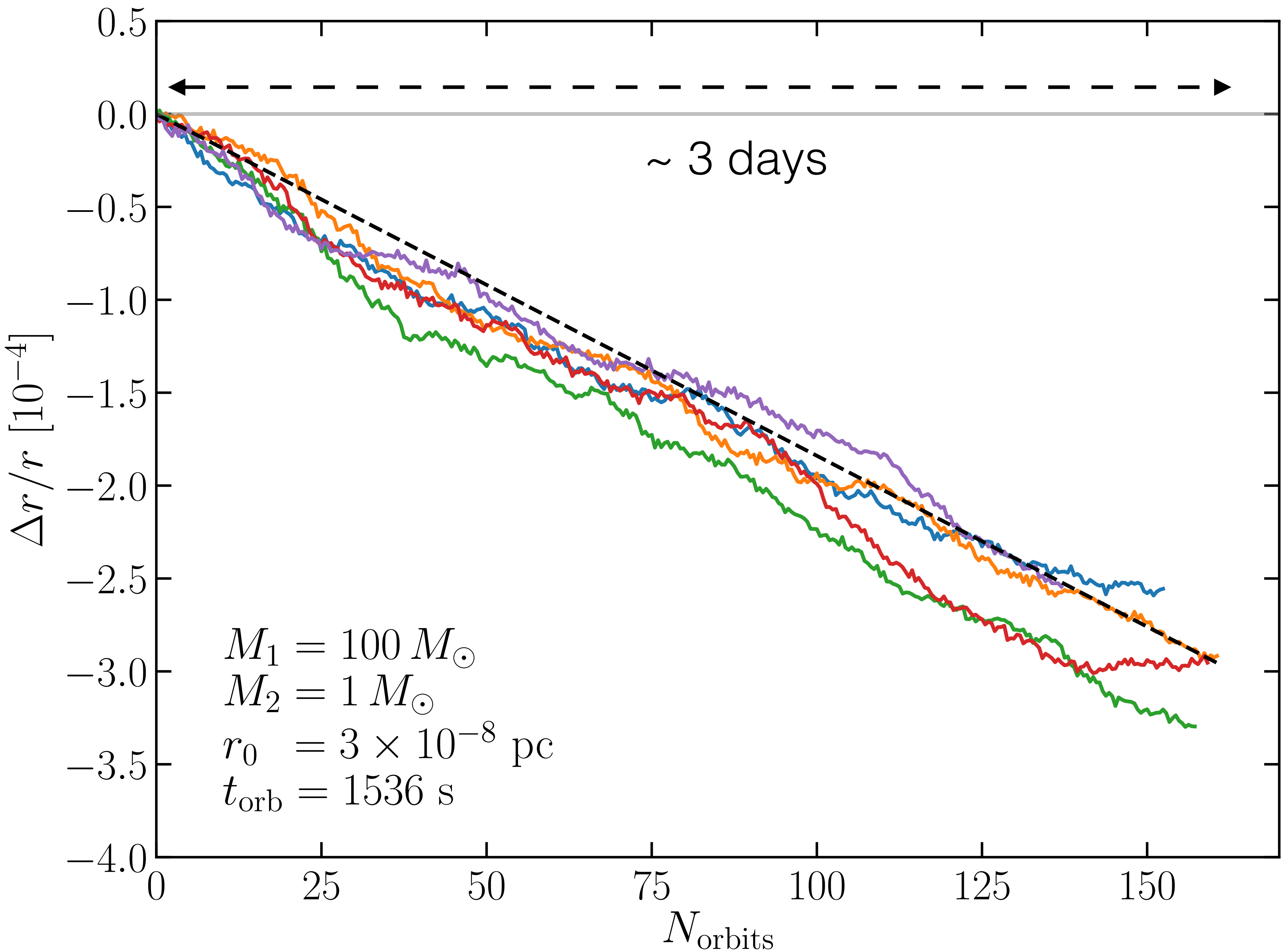
N-body Simulations



[astro-ph/0505010]



N-body Results



Allows us to check assumptions and fix normalisation of DF force (lnΛ), but can't simulate the whole 5 year inspiral!

Self-consistent evolution

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{v}} \equiv f(\mathcal{E})$$

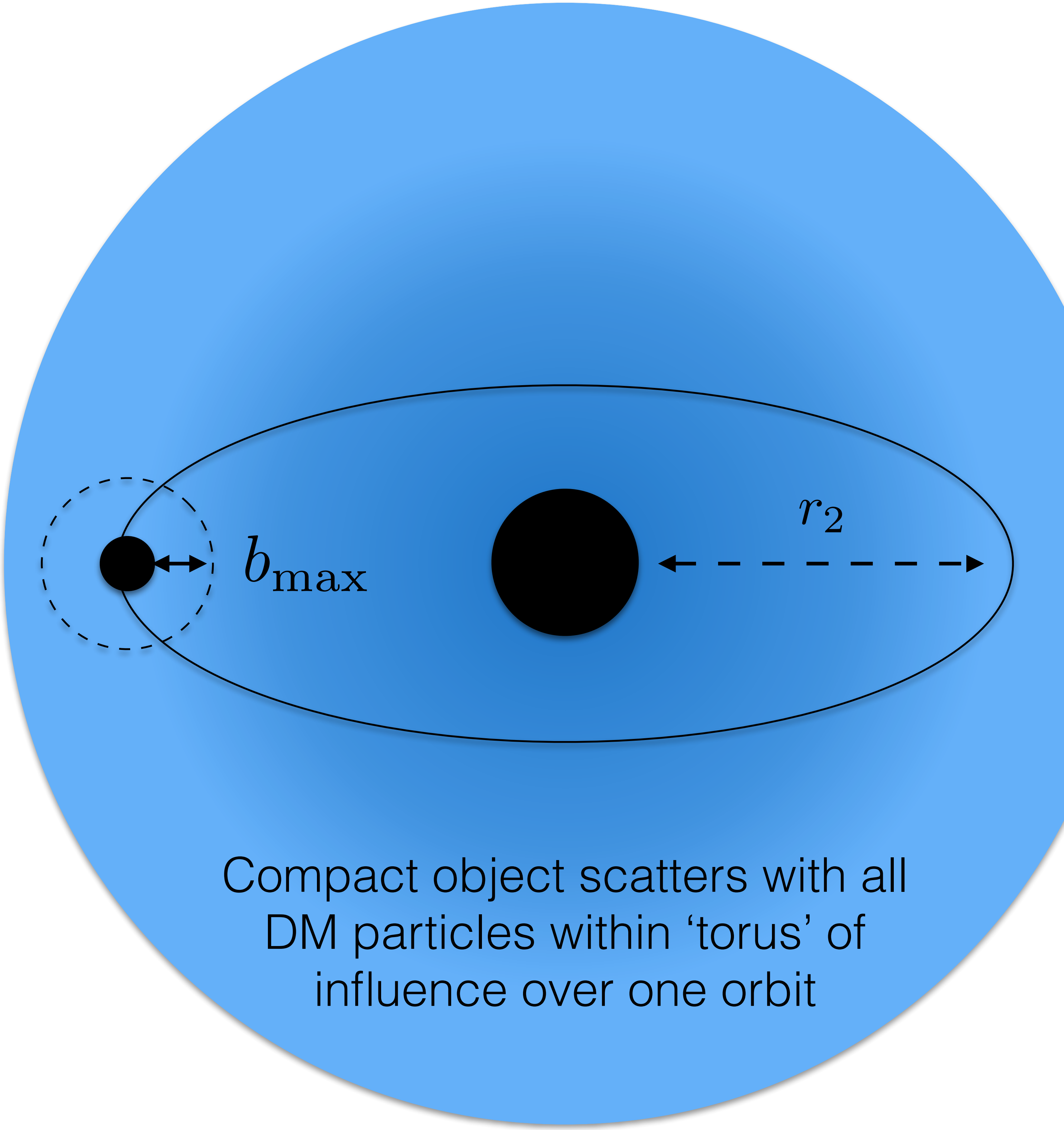
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a 'kick' through gravitational scattering

$$\mathcal{E} \to \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int \mathrm{d}^3\mathbf{v}\, f(\mathcal{E})$$



Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$\Delta f(\mathcal{E}) = -p_\mathcal{E} f(\mathcal{E}) + \int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}}\right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E})\, \mathrm{d}\Delta\mathcal{E}$$

$P_\mathcal{E}(\Delta\mathcal{E})$ - probability for a particle with energy $\mathcal{E}$ to scatter and receive a 'kick' $\Delta\mathcal{E}$

$$p_\mathcal{E} = \int P_\mathcal{E}(\Delta\mathcal{E})\, \mathrm{d}\Delta\mathcal{E}$$

- total probability for a particle with energy $\mathcal{E}$ to scatter

Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

Particles scattering from
$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

$$\Delta f(\mathcal{E}) = \boxed{-p_\mathcal{E} f(\mathcal{E}) +} \boxed{\int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}}\right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E})\,\mathrm{d}\Delta\mathcal{E}}$$

Particles scattering from
$\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

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- total probability for a particle with energy $\mathcal{E}$ to scatter

Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$T_\mathrm{orb} \frac{f(\mathcal{E})}{\mathrm{d}t} = \boxed{-p_\mathcal{E} f(\mathcal{E}) +} \boxed{\int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}}\right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E})\,\mathrm{d}\Delta\mathcal{E}}$$

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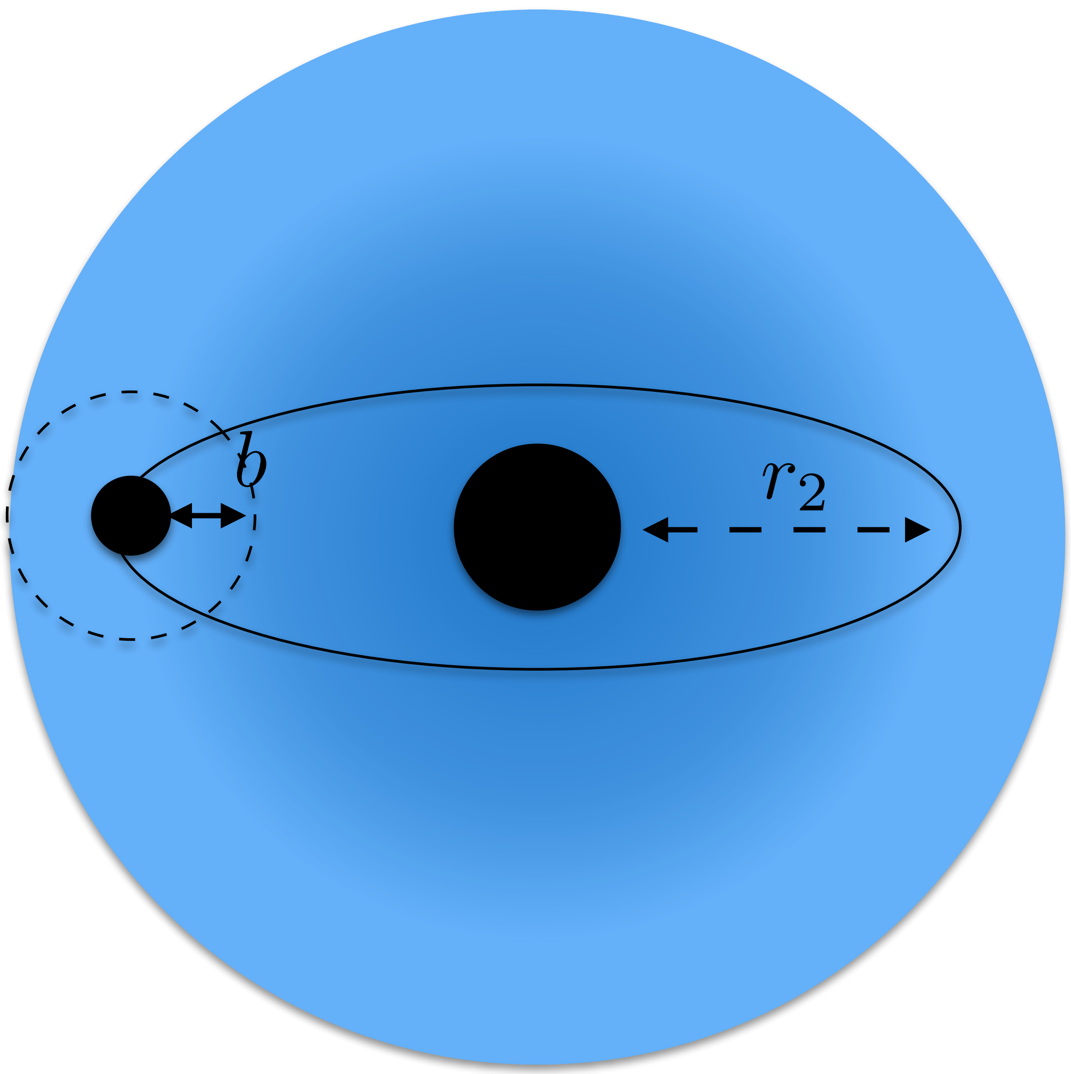
- total probability for a particle with energy $\mathcal{E}$ to scatter

Scattering probability $P_{\mathcal{E}}(\Delta\mathcal{E})$

Two body scattering problem relates energy exchange to impact parameter:

$$\Delta\mathcal{E}(b) = -2v_0^2\left[1 + \frac{b^2 {v_0}^4}{G^2 {m_2}^2}\right]^{-1}$$



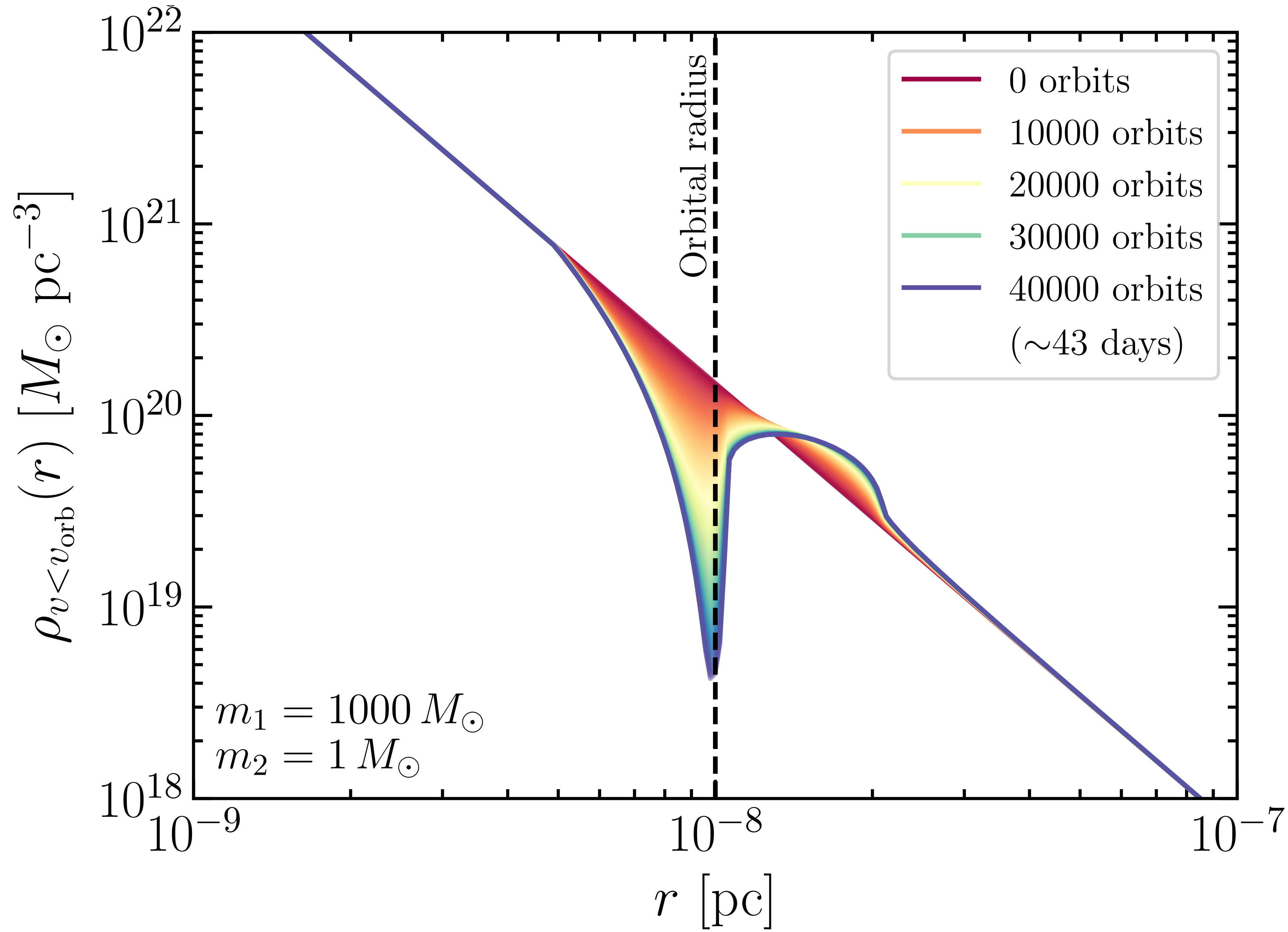
Integrate over the surface of the 'torus of influence'

Working to first order in b/r, $P_{\mathcal{E}}(\Delta\mathcal{E})$ can be written in terms of elliptic integrals

Code available online:
github.com/bradkav/HaloFeedback

Evolution of density profile

How does the DM halo 'react' to the orbiting compact object?



Subtlety: plotting here only 'slow moving' particles

Evolution of density profile

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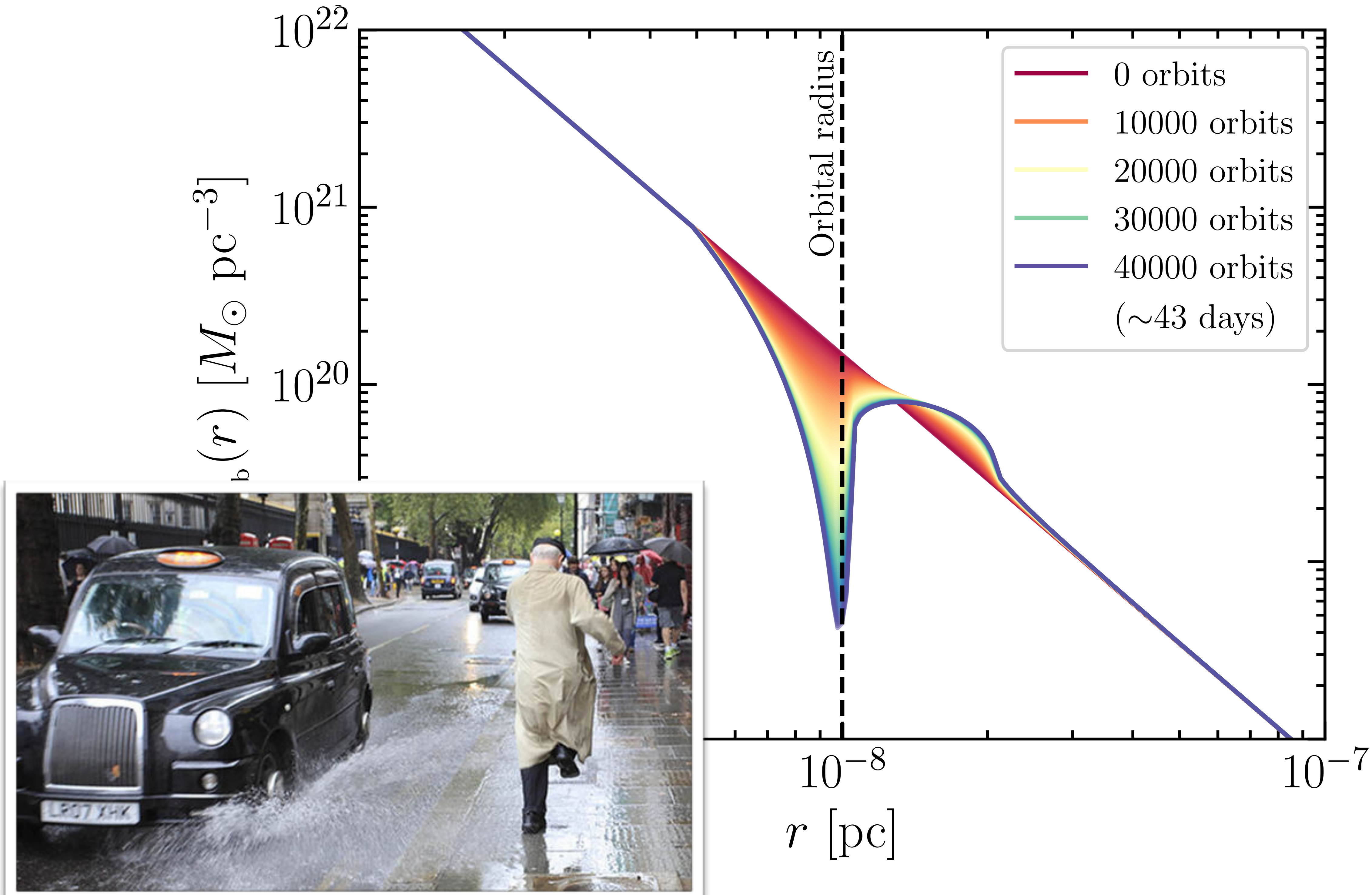
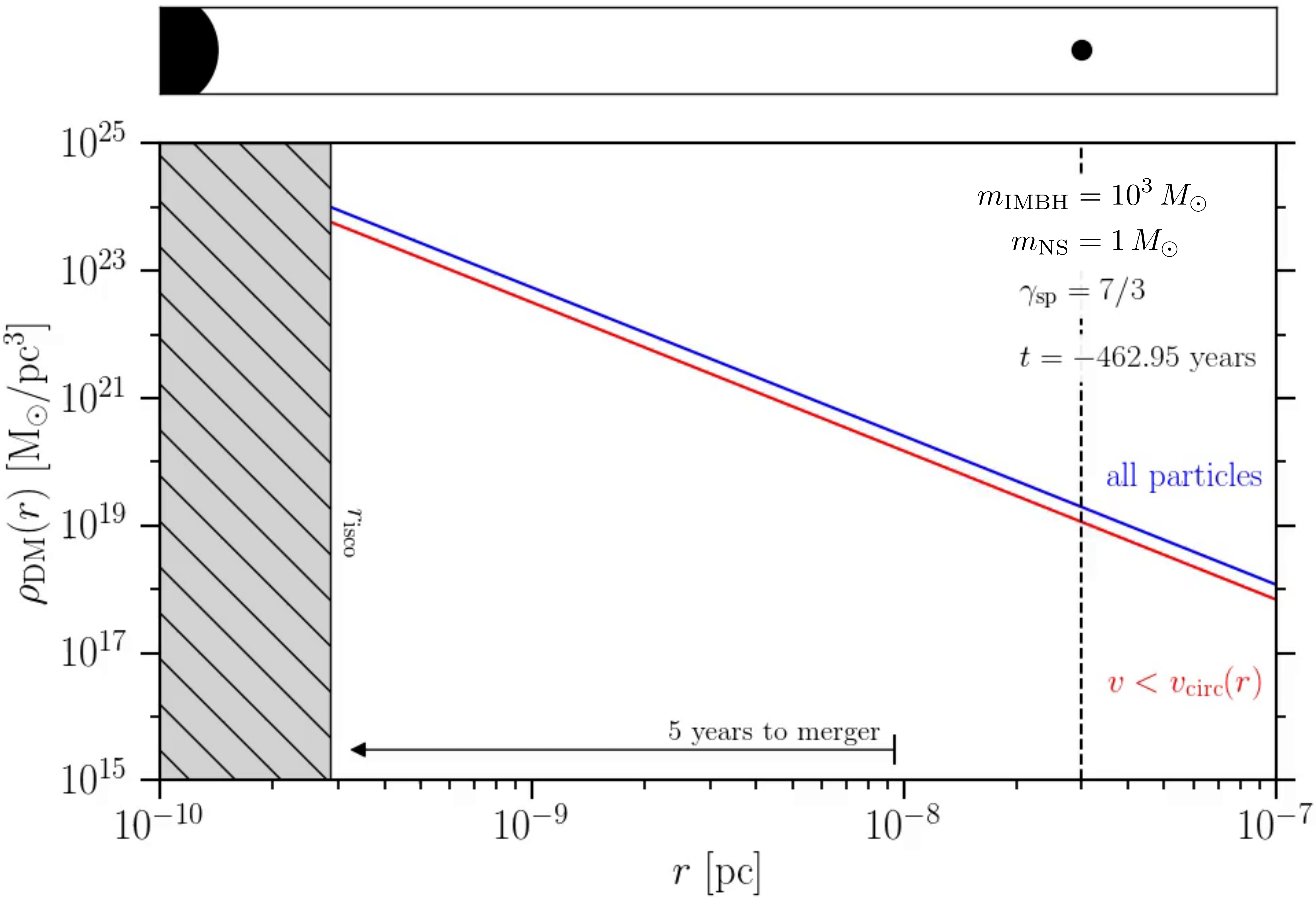


Subtlety: plotting here only ‘slow moving’ particles

Full evolution of the system

Movies: tinyurl.com/GW4DM



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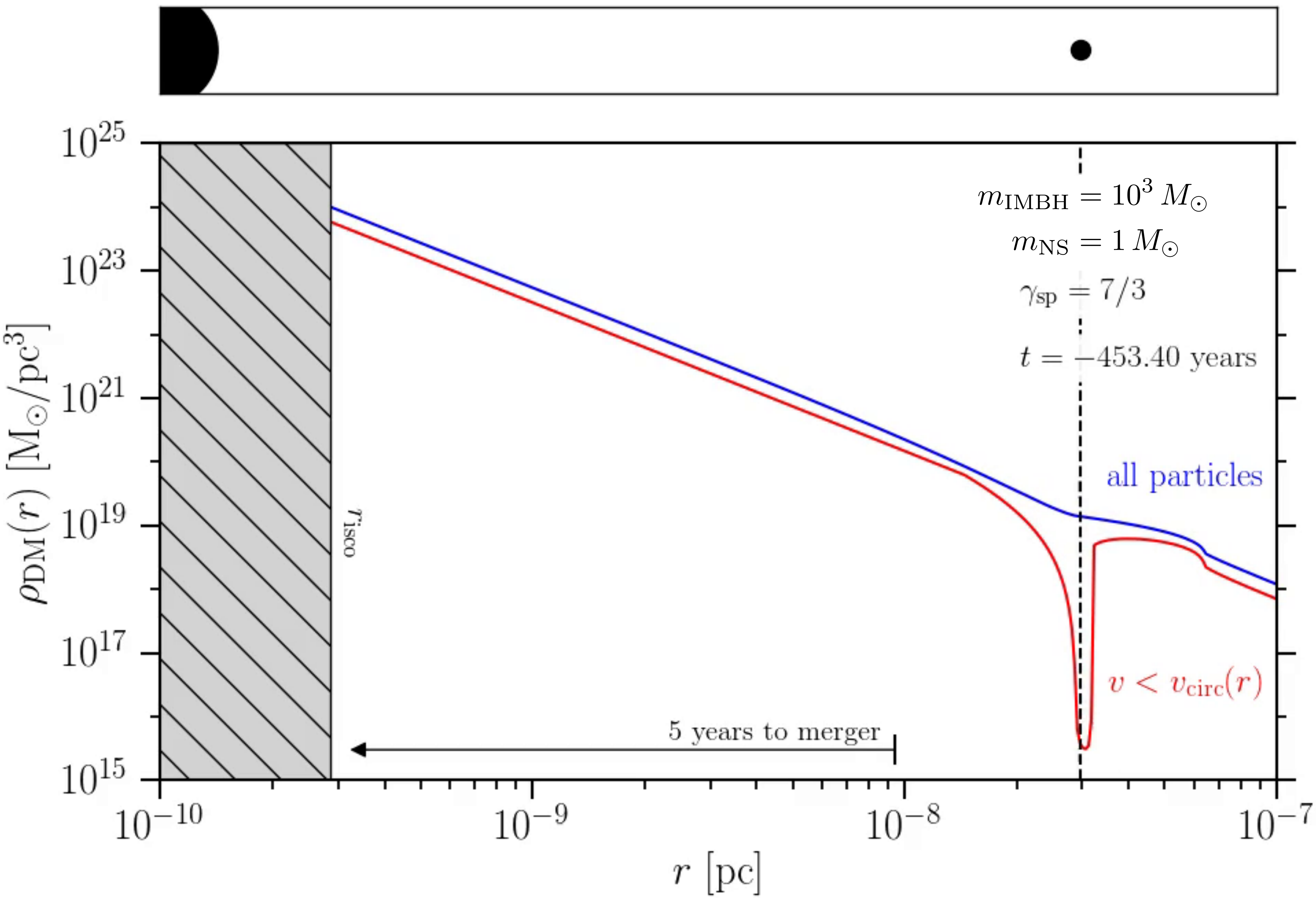

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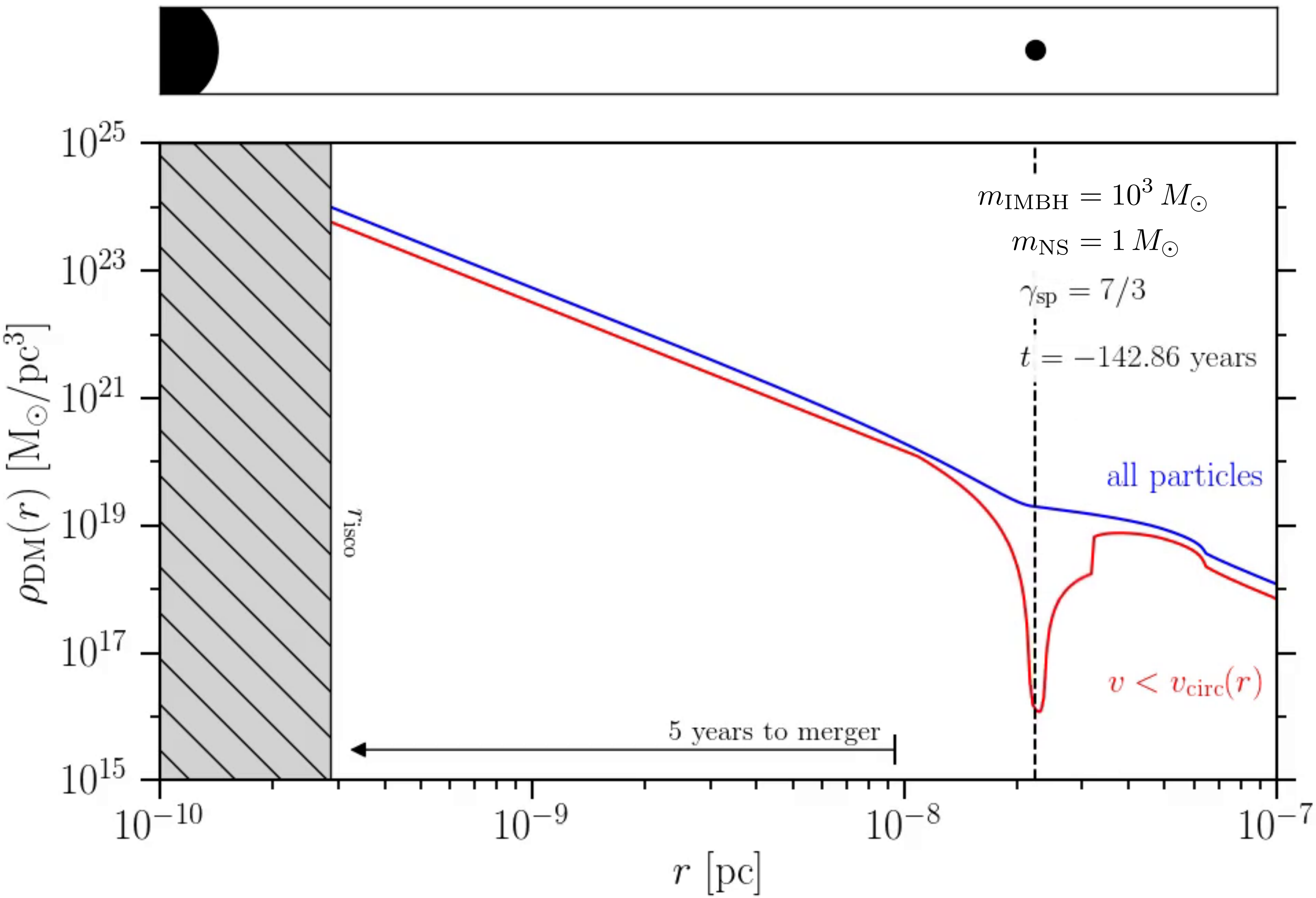

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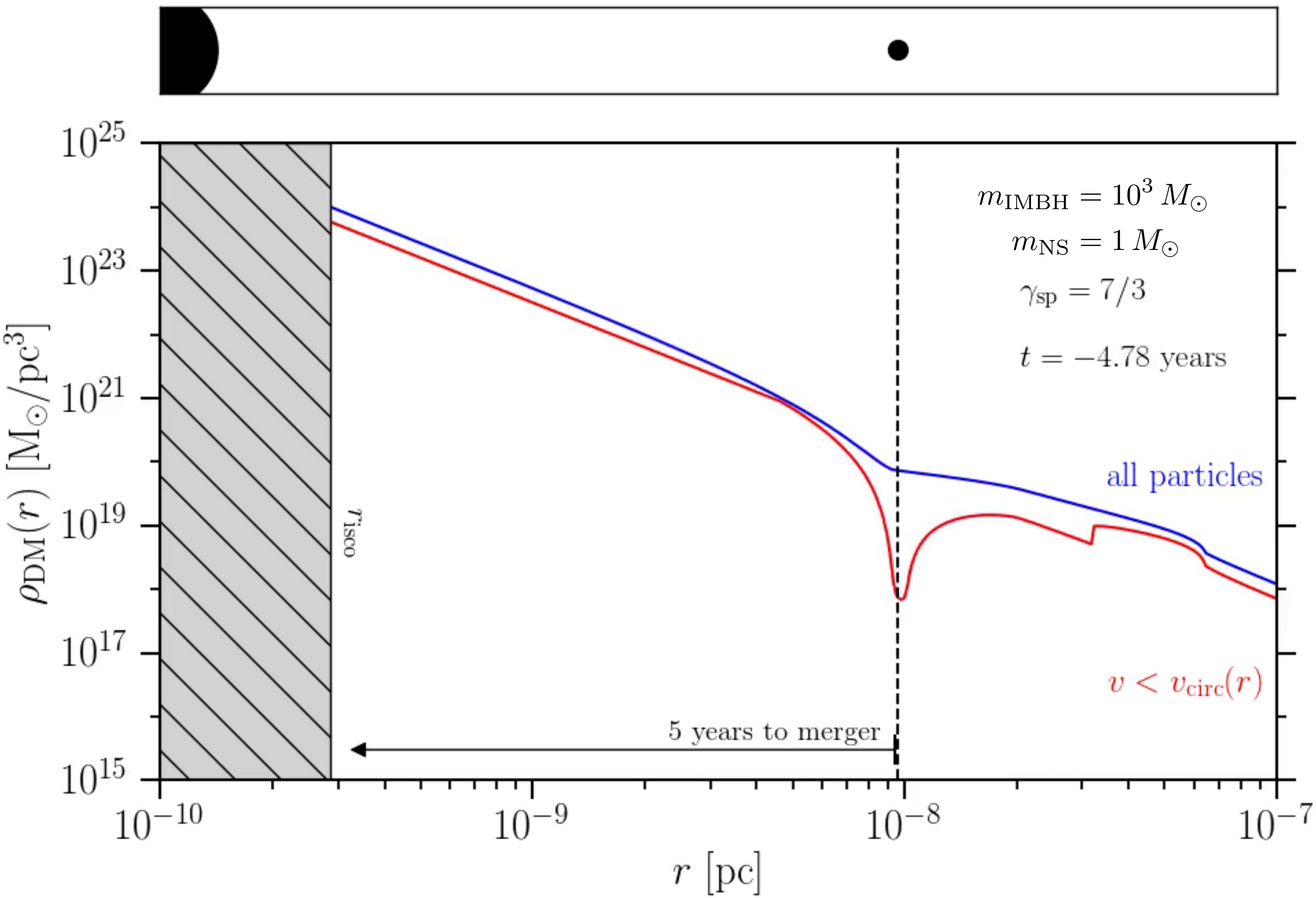

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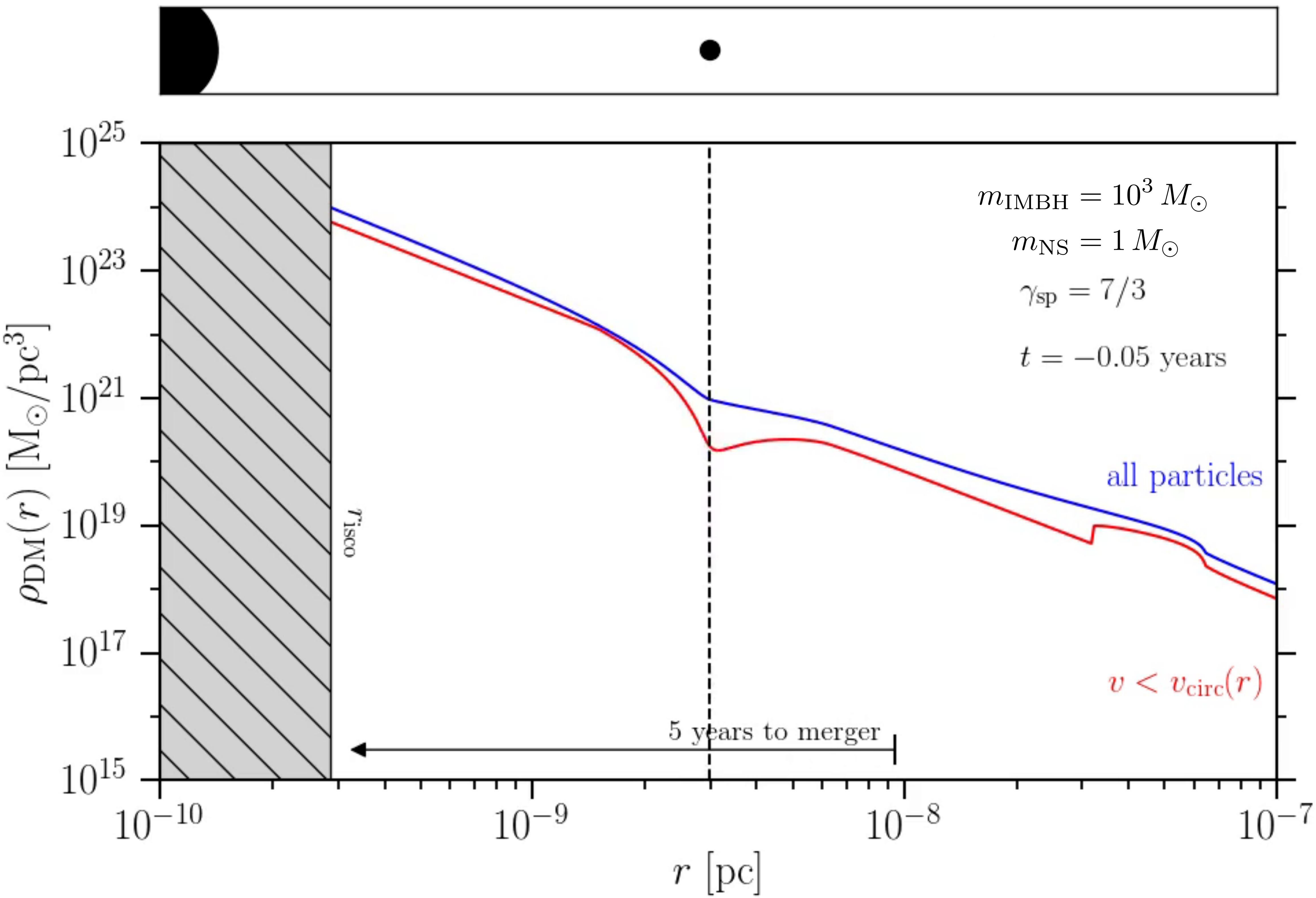

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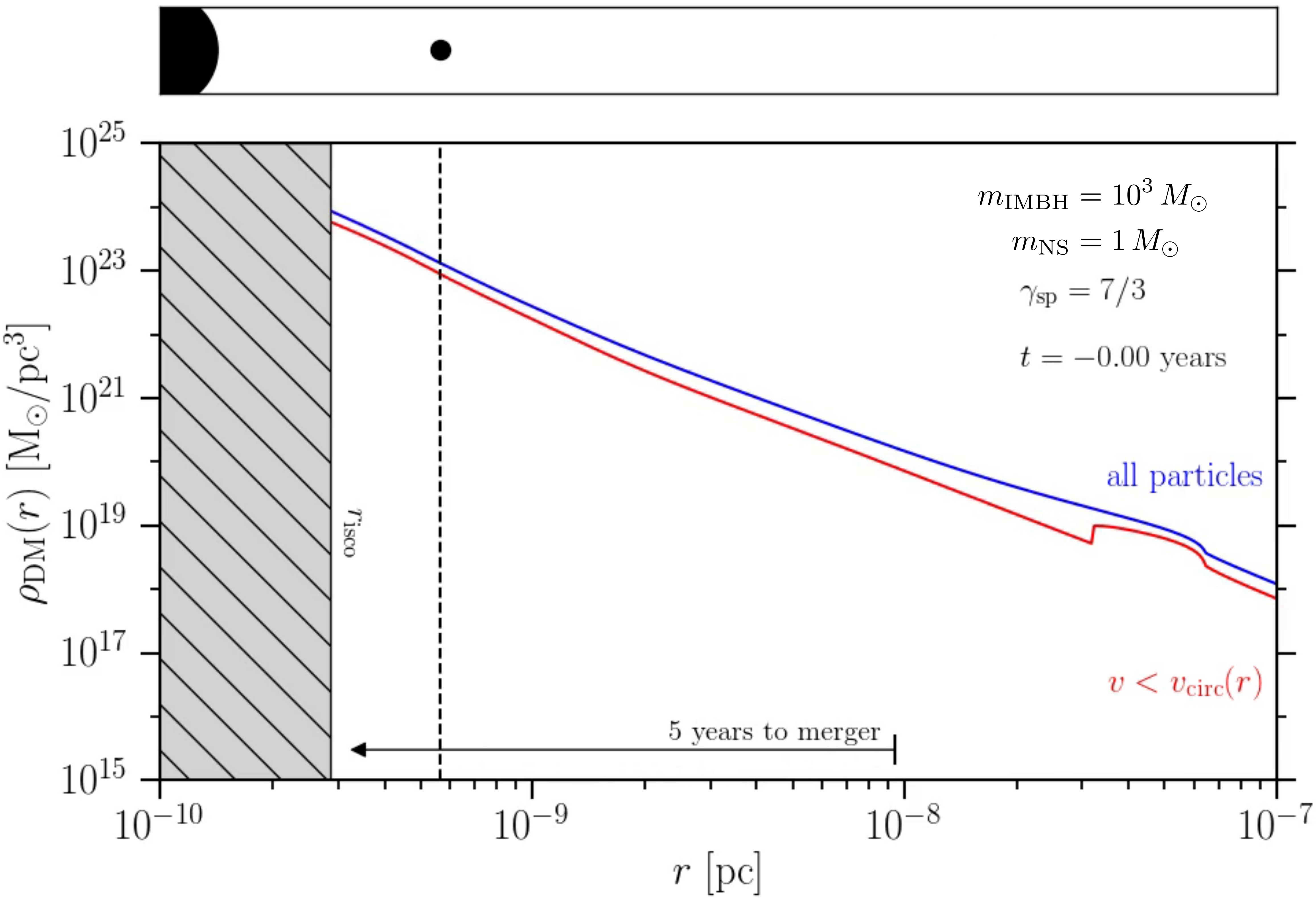
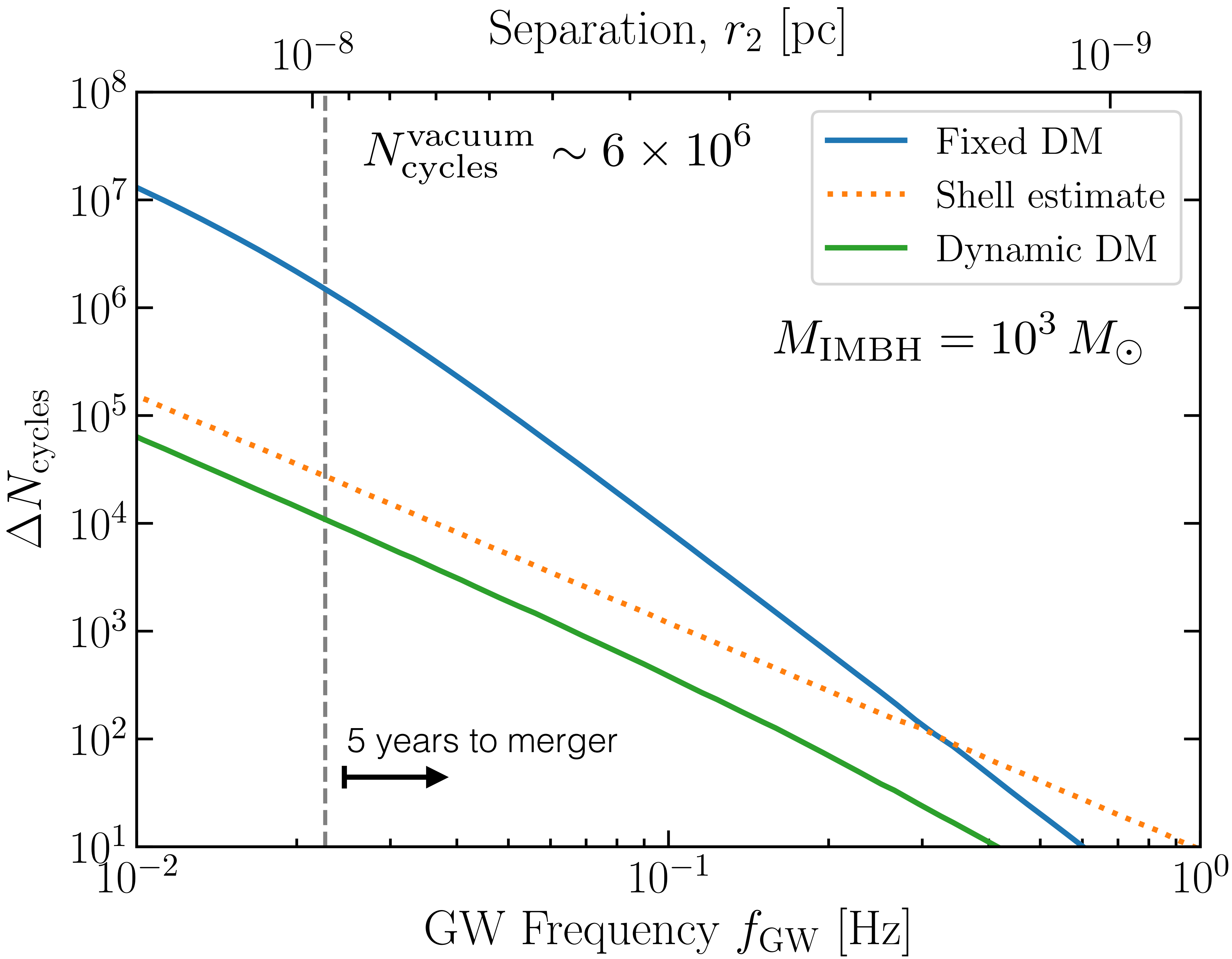


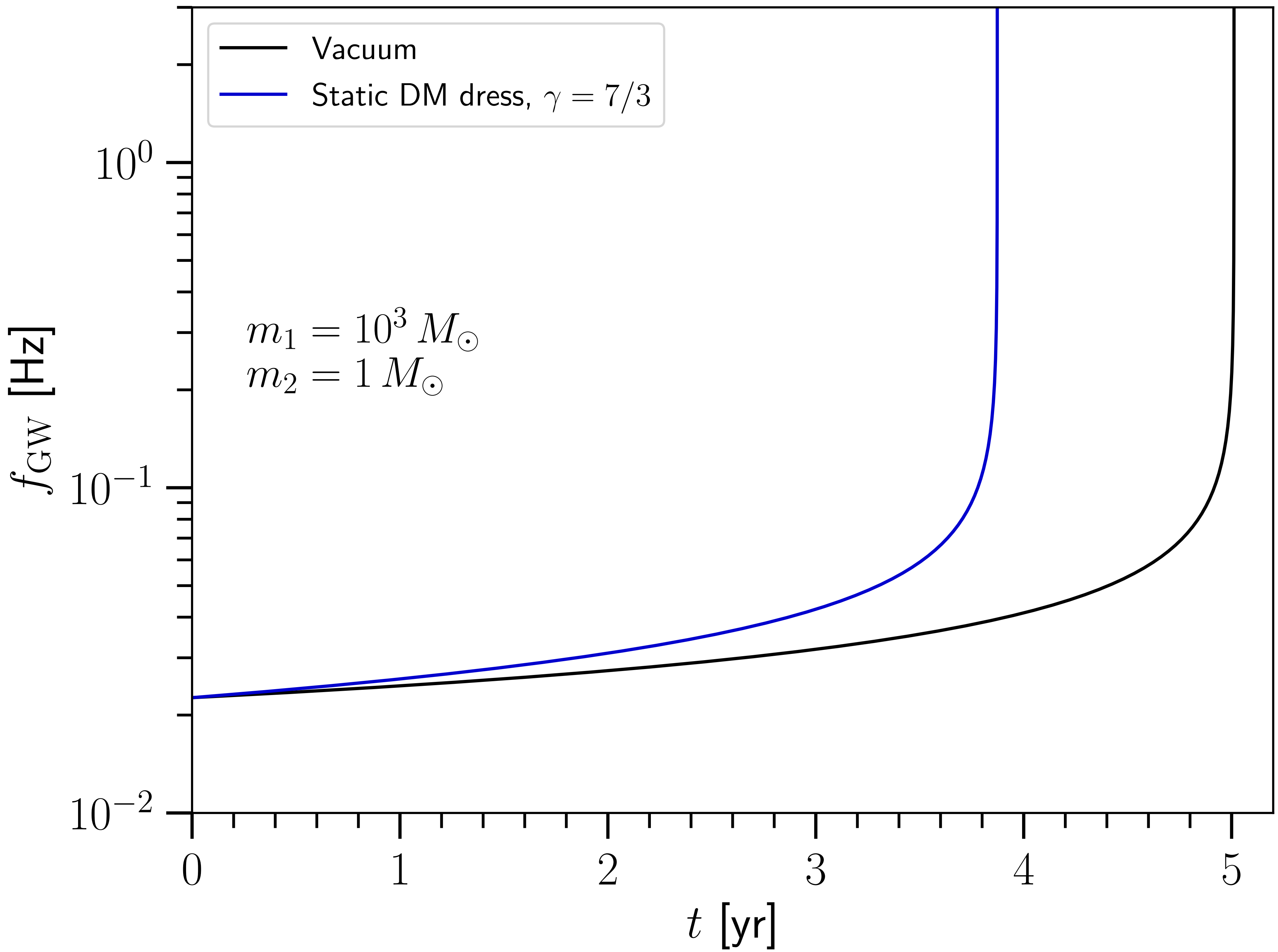
Self-consistent results



$$\Delta N_{\rm cycles}(\text{static}) \approx 10^6 \;\rightarrow\; \Delta N_{\rm cycles}(\text{dynamic}) \approx 10^4$$

Spectrograms: $m_\mathrm{IMBH} = 10^3\,M_\odot$

$$\rho_\mathrm{DM}(r) = \rho_\mathrm{sp}\left(\frac{r_\mathrm{sp}}{r}\right)^{\gamma_\mathrm{sp}}$$



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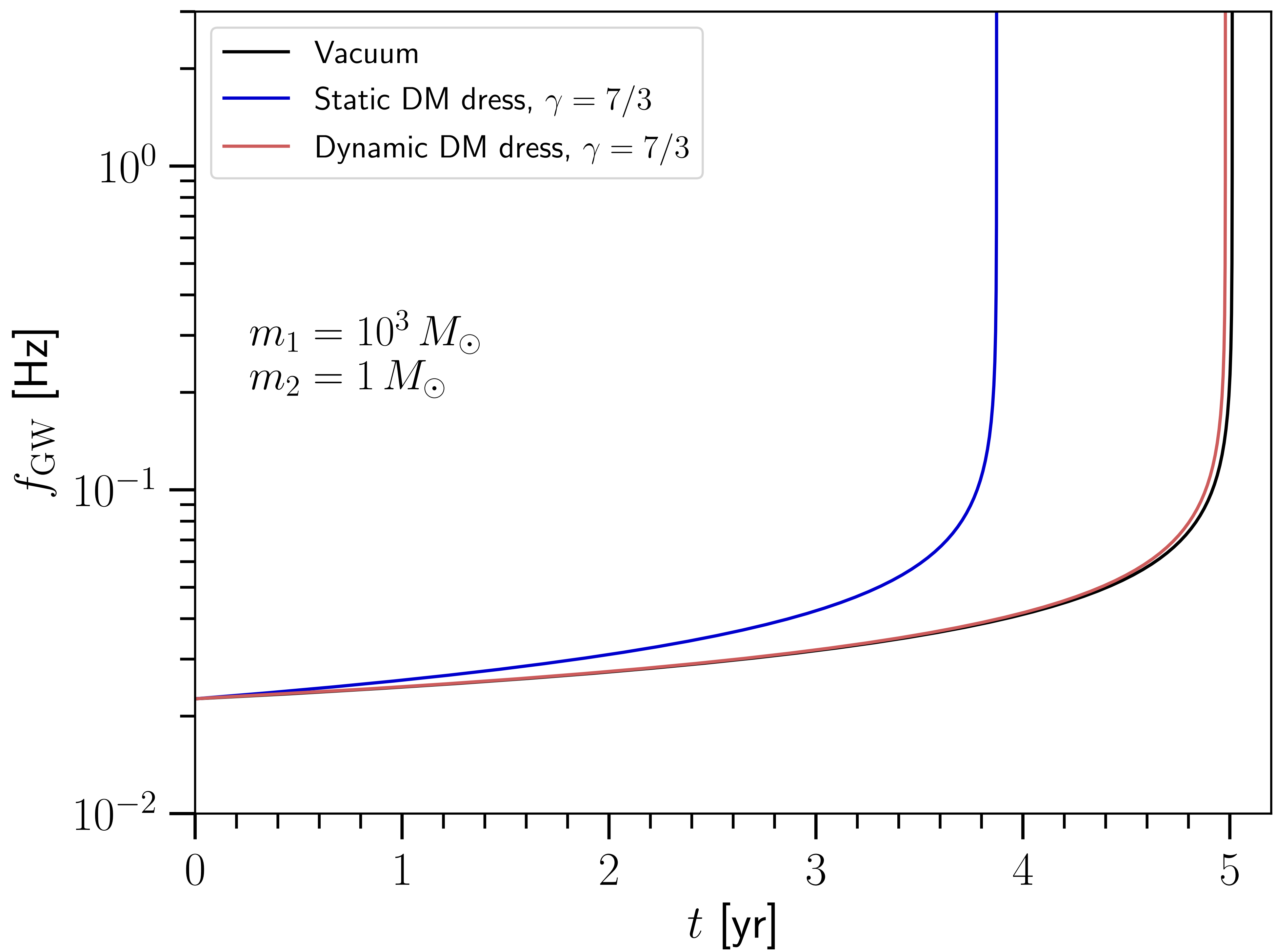
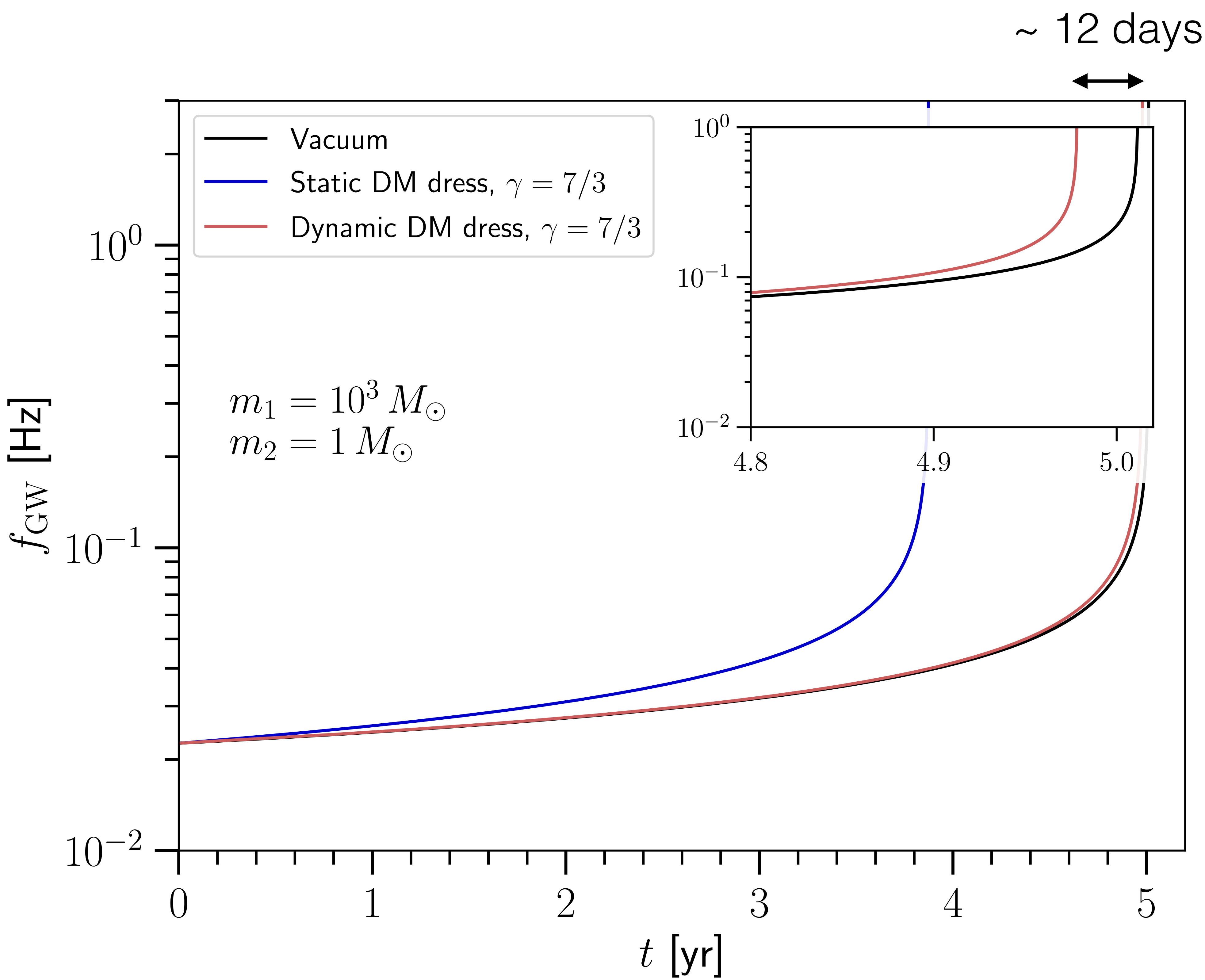


Spectrograms: $m_{\rm IMBH} = 10^3\,M_\odot$

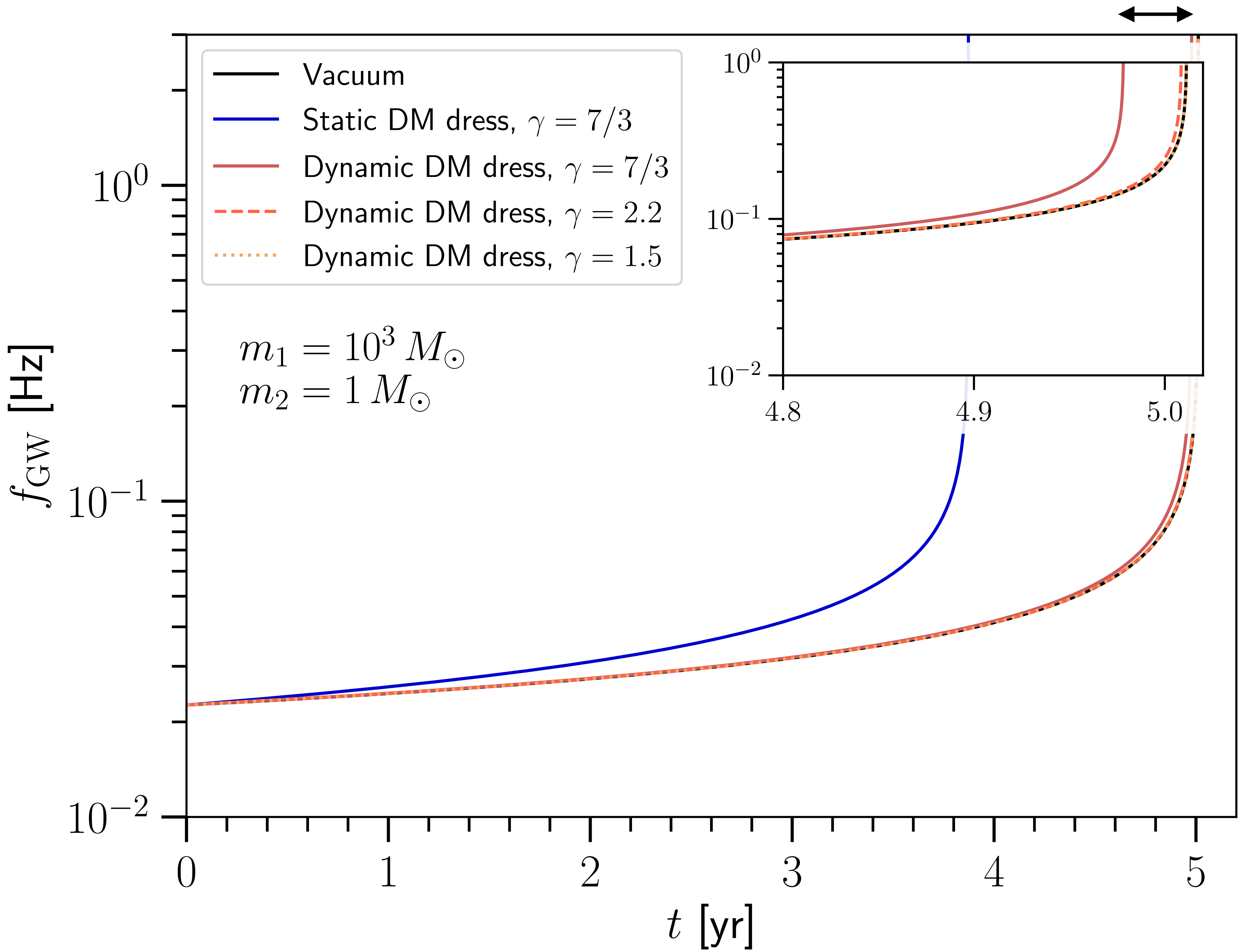
$$\rho_{\rm DM}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$



Spectrograms: $m_{\rm IMBH} = 10^3\,M_\odot$

$$\rho_{\rm DM}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$

~ 12 days

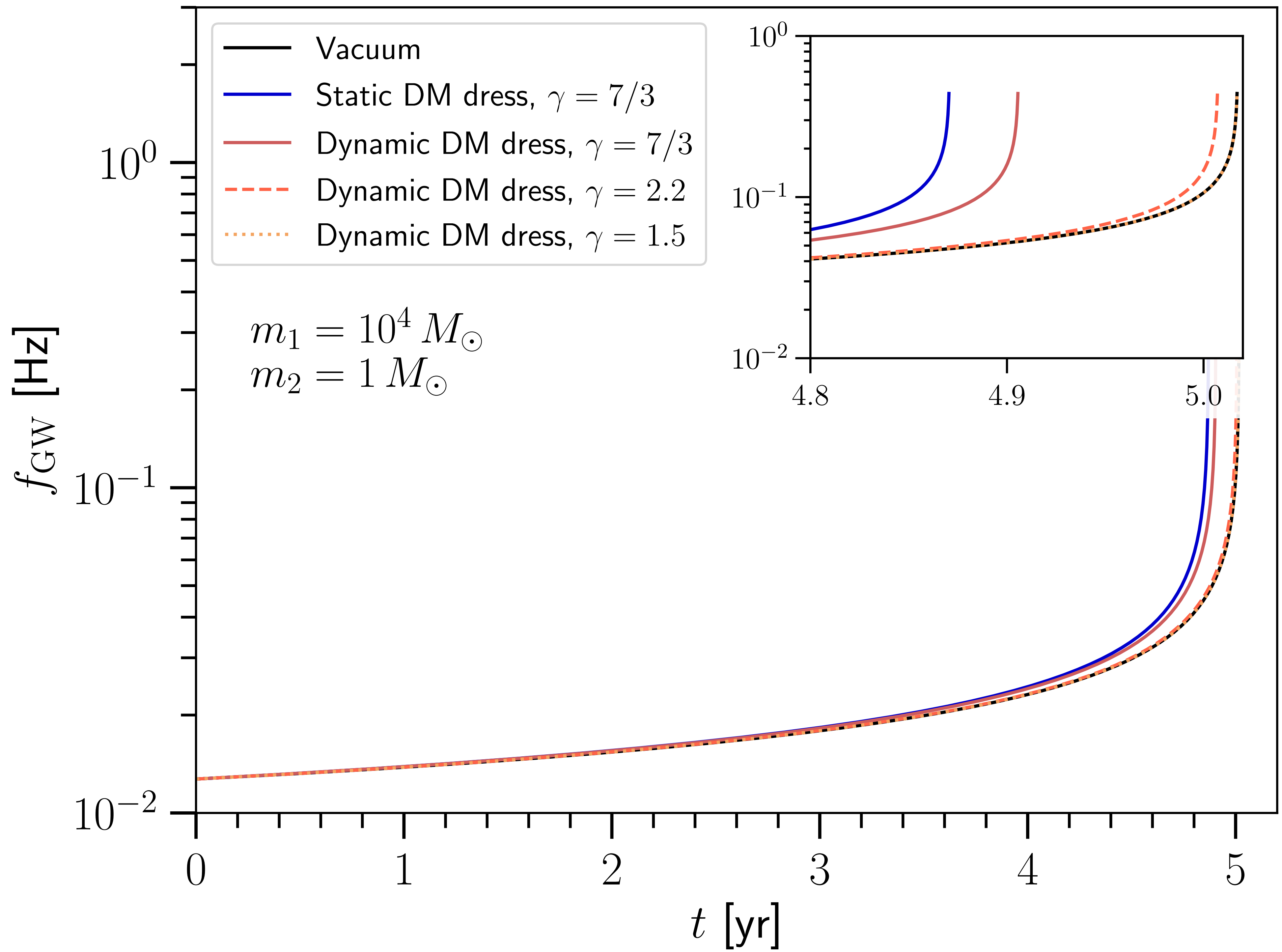


NB: $7/3 \approx 2.333$

Spectrograms: $m_{\rm IMBH} = 10^4\,M_\odot$

$$\rho_{\rm DM}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$

As we increase the IMBH mass, the correction from having a dynamic DM halo decreases (but can still be very relevant)



NB: $7/3 \approx 2.333$

Detectability

$$\Delta N_{\mathrm{cycles}}(\mathrm{static}) \approx 10^6 \;\rightarrow\; \Delta N_{\mathrm{cycles}}(\mathrm{dynamic}) \approx 10^4$$

In many systems, a more realistic treatment leads to a huge reduction in the size of the 'de-phasing' effect.

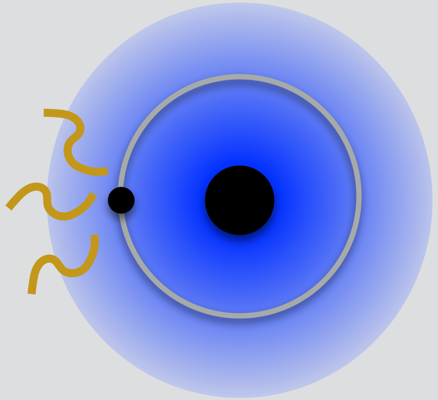
Even for very massive BHs, small corrections can spoil our ability to find the signal in data (so they must be accounted for)

The 'rule of thumb': LISA should be able to detect a de-phasing as small as one radian.

For a 1000 or 10000 solar mass IMBH, a de-phasing of ~10,000 cycles would still be easily detectable.

GW + EM signals of
QCD axion Dark Matter

[Edwards, Chianese, **BJK**, Nissanke & Weniger,
Phys. Rev. Lett. 124, 161101, 1905.04686]

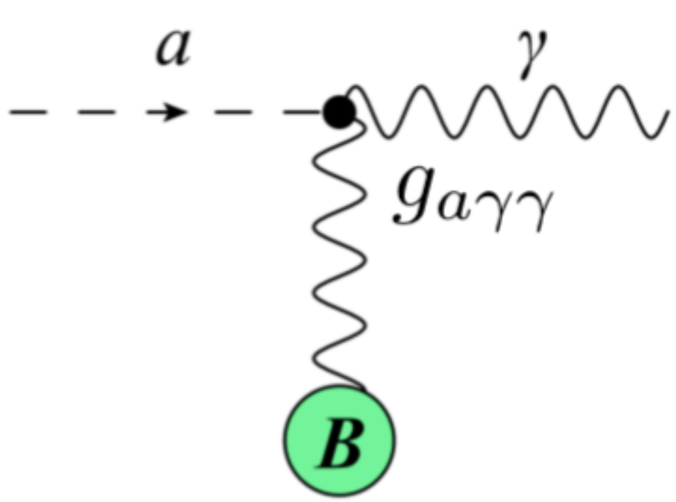


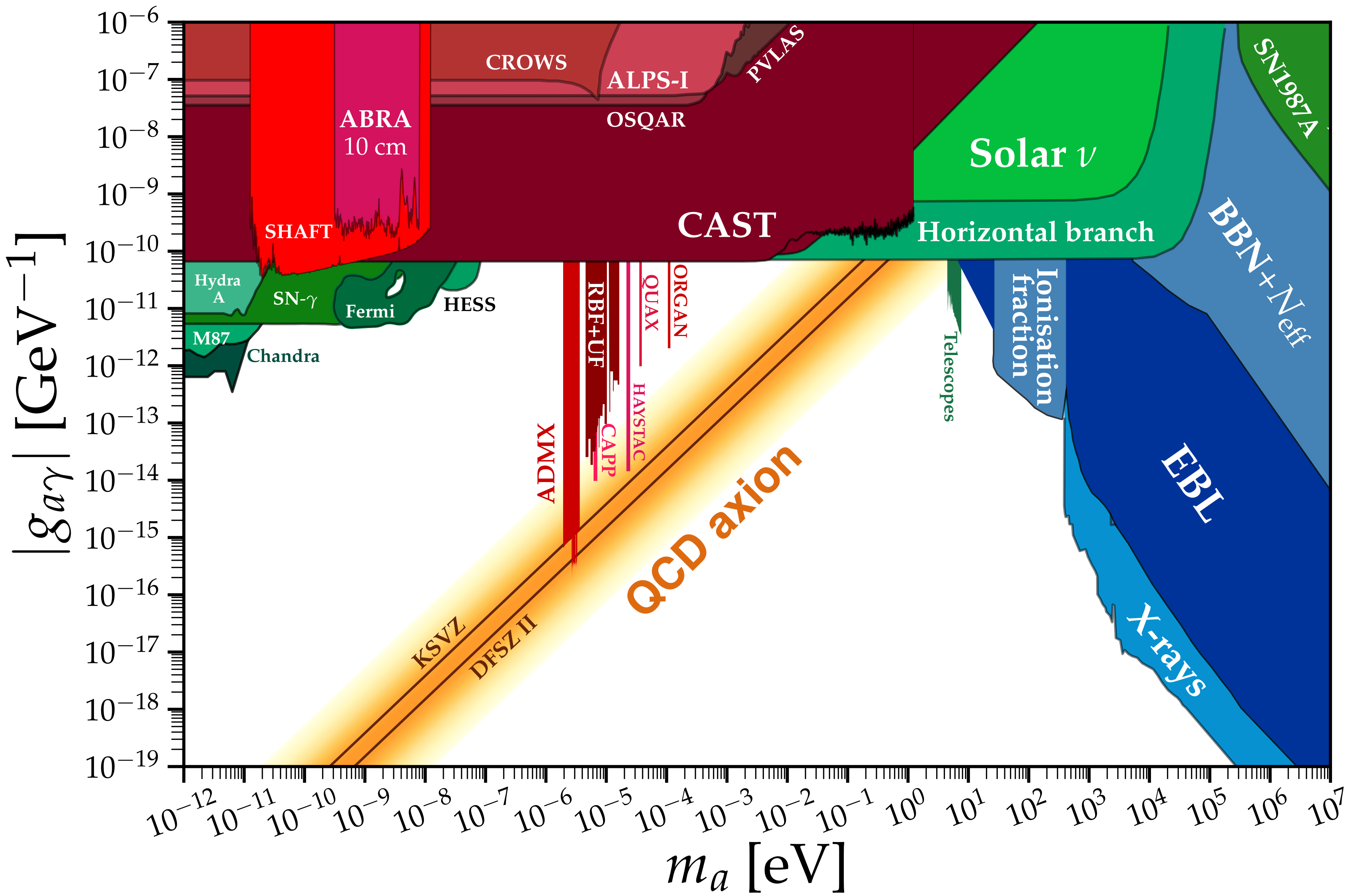
Axion Dark Matter

[Slide credit: Marco Chianese]

Dark Matter could be in the form of light pseudo scalar 'axions',
which may convert to photons (and vice versa) in an external magnetic field:

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$
$$= -\frac{1}{4} g_{a\gamma\gamma} a \boldsymbol{E} \cdot \boldsymbol{B}$$





[O'Hare, https://cajohare.github.io/AxionLimits/]

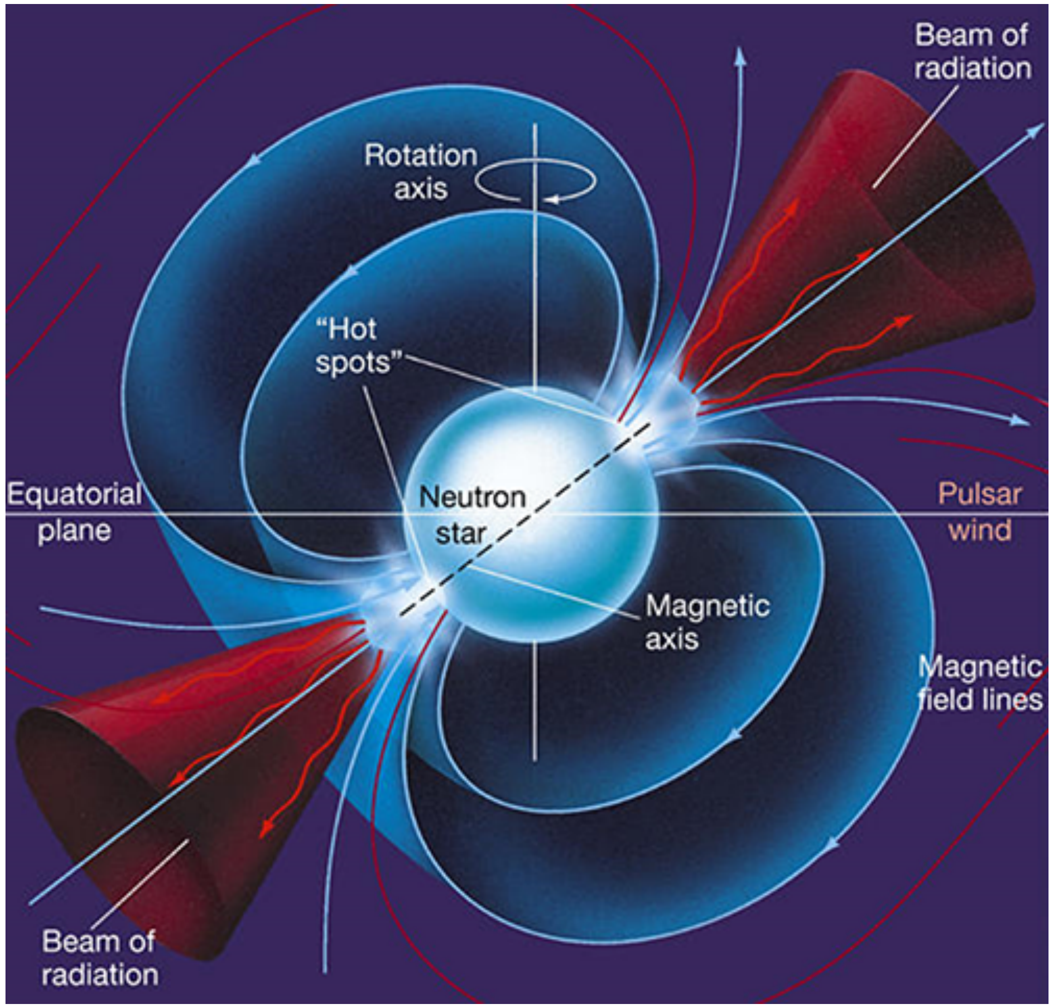
Axions and neutron stars

Old neutron stars can have extremely high magnetic fields:

$$B_0 = 10^{12} - 10^{15}\,G$$

Surrounded by a dense plasma which allows 'resonant' conversion when axion mass matches plasma mass:

$$\omega_p\left(B_0, P\right) = m_a/2\pi$$



Magnetic field and plasma frequency varies as a function of radius from NS; NS can effectively 'scan' over a range of axion masses.

[1803.08230, 1804.03145, 1811.01020, 1910.11907]

Axions and neutron stars

Produce a photon with axion energy $m_a \sim 10^{-6}\,\mathrm{eV} \sim 240\,\mathrm{MHz}$

Radio Signal

Conversion happens at a radius r_c, with probability: $p_{a\gamma} \propto \frac{g_{a\gamma\gamma}^2 B\left(r_c\right)^2}{2v_c}$

Radiated power is given by: $\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Omega} \sim 2 \times p_{a\gamma}\rho_{\mathrm{DM}}\left(r_c\right) v_c r_c^2$

Probe axions in the mass range

$m_a \sim 10^{-7}\,\mathrm{eV}$ up to $m_a \sim 10^{-5}\,\mathrm{eV}$

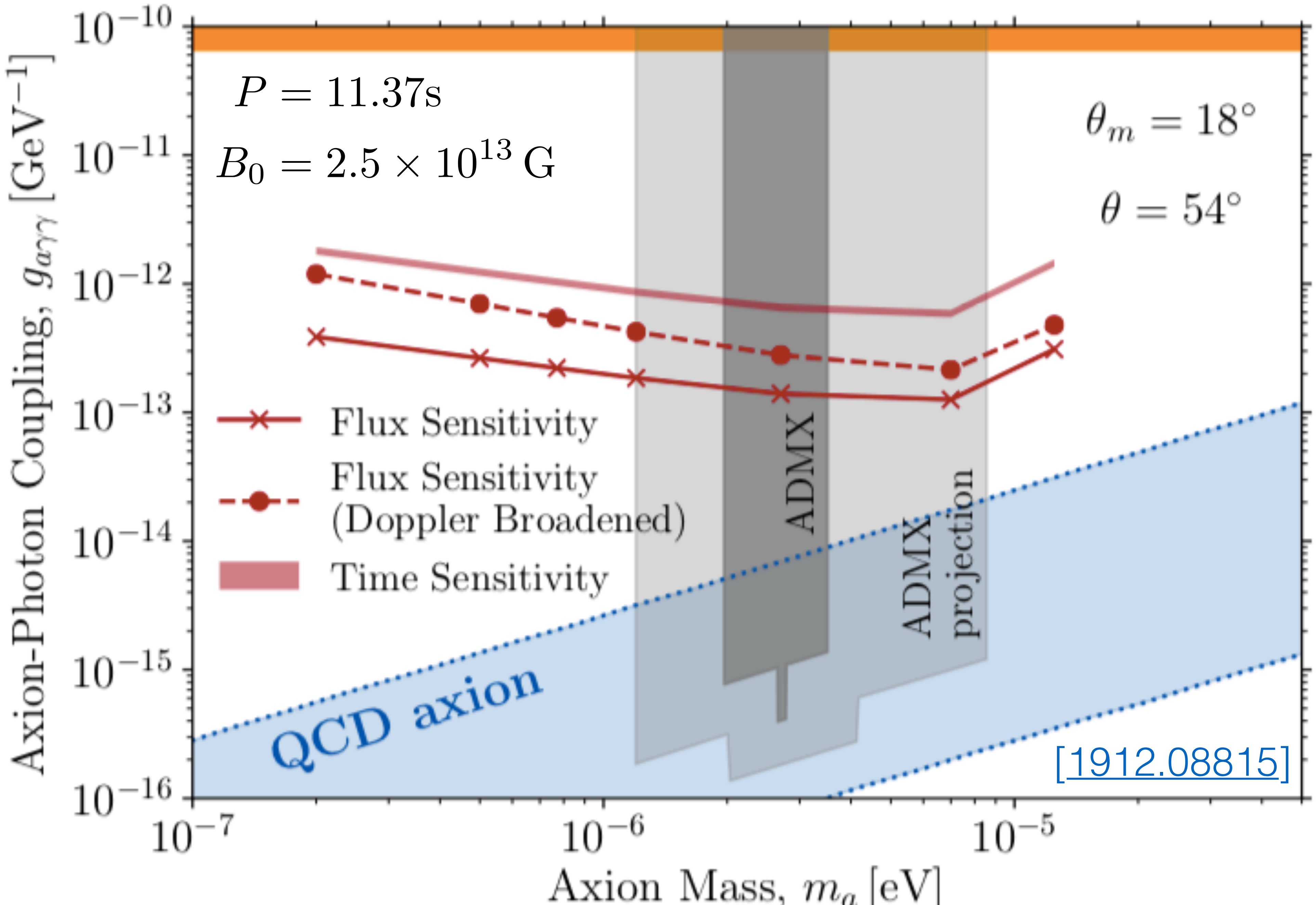
Frequency range of radio telescopes

Require conversion *outside* NS

[1803.08230, 1804.03145, 1811.01020, 1910.11907]

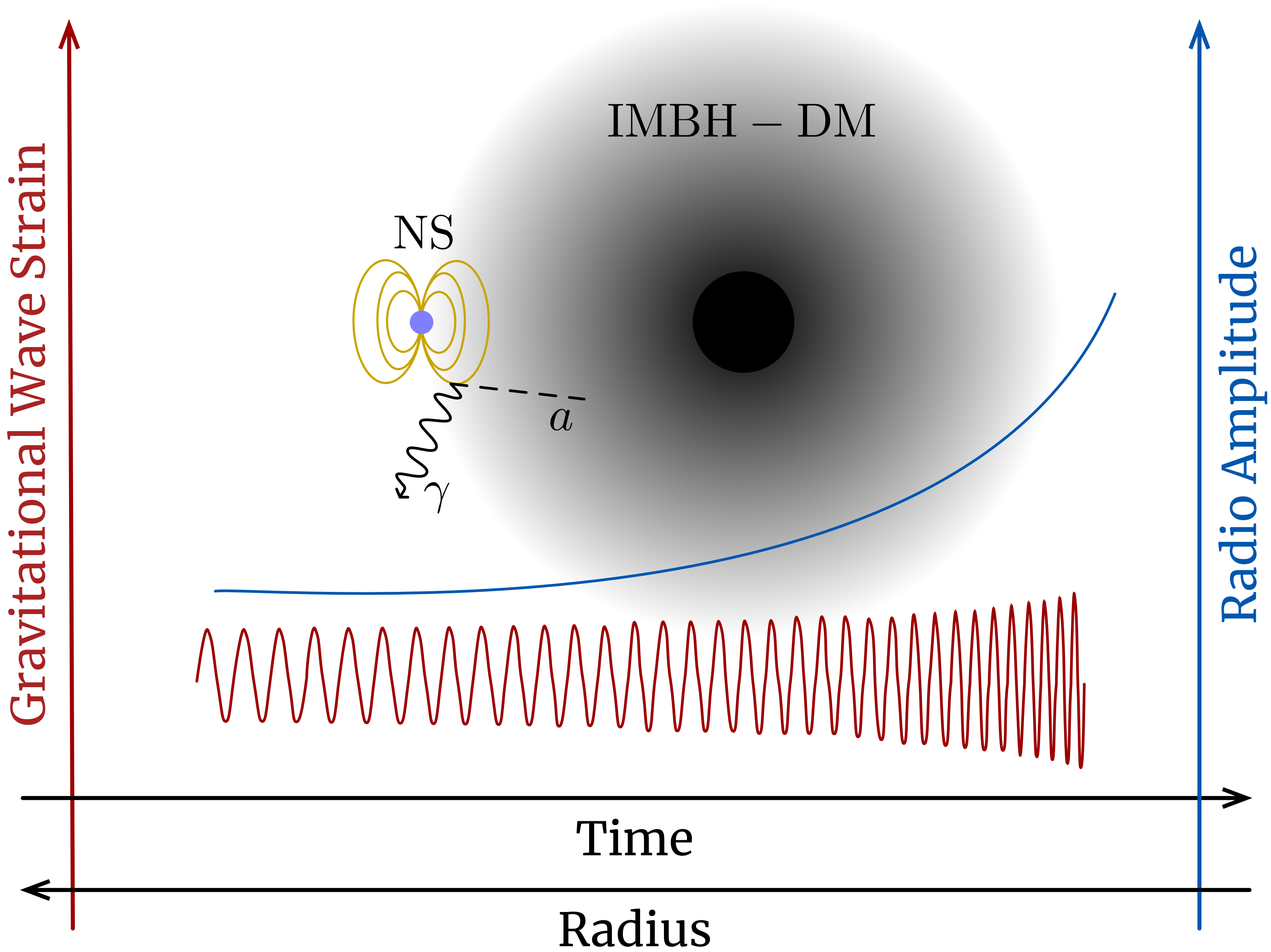
Axion coupling sensitivity

Consider a single isolated NS (J0806.4-412).
How strong does the coupling have to be to give a detectable radio signal in SKA?



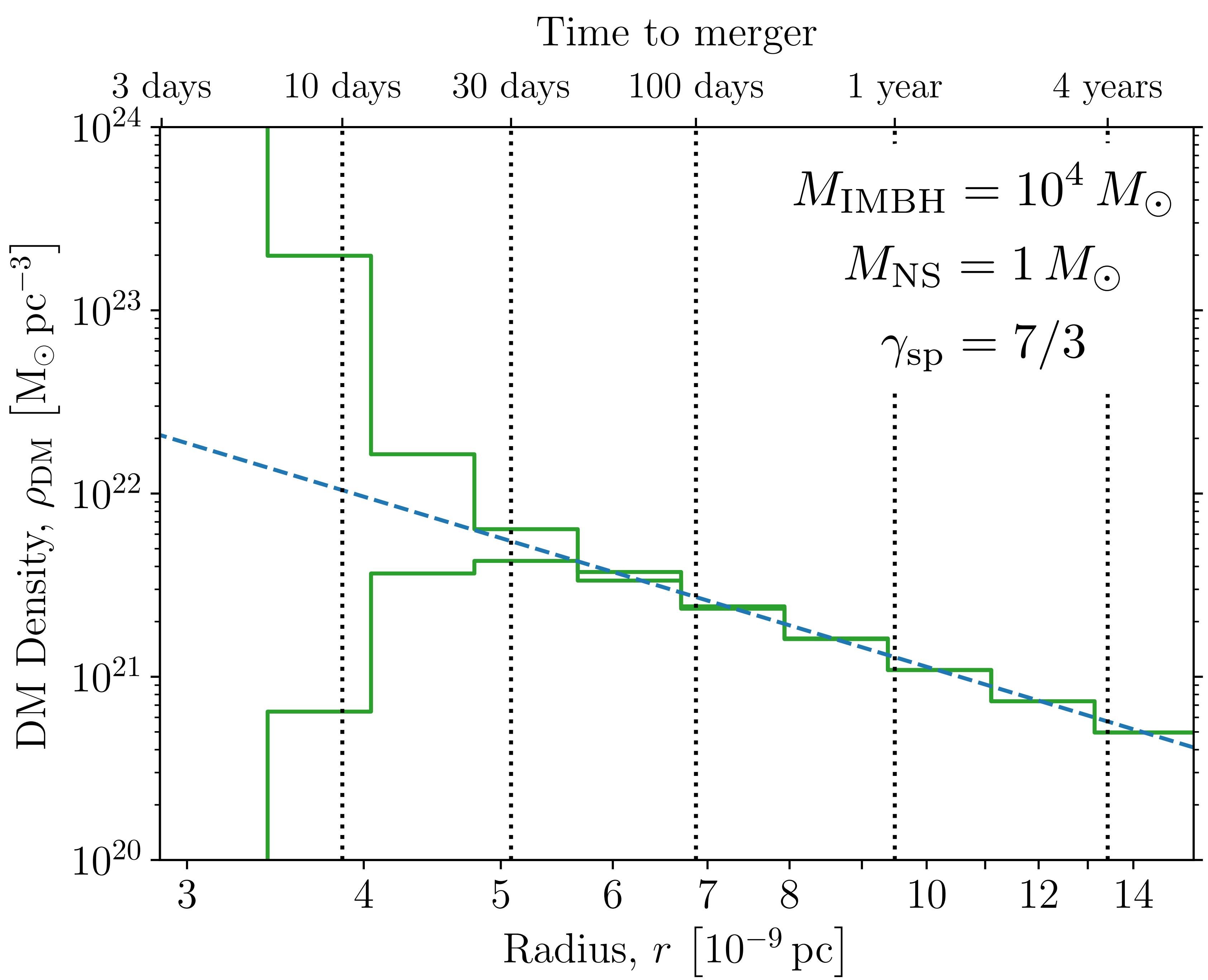
[See also recent limits from 2004.00011]

A unique signature



[Edwards, Chianese, **BJK**, Nissanke & Weniger, 1905.04686]

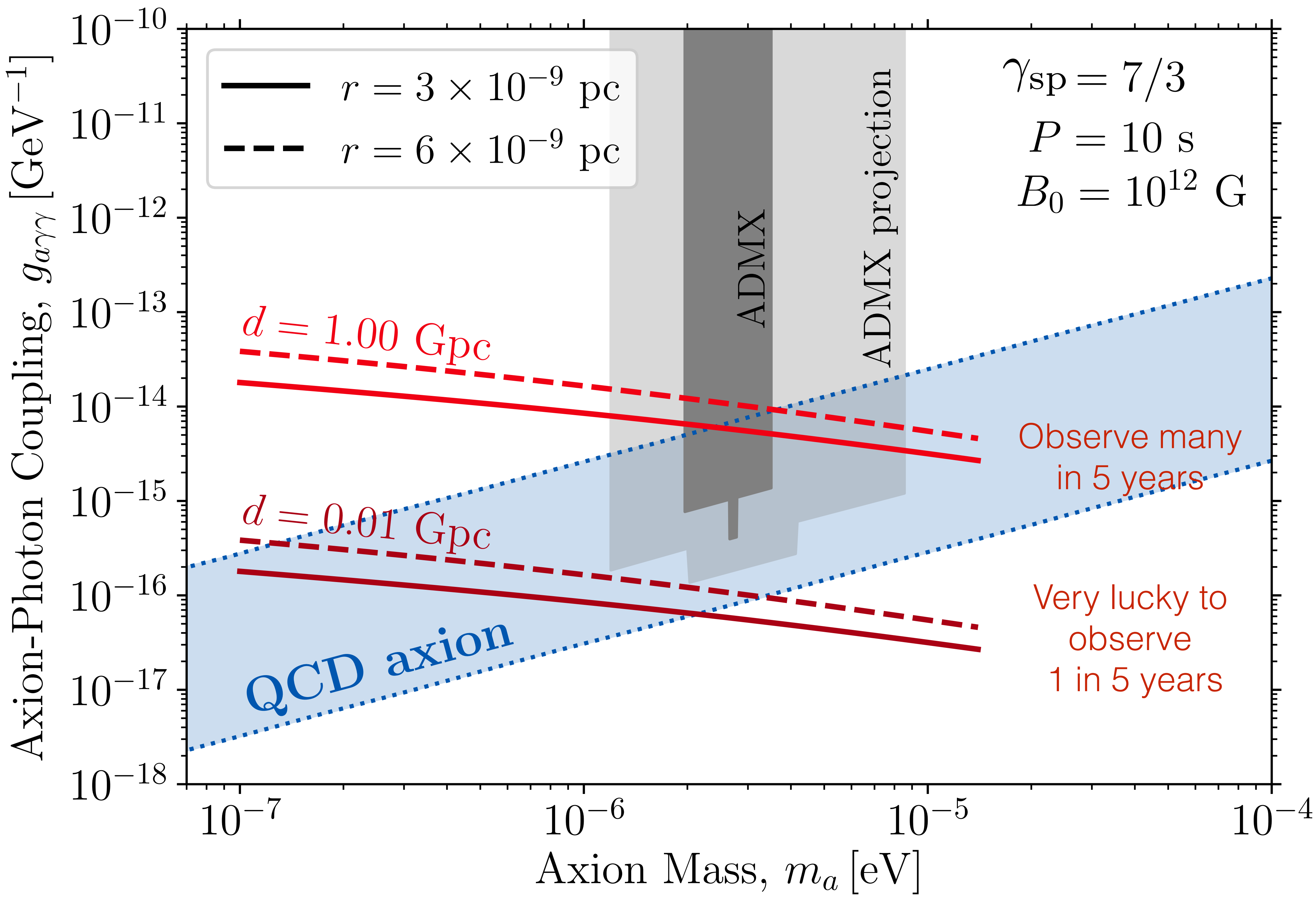
Density reconstruction



$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Omega} \sim 2 \times p_{a\gamma}\rho_{\mathrm{DM}}\left(r_c\right) v_c r_c^2 \qquad\qquad p_{a\gamma} \propto \frac{g_{a\gamma\gamma}^2 B\left(r_c\right)^2}{2 v_c}$$

QCD Axion Reach

SKA should be able to probe QCD axion DM in the range 10^{-7} - 10^{-5} eV.



[Edwards, Chianese, **BJK**, Nissanke & Weniger, 1905.04686]

Promising Signal

Dark Matter de-phasing is a very exciting signal. It's on long timescales (with LISA planned for 2030s), but it would allow us to:

Detect Dark Matter in Gravitational waves

[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]

Probe the nature of Dark Matter

[1906.11845]

Predict EM signals from Dark Matter

[Edwards, Chianese, **BJK**, Nissanke & Weniger, 1905.04686]

But it can be very difficult to extract from the noise, and must be modelled very carefully…

Plans for the future

Improved modelling

- Injection and evolution of angular momentum in the DM halo
- Post-Newtonian corrections
- Better N-body approaches [AMUSE?]

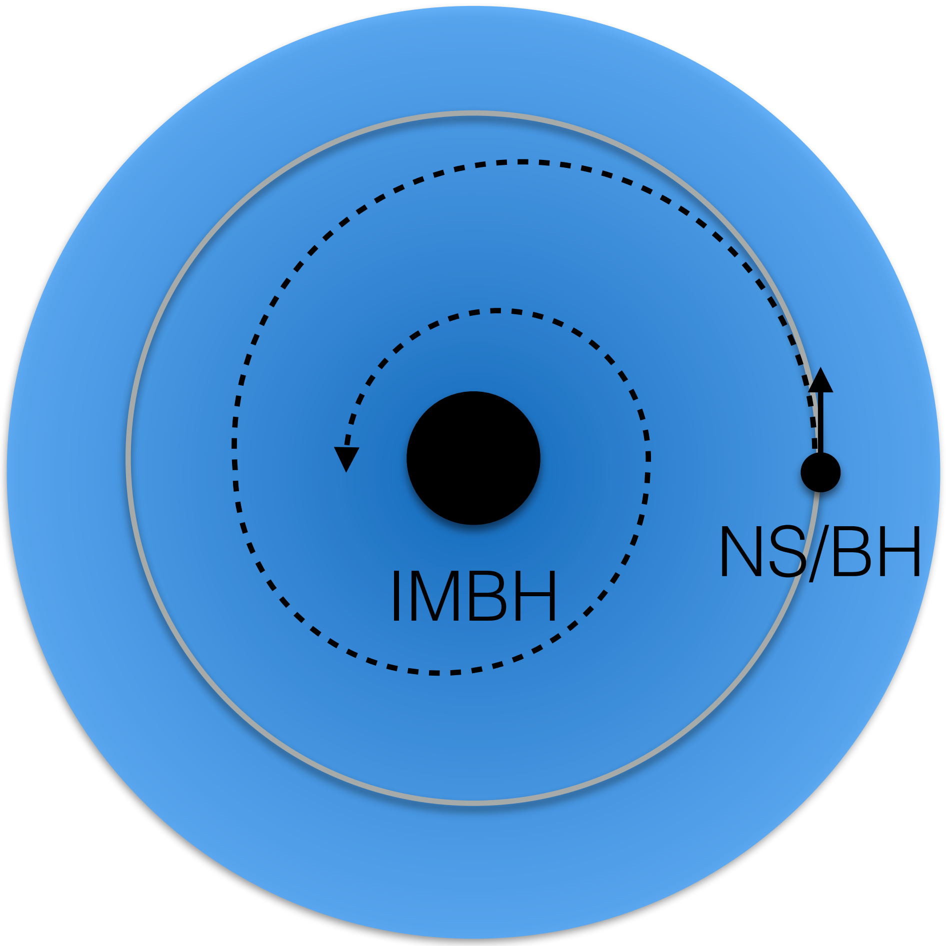
Detection methods

- Producing template banks for LISA searches
- Incoherent searches for continuous GWs
- 'General' de-phased waveform templates [2004.06729]

Detection prospects

- How many IMRI systems form? How many with NSs?
- How many systems have a (surviving) spike?
- Prospects for observing a bright enough radio signal

Conclusions



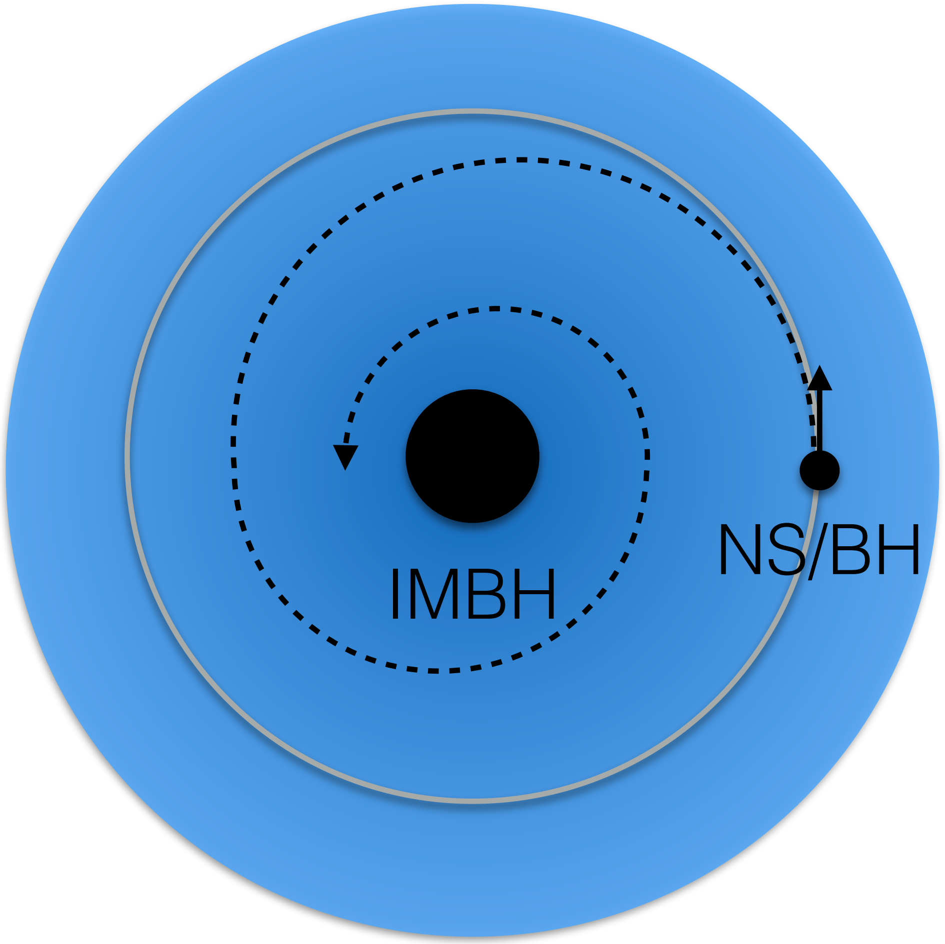
Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**

For light IMBHs, the **correction due to DM halo feedback** can be huge!

More calculations are needed to build template banks, assess detectability and parameter reconstruction

This work could pave the way towards a **multi-messenger detection** of (Axion?) Dark Matter

Conclusions



Dark Matter 'de-phasing' is an extremely promising GW signature, which needs to be **modelled carefully**

For light IMBHs, the **correction due to DM halo feedback** can be huge!

More calculations are needed to build template banks, assess detectability and parameter reconstruction

This work could pave the way towards a **multi-messenger detection** of (Axion?) Dark Matter

Thank you!

Backup Slides

Parameter Reconstruction

Prospects for parameter reconstruction in the *static DM* case:

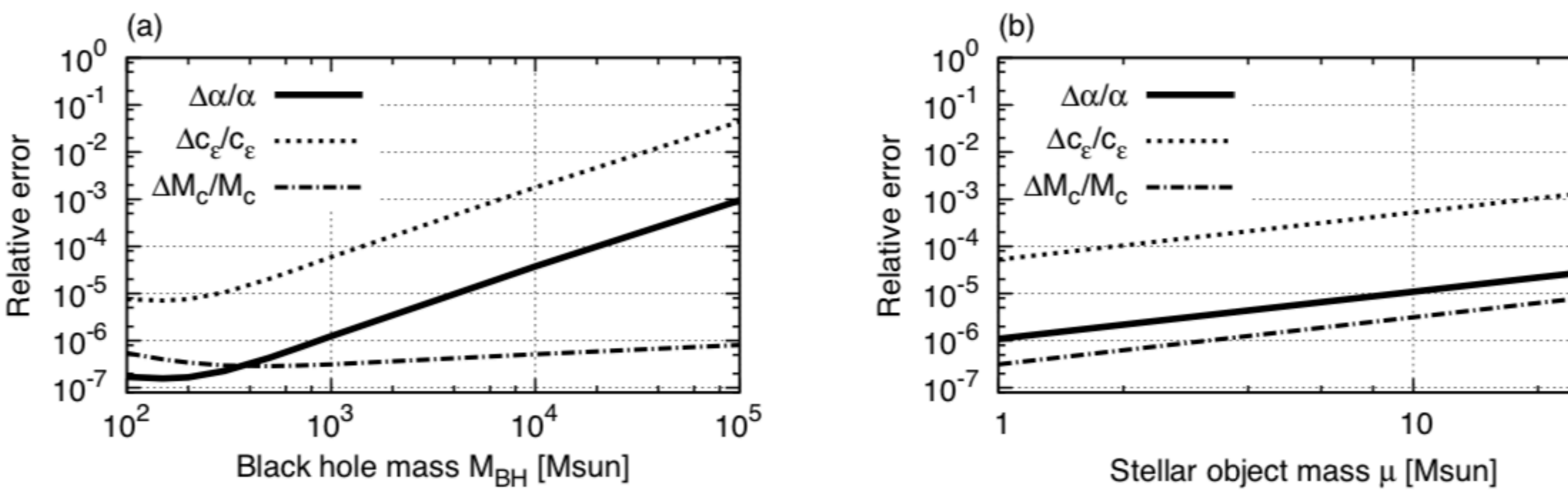


FIG. 4: The relative errors of the parameters in the phase $\tilde{\Phi}(f)$ versus (a) the central BH mass $M_{\rm BH}$ and (b) the stellar mass object mass μ for $S/N = 10$ and $\alpha = 7/3$. For this plot, $\rho_{\rm sp}$ and $r_{\rm sp}$ are taken from the table I. The other parameter is fixed to be $\mu = 1M_\odot$ in the left and $M_{\rm BH} = 10^3 M_\odot$ in the right, respectively. Note that the both axes are in the logarithmic scales. The solid line, the dashed line, the dashed-dotted line correspond to $\Delta\alpha/\alpha, \Delta c_\varepsilon/c_\varepsilon, \Delta M_c/M_c$ respectively.

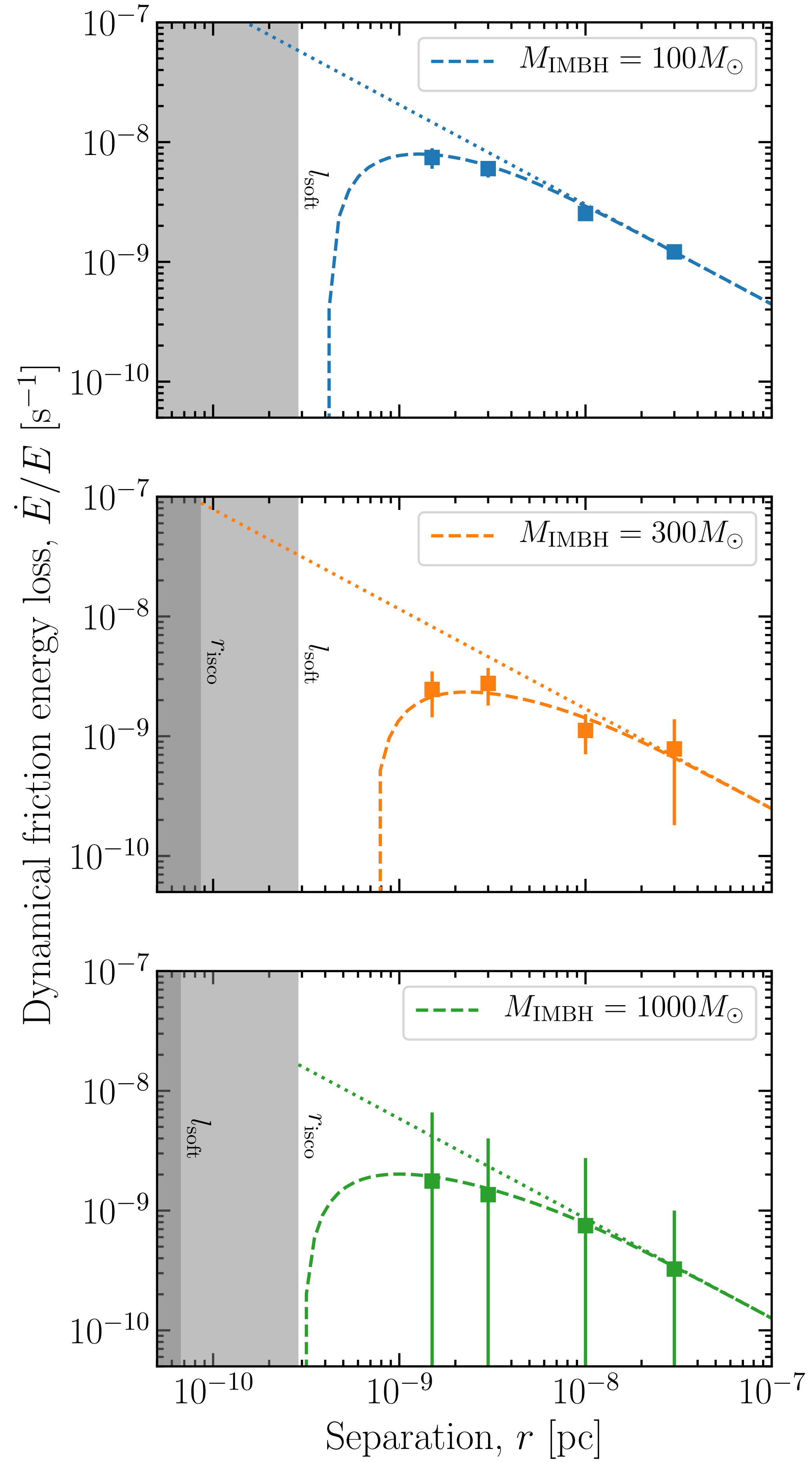
[Eda et al. 1301.5971, 1408.3534]

N-body results

Dependence of dynamical friction force on mass and separation matches expectations

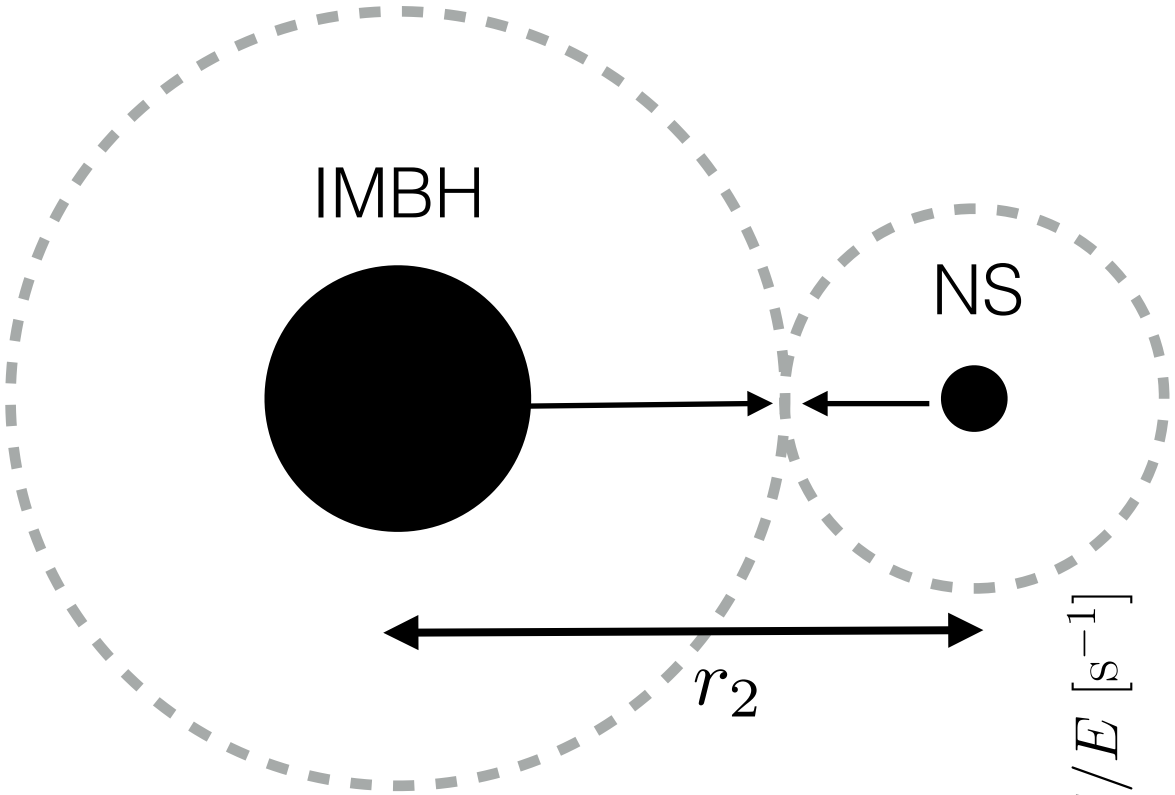
Dynamical friction traces local DM density (to better than 1%)

Drop off in DF force at small separations due to softening of simulations



N-body results

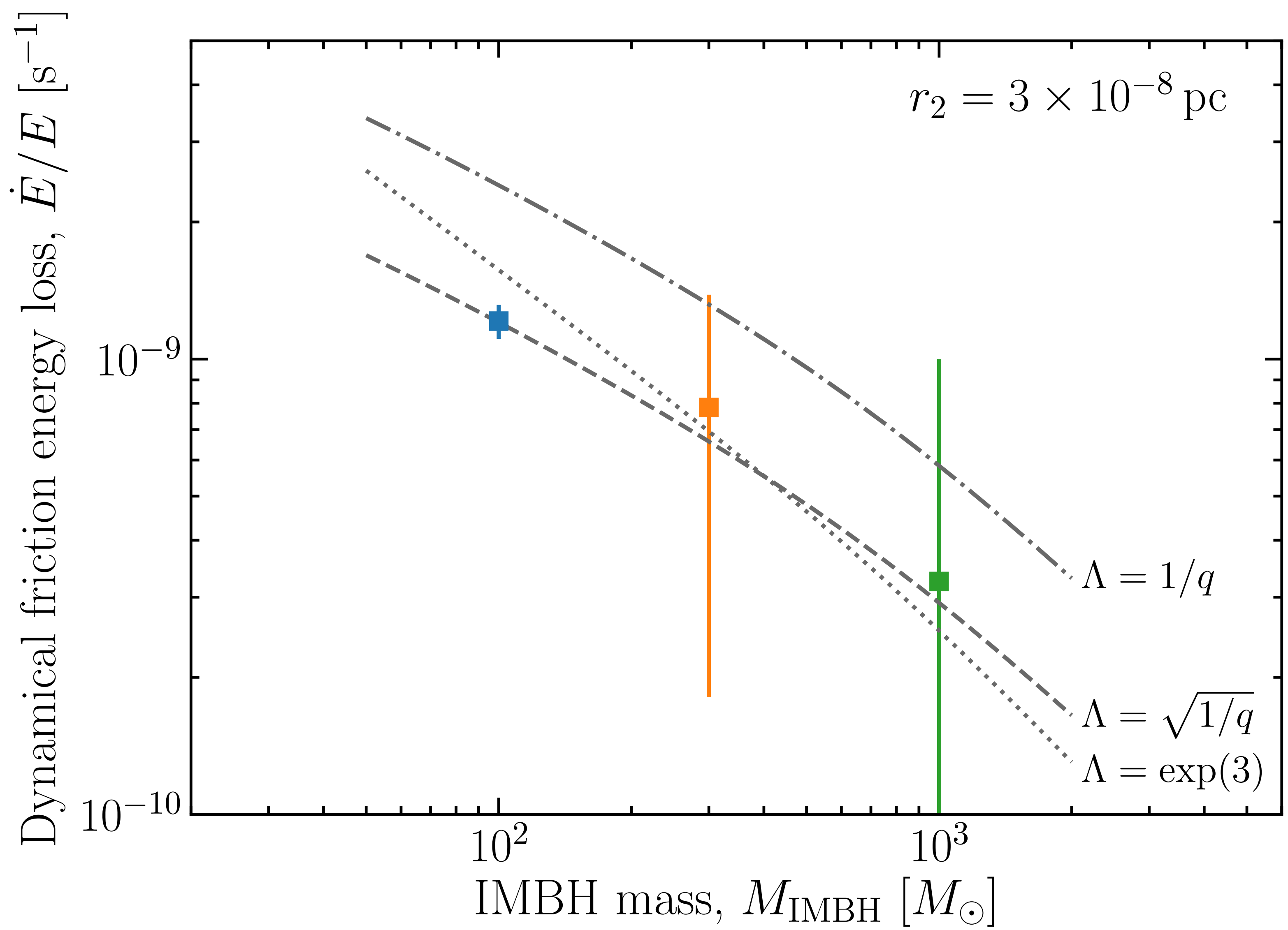
$$q \equiv m_{\rm NS}/m_{\rm IMBH} \ll 1$$



Allows us to calibrate the maximum impact parameter; tells us which particles scatter with the NS.

$$b_{\rm max} = \sqrt{q}\, r_2 \sim 3\%\, r_2$$

$$\Lambda = b_{\rm max} \frac{v_0^2}{G m_{\rm NS}} = \frac{b_{\rm max}}{q\, r_2} = 1/\sqrt{q}$$



Assumptions

- Spherical symmetry and isotropy of the DM halo
- DM particles only scatter within an impact parameter

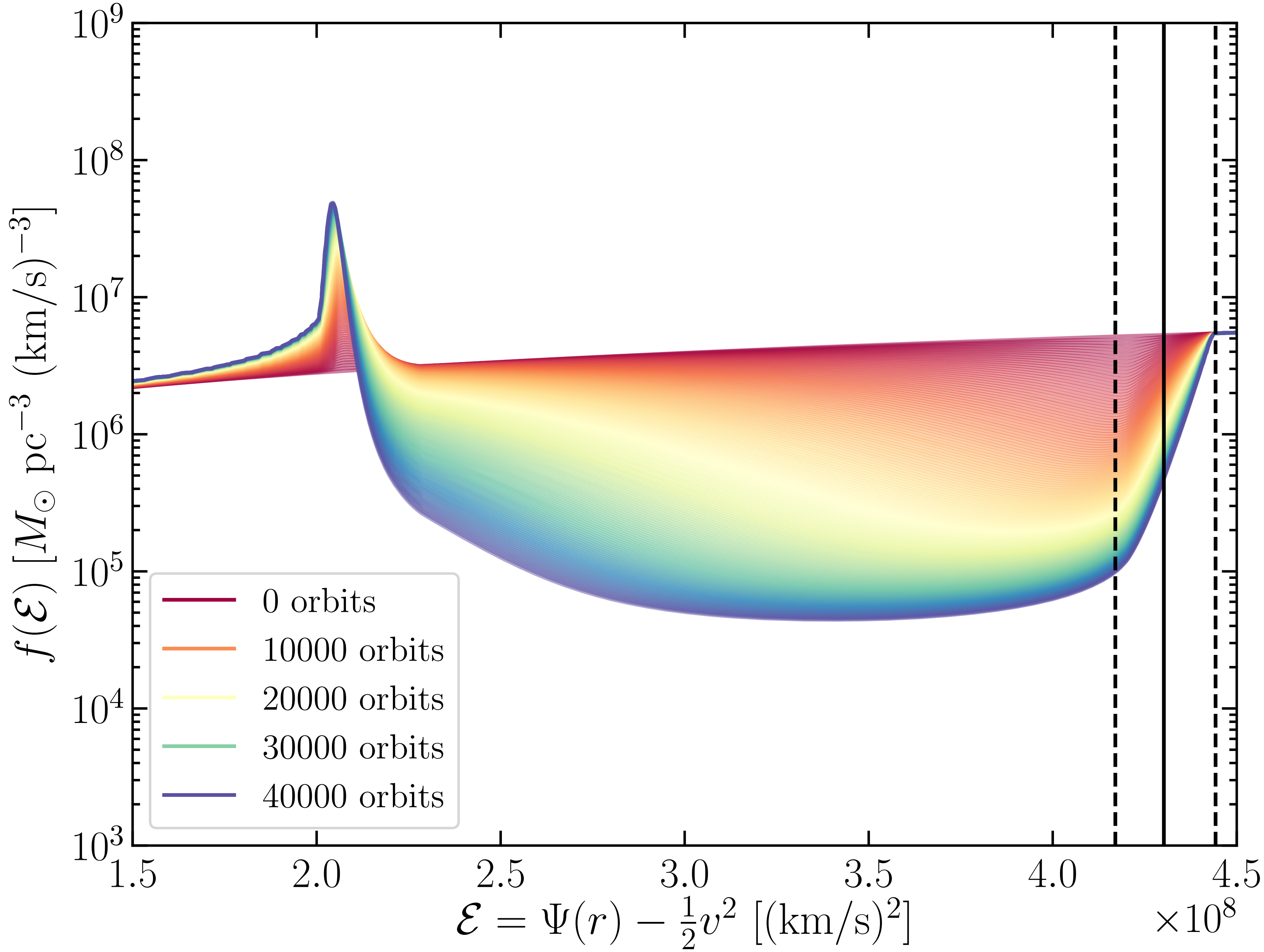
$$b < b_\mathrm{max} = \Lambda \times G_N M_\mathrm{NS}/v_\mathrm{NS}^2$$

- DM distribution is 'locally' uniform

$$b_\mathrm{max} \ll r_0$$

- Halo 'relaxation' is instantaneous
- Orbital properties evolve slowly compared to the orbital period

Distribution function



Self-consistently reconstruct density from distribution function:

$$\rho(r) = 4\pi \int_0^{v_{\mathrm{max}}(r)} v^2 f\left(\mathcal{E}\right) \mathrm{d}v$$

Numbers of cycles

$m_1 = 10^3\,M_\odot$, $N_{\rm cycles} = 5.71 \times 10^6$ in vacuum

	$\gamma_{\rm sp} = 1.5$	$\gamma_{\rm sp} = 2.2$	$\gamma_{\rm sp} = 2.3$	$\gamma_{\rm sp} = 2.\overline{3}$
Static	< 1	2.4×10^4	1.6×10^5	2.9×10^5
Dynamic	< 1	2.7×10^2	1.9×10^3	3.5×10^3

$m_1 = 10^4\,M_\odot$, $N_{\rm cycles} = 3.20 \times 10^6$ in vacuum

	$\gamma_{\rm sp} = 1.5$	$\gamma_{\rm sp} = 2.2$	$\gamma_{\rm sp} = 2.3$	$\gamma_{\rm sp} = 2.\overline{3}$
Static	< 1	1.4×10^3	8.7×10^3	1.6×10^4
Dynamic	< 1	6.2×10^2	4.0×10^3	7.4×10^3

TABLE I. **Change in the number of cycles $\Delta N_{\rm cycles}$ during the inspiral.** Change in the total number of GW cycles due to dynamical friction, starting 5 years from the merger.